MICROECONOMICS II
Problem set 1
Universitat Pompeu Fabra – Winter 2005
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1. Among three candidates $\{a, b, c\}$, a society $1, 2, 3$ must elect one. The voting rule is plurality voting (the candidate with largest number wins) and Player 1 breaks ties.

   In other words, the strategy sets are $S_1 = S_2 = S_3 = \{a, b, c\}$, and if the agents cast the votes $\{x_1, x_2, x_3\}$, the elected candidate is $\pi(x_1, x_2, x_3) = \begin{cases} x_2 \text{ if } x_2 = x_3 \\ x_1 \text{ if } x_2 \neq x_3 \end{cases}$

   Suppose now that the preferences of the voters for the various candidates displays what is called a *Condorcet effect*, that is:

$$\begin{aligned} u_1(c) &< u_1(b) < u_1(a) \\ u_2(b) &< u_2(a) < u_2(c) \\ u_3(a) &< u_3(c) < u_3(b) \end{aligned}$$

   (a) What are strategies the weakly undominated for each player?

   (b) Suppose it is common knowledge that no player uses weakly undominated strategies. Which strategies are now weakly undominated for each player?

   (c) Are there pure strategy Nash equilibria where some player uses dominated strategies?

2. Consider the following game:

	L	C	R
U	$-v_1, v_1$	$v_1, -v_1$	$v_1, -v_1$
M	$v_2, -v_2$	$-v_2, v_2$	$v_2, -v_2$
D	$v_3, -v_3$	$v_3, -v_3$	$-v_3, v_3$

   (a) Solve for its unique mixed strategy Nash equilibrium.

   (b) This game was born with a bellicist interpretation. Can you guess what that was, and provide an alternative that does not involve bodily harm for anyone?

3. Consider the following game :

	L	C	R
T	$2, 2$	$2, 2$	$2, 2$
M	$3, 3$	$0, 2$	$0, 0$
B	$0, 0$	$3, 2$	$0, 3$

   (a) What are the pure-strategy Nash equilibria of this game?

(b) Show that there are no Nash equilibria where player 1 uses both T and M with strictly positive probability.

(c) Show that there are no Nash equilibria where player 1 uses both T and B with strictly positive probability.

(d) Show that there are no Nash equilibria where player 1 uses both M and B with strictly positive probability.

(e) Parts (b), (c) and (d) show that in all Nash equilibria some strategy for player 1 is played with probability 1. What are the Nash equilibria in which Player 2 plays nontrivial mixed strategies?

4. There are ten locations, with respective values $a_1 < a_2 < ... < a_{10}$. Player i ($i = 1, 2$) is endowed with n_i soldiers ($n_i < 10$) and must allocate them among the locations. To each particular location he can allocate no more than one soldier. The payoff at location p is a_p to the player whose soldier is unchallenged, and $-a_p$ to his opponent, unless both have a soldier at p or noone has, in which case the payoff is zero to both. The total payoff is obtained by summing up local payoffs.

(a) Show that in this game both players have a unique strategy which weakly dominates all others. Does the strategy profile where both agents use their dominating strategy constitute a Nash equilibrium (please justify your answer, positive or negative)?