Analysis of supergames: factors which facilitate collusion

By specifying the game, richer implications as to the factors which make collusion more or less likely in a given industry.

Repeated games with infinite horizon and trigger strategies

- A1 There exist n identical firms;
- A2 Homogeneous good and same cost c;
- A3 In each period t, firms set prices simultaneously and independently;
- A4 The game is played an infinite number of times [or firms have a discount factor d and the probability that the market still exists next period is $\phi \in (0, 1)$, then by setting $\delta = d \cdot \phi$ the analysis holds];
- A5 There are no capacity constraints;
- A6 Demand is such that
  - (i) if $p_i = p_j = p \quad \forall j \neq i, \forall i$
    $\Rightarrow D_i = \frac{D(p)}{n}$ and $\pi_i = \frac{\pi(p)}{n}$
  - (ii) if $p_i < p_j \quad \forall j \neq i$
    $\Rightarrow D_i = D(p_i)$ and $\pi_i = \pi(p_i)$
  - (iii) if $p_i > p_K \quad (K \in 1, \ldots, n)$
    $\Rightarrow D_i = 0$ and $\pi_i = 0$;
- A7 Each firm wants to maximise its present discounted value of profits;
- A8 No physical link between periods, but strategies depend on the history of past prices.

Consider now the following "TRIGGER STRATEGIES"

- Each firm sets p_m at $t = 0$.

- It sets p_m at time t if all firms have set p_m in every period before t.

- Otherwise, each firm sets $p = c$ forever (NASH REVERSAL).

This set of strategies represents an equilibrium (which gives a collusive outcome through purely non–cooperative behaviour) if δ is large enough.

To see this result, rewrite (1) as:

$$\underbrace{\frac{\pi(p_m)}{n}(1+\delta+\delta^2+\ldots)}_{\text{"choosing the collusive strategies"}} \geq \underbrace{\pi(p_m)}_{\text{"deviation profit"}} + \underbrace{\delta 0+\delta^2 0+\delta^3 0+\ldots}_{\text{"punishment profits"}}$$

Since $1+\delta+\delta^2+\ldots=\sum_{t=0}^{\infty}\delta^t=\frac{1}{1-\delta}$,

$$\boxed{\delta \geq 1-\frac{1}{n}}.$$

Note that if $n \nearrow$ the ICC is tighter $\Rightarrow$ collusion is less likely.

$n=2 \quad \Rightarrow \delta \geq \frac{1}{2}$ (textbook case)
$n \rightarrow \infty \quad \Rightarrow \delta \geq 1$ but $\delta \in [0,1]$!]

THE LARGER THE NUMBER OF FIRMS IN THE INDUSTRY, THE MORE DIFFICULT TO REACH COLLUSION!

Other variables which affect collusion

- Small, regular orders facilitate collusion: an unusually large order would increase the temptation to deviate, as $\pi(D)$ becomes larger, other payoffs being unchanged.

- High frequency of market contacts also facilitate collusion. Consider a market which meets every two periods. The ICC becomes:
$\frac{\pi(p_m)}{n} + \frac{\delta^2 \pi(p_m)}{n} + \frac{\delta^4 \pi(p_m)}{n} + \cdots \geq \pi(p_m)$,
write $\delta^2 = d$. Then it is (as before): $d \geq 1 - \frac{1}{n}$,
whence: $\boxed{\delta \geq \sqrt{1 - \frac{1}{n}}}$.
Since $\sqrt{x} \geq x$ for $x \in [0, 1]$, and since $(1 - \frac{1}{n}) \in [0, 1]$,
then $\sqrt{1 - \frac{1}{n}} \geq 1 - \frac{1}{n}$: the ICC is tighter and collusion more difficult.

- Immediate identification of deviation also helps collusion. If a deviation can be observed and punished with a delay of two periods, then ICC becomes:

  $$\frac{\pi(p_m)}{n}\left(\frac{1}{1-\delta}\right) \geq \pi(p_m) + \delta\pi(p_m),$$

  while, when the deviation is detected in the following period, one has:

  $$\frac{\pi(p_m)}{n}\left(\frac{1}{1-\delta}\right) \geq \pi(p_m).$$

  In the latter case collusion is easier to sustain (as the ICC is laxer).

  $\Rightarrow$ Improved observability helps collusion.

