# Informational Cascades Elicit Private Information

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**Abstract**

This paper introduces cheap talk in a dynamic investment model with information externalities. I first show how the credibility of cheap talk statements is adversely affected when players can postpone their investment decisions. I next show how informational cascades help in restoring this credibility. Subsidising investments favours truthful revelation of private information. A more able sender has more incentives to truthfully reveal her private information than a less able one.

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# 1 Introduction

In recent years a growing number of theories have highlighted the importance of private information in understanding many important real-world issues. For example, Morris and Shin (2000) and Corsetti, Dasgupta, Morris and Shin (2001) stress the role played by private information when analysing currency crises. Postlewaite and Vives (1987), Goldstein and Pauzner (2000) and Chari and Jagannathan (1988) analyse bank runs in a private-information set-up. In a different class of models Banerjee (1992), Bikhchandani, Hirschleifer and Welch (BHW,1992) and a.o. Chamley and Gale (CG,1994) analyse investment behaviour when all investors possess some private information concerning the profitability of an investment option. They show how in their set-up everyone can end up investing when actually no one should have done so. Avery and Zemsky (1998) modify BHW's model to explain price bubbles in financial markets...

All the papers mentioned above crucially assume that everyone possesses some information concerning "the realized state of the world" (which determines the profitability of investing, attacking the currency, running to the bank, ...). If everyone could truthfully exchange their private information, they would be able to take their payoff-relevant action on the basis of more information and achieve a higher payoff[1] (and this is even true in Avery and Zemsky's explanation of financial bubbles).

Where does private information then come from? Stated differently, why should we rule out profitable preplay communication? Moreover, those models seem at odds with casual observations of everyday life in which a lot of private information is simply transmitted through cheap talk. For example, in a recent Financial Times interview Mr. Dan Peterson, CEO of a telecommunications firm pictured himself as an optimist when talking about the growth potential of his sector by declaring: "An economic slowdown does not presage a slowdown in the pace of technical innovation."[2] Similarly, Mr. Didier Bellens, chief executive of the RTL Group, drew a gloomy picture of the European advertising market by asserting that it would probably grow by just two per cent in 2001.[3] Some institutions are even specialised

[1] For example in Morris and Shin's account of currency crises players would benefit from truthful revelation by enabling them (i) to attack the currency in the "right" state of the world and (ii) to increase the probability of launching a successful attack by coordinating their actions.

[2] See Financial Times, July the 18th, 2001

[3] At that time it was generally believed that it would grow by eight per cent. See Financial Times, June the 19th, 2001.

in collecting and summarising the opinions of a large number of market participants. For example, the Munich-based IFO institute for economic research releases a quarterly index reflecting the business confidence of the average German investor. Similar institutions are also at work in the other developed economies. Some economists are aware of this tension between our theoretical assumptions and casual observations of everyday life. For example, Robert Schiller (1995) while discussing the current theories of herd behaviour wrote:

> Human behavior common to all human societies involves a tendency for an idle free-flowing exchange of ideas and thoughts; we call this "conversation". (p.183)

Obviously, the examples I listed above all raise one natural question: Why should investors truthfully reveal their private information? How credible is Mr. Bellen's pessimism? Cheap talk can be ruled out on the basis of competition effects[4]. However, competition effects are absent in the papers I mentioned in my first paragraph. Profitable preplay communication can also be ruled out by the claim that every game of cheap talk always possesses a pooling (or babbling) equilibrium. However, as argued by Farrel and Rabin (1996), it remains to be seen whether the pooling equilibrium constitutes a natural focal point in a game of cheap talk. This last claim would actually be more convincing if one could show that, were we to allow for preplay communication, the game of cheap talk would then be characterised by a unique pooling equilibrium. Conversely, if one could show the existence of a separating equilibrium (e.g. an equilibrium in which an optimistic player sends the message "I am an optimist", while a pessimistic player sends the message "I am a pessimist") this should enable us to address more ambitious questions like: "What could be done (if anything) to promote efficient exchange of private information?" and "Does the ability of the sender influence her incentives for truthful revelation?" This paper addresses the questions and concerns raised in this paragraph.

