MICROECONOMICS II
Problem set 4
Universitat Pompeu Fabra – Winter 2005
Professor: Antonio Cabrales

1. Players 1 and 2 bargain over an item whose value for player 1 is either 0 or 3, with equal probabilities. Player 1 knows the value of the object, while player 2 is informed of this value only after he purchases it. The value of the object to player 2 is its value to player 1 plus 2. The bargaining procedure is the following: player 1 makes an offer which player 2 either accepts or rejects; in the event of rejection player 1 makes another offer, which player 2 either accepts or rejects. If no offer is accepted, then player 1 is left with the object and obtains a payoff equal to its value; player 2's payoff is 0. Show that there is a sequential equilibrium in pure strategies in which there is no deal when player 1's valuation is 3, while the object is sold at the price of two in the first period when player 1's valuation is 0.

2. Consider an infinitely repeated game in which 2 players play the following stagE-game every period:

	A	D
A	2, 3	1, 5
D	0, 1	0, 1

   Both players discount the future with discount factor $\delta = 1/2$. Show that the outcome (A, A) repeated infinitely is not part of the equilibrium path for any subgame-perfect equilibrium.

3. This game is played by an infinite number of individuals who live, in overlapping generations, two periods each. In each period there is a young individual and an old individual. Each individual works when she is young and obtains two units of a consumption good. If she consumes both units, she gets today a utility of 3. If she consumes just one unit, she gets a utility of 2. The good is perishable, so she cannot keep it until the next period, but she can give one unit to the old who lives in her first period. The old cannot work, and she consumes only if the young gives her one unit of the good she produces. Not consuming gives her a utility of 0 and consuming gives her a utility of 2. The utility of an individual is the sum of utilities in the two periods in which she lives.

   The only actions of an individual are, then, giving or not one unit of consumption good, when they are young. They can condition their actions on the history, as all the previous actions that happened in this game are common knowledge by every player.

   (a) Describe a subgame perfect equilibrium in which each individual consumes only what she produces.

   (b) Describe a subgame perfect equilibrium in which each individual (except the first) consumes (in equilibrium) a unit of the good in every period when they live (the first individual consumes two units as there is no "old" individual around).

   (c) Now assume that there is a probability p_t that the young player does not arrive to her old age (she nevertheless has her daughter before leaving this world).

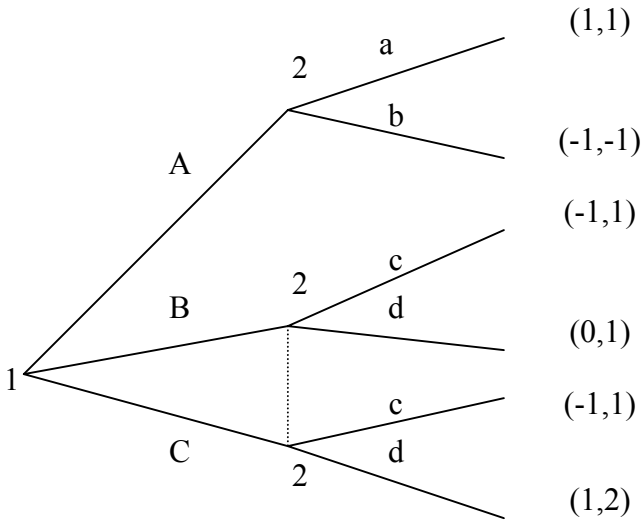
The probability p_t evolves as a autoregressive process

$$p_t = \begin{cases} 0.99999p_{t-1} + \varepsilon_t & \text{if} \quad 0 \leq p_{t-1} + \varepsilon_t \leq 1 \\ 0 & \text{if} \quad 0.99999p_{t-1} + \varepsilon_t < 0 \\ 1 & \text{if} \quad 0.99999p_{t-1} + \varepsilon_t > 1 \end{cases}$$

where $\varepsilon_t \tilde{} N(0.9999999999, 10^{-25})$. The initial $p_0 = 1$. Under these circumstances, does there still exist an equilibrium as in part (b). Why?

(d) A model like this has sometimes been proposed to support the sustainability of pay-as-you-go social security, with selfish agents. Do you think this is a good model? What does part (c) say about the robustness of the said model?

4. Consider the extensive-form game depicted below and find all pure-strategy profiles that constitute (i) a Nash equilibrium, (ii) a subgame-perfect equilibrium, (iii) a sequential equilibrium.


(1,1)
a
2
b
(-1,-1)
A
(-1,1)
c
2
B
d
(0,1)
1
c
(-1,1)
C
d
2
(1,2)