- Collusion and demand evolution

  Consider the following situation:

  - At $t = 0$: $D(p)$; $\pi(p)$
  - At time t, $\theta^t D(p)$;
    $\theta^t \pi(p) \quad t = 1, 2, \ldots$.
    The ICC can be rewritten:
    $\frac{\pi(p_m)}{n} + \delta\theta \frac{\pi(p_m)}{n} + \frac{\delta^2\theta^2\pi(p_m)}{n} + \cdots$
    $\geq \pi(p_m)$

    $$\boxed{\delta\theta \geq 1 - \frac{1}{n}}.$$

  - If $\theta > 1$ (demand growth). This relaxes the IC and makes collusion easier (the expected rise in future demand increases the future cost of a deviation).
  - If $\theta < 1$ (demand decline). This makes collusion less sustainable, as the temptation to deviate is stronger (the future cost of deviation is lower).

- However, in Rotemberg–Saloner, “price wars" occur during booms. This is because in each period demand has a probability $\frac{1}{2}$ to be low and probability $\frac{1}{2}$ to be high, and a high demand today doesn't increase the probability of high demand tomorrow. In this situation, a high demand (boom) today is like a one–off large order, and raises the incentive to deviate $\Rightarrow$ collusion more difficult during “booms".

- Also, contrast with Green–Porter (see below), where unexpected low demand would trigger the punishment phase (but in Green–Porter notice that we talk of unexpected change in demand).

Symmetry helps collusion

- Market A: Firm 1 (resp. 2) has share $s_1^A = \lambda$ (resp. $s_2^A = 1 - \lambda$).

- $\lambda > \frac{1}{2}$: firm 1 "large"; firm 2 is "small".

- Firms are otherwise identical.

- Usual infinitely repeated Bertrand game.

- ICs for firm $i = 1, 2$:

$$\frac{s_i^A (p_m - c) Q(p_m)}{1 - \delta} - (p_m - c) Q(p_m) \geq 0,$$

- Therefore: $IC_1^A : \frac{\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq 1 - \lambda$

- IC_2^A : $\frac{1-\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq \lambda$ (binding IC of small firm).

- Higher incentive to deviate for a small firm: higher additional share by decreasing prices.

- The higher asymmetry the more stringent the IC of the smallest firm.

Multimarket contacts

- Market B: Firm 2 (resp. 1) with share $s_2^B = \lambda$ (resp. $s_1^B = 1 - \lambda$): reversed market positions

- ICs in market $j = A, B$ considered in isolation:

$$\frac{s_i^j \, (p_m - c) \, Q(p_m)}{1 - \delta} - (p_m - c) \, Q(p_m) \geq 0,$$

- $IC_2^B : \frac{\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq 1 - \lambda$

- $IC_1^B : \frac{1-\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq \lambda$.

- By considering markets in isolation (or assuming that firms 1 and 2 in the two markets are different) collusion arises if $\delta \geq \lambda > 1/2$.

Multimarket, cont'd

- If firm sells in two markets, IC considers both of them:

$$\frac{s_i^A(p_m-c)Q(p_m)}{1-\delta}+\frac{s_i^B(p_m-c)Q(p_m)}{1-\delta}$$
$$-2\left(p_m-c\right)Q(p_m)\geq 0,$$

or:

$$\frac{(1-\lambda)(p_m-c)Q(p_m)}{1-\delta}+\frac{\lambda(p_m-c)Q(p_m)}{1-\delta}$$
$$-2\left(p_m-c\right)Q(p_m)\geq 0.$$

- Each IC simplifies to: $\delta \geq \frac{1}{2}$.

- Multimarket contacts help collusion, as critical discount factor is lower: $\frac{1}{2} < \lambda$.

- Firms pool their ICs and use slackness of IC in one market to enforce more collusion in the other.

- In this example, multi-market contacts restore symmetry in markets which are asymmetric.

A problem with supergames: multiple equilibria

Supergames admit a continuum of equilibrium solutions.

Consider the same game as above, but with the following trigger strategy:

(1)	Each firm sets $p \in [c, p_m]$ at $t = 0$;
(2)	It sets p at period t if all the firms have set p in every period before t;
(3)	Otherwise, it sets $p = c$ forever.

It is easy to check that this set of strategies is an equilibrium at exactly the same condition as before, that is: $\delta \geq 1 - \frac{1}{n}$.

The ICC can be written as:
$\frac{\pi(p)}{n}(1 + \delta + \delta^2 + \cdots) \geq \pi(p) + 0 + 0 + \cdots$.

From which one obtains this condition: $\delta \geq 1 - \frac{1}{n}$.

$\Rightarrow$ Any price between the competition and the monopoly price can be sustained at equilibrium.

- By acting non–cooperatively, firms <u>might</u> arrive at a collusive outcome. But this is just one of the many possible outcomes. This raises at least two questions:

1. What is the prediction power of supergames?

2. How is the equilibrium price “chosen"?

A technical note: optimal punishments

In many situations, setting Nash strategies forever is not the optimal punishment. Harsher punishments might increase the future loss of a deviation, and thus sustain the collusive price for a wider range of discount factor values.