I introduce therefore cheap talk in two different investment models. First, I add cheap talk to a *static* investment model. All players must take an investment decision and possess a private, imperfect signal concerning the future state of the world. Investment is only profitable in the good state. For the sake of simplicity, I assume that the returns of the investment project only depend on the state of the world. Hence, for efficiency reasons one would want to have all players truthfully

[4]For example, cheap talk does not work in a Cournot-duopoly because one firm always wants to send a message in the hope to reduce the output of the other firm.

exchanging their signals. In the first stage one randomly drawn player (the sender) is asked to divulge her private information (i.e. her signal) to the other players (the receivers). In the second stage all players decide whether to invest or not. Second, I add cheap talk to a *dynamic* investment model. The set-up of this dynamic model is similar to the one described above except that in the second stage players now play a waiting game similar to the one studied in CG and Chamley (1997). Players can invest in two periods. In the second period, everyone observes how many players invested in the first period, and, those who haven't invested at time one, decide then whether to carry out their investment plans or not.

Two main conclusions can be drawn out of this paper. First, I show how the credibility of cheap talk statements may be adversely affected when investors can postpone their investment decisions. Second, I show how, in the dynamic investment model, an informational cascade can help in restoring separating equilibria.

My first main conclusion rests on the comparison between proposition (2) and proposition (3). Proposition (2) summarises the main result drawn out of my static investment model and states that, independently of the cost of the investment project, there always exists a separating equilibrium. The intuition is straightforward: if the sender is optimistic (pessimistic) she will, independently of her message, (not) invest in the second stage. Therefore she cannot gain by deviating. Unfortunately, this insight does not hold anymore once we work with a dynamic investment model in the second stage. In particular, proposition (3) shows that, for high enough an investment cost, the unique equilibrium is then the pooling one. Stated differently, for high enough an investment cost, it is without loss of generality to assume away efficient preplay communication. The intuition behind this result goes as follows: in my model expected payoffs are driven by the relative number of optimists in the economy (the higher the proportion of optimists in the population, the higher the probability that the world is in the good state). At time two all players observe the number of period-one investments and use this knowledge to get an "idea" of the proportion of optimists in the economy. This updating process depends on the period-one investment strategies[5] (which on their turn are affected by the message sent in the first period). Both sender's types want to send the message which would allow them to get the "best possible idea" about the proportion of optimists in the

[5]For example, upon observing $k$ period-one investments, players compute different posteriors if pessimists invested (at time one) with zero probability and optimists with a probability equal to one, than if pessimists invested with the same probability as the optimists.

population. From Blackwell's value of information theorem (1951), we know that inferring the proportion of optimists in the economy on the basis of the period-one investments is easier, the higher the probability with which optimists invest, and the lower the probability with which pessimists invest (and this is true for both sender's types). If the investment cost is high enough, pessimists will - independently of the sender's message and strategy - never invest in the first period. Therefore, both sender's types want to send the message which makes the optimists invest with as large a probability as possible. Thus both sender's types share the same preferences over the optimists' actions, and therefore no information can be transmitted through cheap talk.

My second main conclusion is summarised in proposition (4) which states that a separating equilibrium exists for low-cost projects[6]. In the separating equilibrium all players invest at time one whenever the sender announces "I am an optimist" (i.e. an informational cascade[7] in which everyone invests is ignited by the arrival of a favourable message). In my model this informational cascade induces a pessimist to send the message "I am a pessimist": if she were to deviate and sent instead the message "I am an optimist", she wouldn't be able to learn anything about the proportion of optimists in the population and would never invest. My analysis also permits me to draw some positive and normative conclusions. In particular, I show that subsidising investments may promote exchange of private information by increasing the range of parameter values in which a separating equilibrium exists. I also show that a more able sender has more incentives to truthfully reveal her private information than a less able one.

Obviously, this is not the first paper to investigate the credibility of cheap talk statements. In a seminal paper Crawford and Sobel (1982) already analysed the issue of information transmission through cheap talk. However, in their model the receiver chooses an action which influences both player's payoffs after having received a message from the informed sender. In my model the sender first sends a message and then plays a (waiting) game with the receivers. Farrel (1987, 1988, 1989) and Baliga and Morris (2000) also assume that both players play a game after having received or sent a message. However, they consider a very different game:

[6]Obviously, proposition (4) is based on the model in which investors face a dynamic investment decision.

[7]All players - irrespective of their private information - take the same action at time one. By definition, this is an informational cascade.

in Farrel (1987,1988) and Baliga and Morris the communication stage is followed by a coordination game, while in Farrel (1989) both players engage in a bargaining game after the communication stage. As I consider a (very) different game[8], I also get very different results: Crawford and Sobel have shown how the credibility of cheap talk statements are undermined when the sender and the receiver have different preferences over the optimal action, Baliga and Morris argued that positive spillovers impede information exchange, while I show how the possibility of postponement may destroy incentives for truthtelling (and how informational cascades help in restoring these incentives).

