

**MICROECONOMICS II**  
**Final Exam**  
**Universitat Pompeu Fabra – Winter quarter 2004**  
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1. (30) With equal probabilities Player 1 is dealt a card  $H$  or a card  $L$ . Player 2 is never dealt a card, and does not get to look at the card of Player 1 until the end of the game. After looking at his card Player 1 decides to either *Play* or *Fold*. If he *Folds*, the game ends, he loses 1 euro and Player 2 wins 1 euro. If he *Plays*, Player 2 must now consider whether to *Play* or *Fold*. If he decides to *Fold*, then he loses 1 euro and Player 1 wins 1 euro. If he *Plays*, then the card is shown. If it is  $H$ , Player 1 wins 4 euros, and Player 2 loses 4 euros. If it is  $L$ , Player 1 loses 4 euros, and Player 2 wins 4 euros.
  - (a) (10) Draw a game tree and the strategic form of this game.
  - (b) (10) Show that player 1 has only 2 undominated strategies.
  - (c) (10) Find the Bayes-Nash equilibria of this game.
  
2. (30) Find the set of sequential equilibria in the following game:

(3,1)                      (-2,0)                      (2,0)                      (-1,1)

3. (30) Consider an infinitely repeated game in which 2 players play the following stage-game every period:

	A	D
A	2, 3	1, 5
D	0, 1	0, 1

Both players discount the future with discount factor  $\delta = 1/2$ . Show that the outcome  $(A, A)$  repeated infinitely is not part of the equilibrium path for any subgame-perfect equilibrium

4. This game is played by an infinite number of individuals who live, in overlapping generations, two periods each. In each period there is a young individual and an

old individual. Each individual works when she is young and obtains two units of a consumption good. If she consumes both units, she gets today a utility of 3. If she consumes just one unit, she gets a utility of 2. The good is perishable, so she cannot keep it until the next period, but she can give one unit to the old who lives in her first period. The old cannot work, and she consumes only if the young gives her one unit of the good she produces. Not consuming gives her a utility of 0 and consuming gives her a utility of 2. The utility of an individual is the sum of utilities in the two periods in which she lives.

The only actions of an individual are, then, giving or not one unit of consumption good, when they are young. They can condition their actions on the history, as all the previous actions that happened in this game are common knowledge by every player.

- (a) (10) Describe a subgame perfect equilibrium in which each individual consumes only what she produces.
  - (b) (20) Describe a subgame perfect equilibrium in which each individual (except the first) consumes (in equilibrium) a unit of the good in every period when they live (the first individual consumes two units as there is no "old" individual around).
5. (30) We have  $n$  agents with *identical* utility functions. These functions are strictly concave, that is, let two consumption bundles  $a^i, b^i$ ,  $i \in H$ , if  $u(a^i) \geq c$ ,  $u(b^i) > c$ , then  $u(\lambda a^i + (1 - \lambda)b^i) > c$ , for  $\lambda \in (0, 1)$ . There is no production and some initial bundle of goods  $r$  for the whole economy. Show that an equal division allocation of the total initial bundle of goods, that is a consumption bundle  $x^i = \frac{1}{n}r$  for all  $i \in H$ , is Pareto-efficient. (Hint: try to show it by contradiction. Suppose there exists an allocation  $y^i$ , for all  $i \in H$ ,  $\sum_{i=1}^n y^i = r$ , which is Pareto superior to  $x^i$ . The definition of Pareto-superiority plus strict concavity will give you a contradiction).
6. (30) Consider an economy with one firm and two consumers. The firm produces wheat  $y$  with labor  $l^y$  so that  $y \leq 2l^y$ . Consumer 1 has preferences such that  $u^1(y, l^c) = \min\{y, 3l^c\}$  and consumer 2 has preferences  $u^2(y, l^c) = y^{1/2}$ . Both consumers have an endowment of labor such that  $l^y + l^c = A$  and no endowment of  $y$ . What is the general competitive equilibrium in this economy (prices, production and consumption for all agents)?