Abreu: A very strong punishment for just one period, followed by a reversal to collusion ("stick and carrot" strategy).

Essential for the optimal punishment equilibria to exist is that two ICCs are respected.

1. A firm does not want to deviate from the collusive path.

2. A firm does not want to deviate from the punishment path.

Stigler's Critique: Secret Price Cuts

For the effectiveness of any punishment strategies (explicit cartels or 'tacit collusion'), it is essential that deviation is detected.

Stigler: Collusive agreements would break down because of secret price cuts.

$\longrightarrow$ Importance of information available to firms.

The supergame models we have seen so far do not address the Stigler's Critique: Whenever a deviant firm undercuts, other firms get zero demand, and know this is due to the deviation.

Green and Porter (1983): There exist secret price cuts. Yet, some collusion still exists .

Green–Porter

- Rival prices are not observable

- Demand is uncertain

Demand	$D = 0$	$\forall p$, with probability α
	$D = D(p)$	with probability $1 - \alpha$
Profit	$\pi = 0$	$\forall p$, with probability α
	$\pi(p) > 0$	with probability $1 - \alpha$

When a firm faces zero demand, it does not know if this. is due to a rival's deviation or to an unexpected negative shock in demand.

$\Rightarrow$ Punishment phases which last forever lose their meaning.

- Firms' strategies involve a "punishment" phase of <u>T periods</u> whenever a decline in (zero) demand is observed.

<u>STRATEGIES</u>

1. Game starts in a collusive phase.
2. Both firms charge p^m until one firm observes zero demand.
3. The following T periods, both firms charge $p = c$.
4. After T periods of punishing, both firms revert to monopoly pricing p^m.

Necessary and sufficient condition for this strategy profile to be an equilibrium:

To show the optimal T, define:

$$\begin{aligned} V^+ &= \text{P.D.V. of a firm's profit at } t, \\ &\quad \text{when there is collusive phase,} \\ V^- &= \text{P.D.V. of a firm's profit at } t, \\ &\quad \text{when in punishment phase.} \end{aligned}$$

$$\begin{aligned} V^+ &= \underbrace{(1-\alpha)\left(\tfrac{\pi^m}{2}+\delta V^+\right)}_{\text{profits when } D>0} + \underbrace{\alpha(\delta V^-)}_{\text{profits when } D=0} \\ V^- &= \delta^T V^+ \end{aligned}$$

By solving this system one obtains:

$$\boxed{V^+ = \frac{(1-\alpha)\pi^m/2}{1-\delta(1-\alpha)-\alpha\delta^{T+1}}}; \quad \boxed{V^- = \frac{(1-\alpha)\delta^T\pi^m/2}{1-\delta(1-\alpha)-\alpha\delta^{T+1}}}.$$

Write the INCENTIVE CONSTRAINT as:

$$V^+ \geq \underbrace{(1-\alpha)(\pi^m + \delta V^-)}_{\text{profits when } D>0} + \underbrace{\alpha(\delta V^-)}_{\text{profits when } D=0}$$

and, by substitution, IC becomes:

$$\boxed{1 \leq 2(1-\alpha)\delta + (2\alpha - 1)\delta^{T+1}} \qquad \text{(IC)}$$

The problem now is:

$$\boxed{\max_T V^+ \qquad \text{subject to} \qquad \text{(IC)}}$$

There is a trade–off in the choice of T:

$$T \uparrow \begin{cases} \delta^{T+1} \downarrow \;\Rightarrow\; \text{RHS of (IC)} \uparrow \;\text{ for }\; \alpha < \frac{1}{2} \\ \delta^{T+1} \downarrow \;\Rightarrow\; V^+ \downarrow \end{cases}$$

(an increase in T makes it easier to satisfy the lower profits).

$\Rightarrow$ The program is satisfied by the smallest T which satisfies the incentive constraint.

Note 1: The punishment period cannot be of negligible duration. Indeed:

$T = 0 \quad \Rightarrow$ (IC) $\quad 1 \leq 2(1-\alpha)\delta + (2\alpha - 1)\delta$

$\iff \qquad \delta \geq 1 \qquad$ impossible!

Note 2: We find the trigger strategies with the case of certainty as a limiting case:

For $T \to \infty \quad \Rightarrow$ (IC") $\quad 1 \leq 2(1-\alpha)\delta$

$\iff \delta \geq \frac{1}{2(1-\alpha)}$. For $\alpha = 0 \Rightarrow \delta \geq \frac{1}{2}$.

Note 3: When α is too high, the opportunity cost of cheating is too low $\Rightarrow$ deviation is optimal (given that one enters punishment phase, better to cheat!)