This paper also belongs to the literature on informational cascades (see a.o. Banerjee (1992), Bikhchandani, Hirschleifer and Welch (BHW,1992), Chamley and Gale (1994), Zhang (1995),...). In those papers an informational cascade develops as a consequence of the early arrival of some public information. In my paper this causality is reversed: it is the anticipation of a future informational cascade which induces the early arrival of some public information (i.e. the signal of the sender which, in the separating equilibrium, gets truthfully transmitted through words). A similar, though different, mechanism can be found in Zhang (1995). In that paper the occurence of an informational cascade reduces a player's gain of waiting and thereby induces her to invest early. However, in his paper the occurence of a future informational cascade only influences the time at which the player with the most precise signal invests. Absent this informational cascade the player with the most precise signal would be the first to invest anyway (and the remaining players would exactly infer the precision of her signal out of the timing of her investment decision). Therefore, in Zhang's paper the informational cascade does not cause the arrival of public information.

This paper is organised as follows. In section two, I present my two-stage game. In the third section, I take the players' posteriors as given and I solve for equilibrium strategies in the waiting game. I next compute equilibrium strategies in the sender-receiver game (section four). I first analyse a benchmark case in which players cannot postpone their investment decisions (section 4.1). Next, I compute equilibrium strategies when the communication stage is followed by the dynamic investment game. In section 4.2, I first show how the credibility of cheap talk may be undermined when players can postpone their investment decisions (proposition

[8] To the best of my knowledge, this is the first paper to add communication to a war-of-attrition type model.

(3)). Next, I show how this credibility can be restored by an informational cascade (proposition (4)). Section five analyses the robustness of proposition (4). Final comments are summarised in the sixth and final section.

# 2 The model

Assume that a population of $N \geq 5$ risk neutral players must decide whether to invest in a risky project or not. The value of the investment project, V, can take two values: $V \in \{1, 0\}$. The cost of the investment project is denoted by c. The state of the world is described by $\Theta \in \{H, L\}$. if $\Theta = H$ the good state prevails and $V = 1$. If $\Theta = L$, the economy is in a bad state and $V = 0$ ($P(\Theta = H) = P(\Theta = L) = \frac{1}{2}$). Each player receives a private, conditionally independent signal concerning the realised state of the world. Formally, player $l$'s signal $s_l \in \{h, l\}$ ($l = 1, ..., N$) where $P(h|H) = P(l|L) = p > \frac{1}{2}$. In this paper, I assume that:[9]

A1: $1 - p < c < p$

A1 implies that a player who received signal $h$ is - a priori - willing to invest, because $P(H|h) = p > c$. Henceforth, I call a player who received a good (bad) signal an (a) optimist (pessimist). A player who received a bad signal computes $P(H|l) = 1 - p$. I analyse the following two-stage game:

-1) The state of nature is realised and all players receive their signals,
0) One randomly drawn player (henceforth player $i$) is asked to report her signal. Her message, $\hat{s}_i \in \{h, l\}$, is made public to all the other players,
1) All players make their investment decisions,
2) All players observe how many persons invested in period one, and those who haven't invested yet make their investment decisions,
3) All players learn the true state of the world. Payoffs are received and the game ends.

In the first stage (time zero) player $i$ (= the sender) influences the posteriors with which the remaining players (= the receivers) will compute equilibrium strategies in the second stage (time one and two). Henceforth we denote the second stage as the

[9]Actually, all my results carry through as long as $c \in [\frac{(1-p)^2}{(1-p)^2+p^2}, p]$. For the sake of expositional clarity, I decided to restrict attention to the parameter range $c \in (1 - p, p)$.

waiting game. At time one, all players must choose an action from the set {invest, wait}. At time two all players who waited at time one must choose an action from the set {invest, not invest}. Each player only possesses one investment opportunity, so a period-one investor cannot invest in a second project at time two. Investments are irreversible. If a player does not invest in any of the two periods, she gets zero. Call $k \in \{0, 1, ..., N\}$ the number of players who invest in the first period. $\delta$ denotes the discount factor.

$h_t$ $(t = 0, 1, 2)$ denotes the history of the game at time $t$. $h_0 = \{\emptyset\}$. $h_1 = \hat{s}_i$ and $h_2 = (\hat{s}_i, k)$. $H_t$ denotes the set of all possible histories at $t$. $H = \bigcup_{t=0}^{2} H_t$. A symmetric behavioural strategy for the receivers is a function $\Lambda : \{h, l\} \times H \to [0, 1]$ with the interpretation that $\Lambda(s_j, h_t)$ represents the probability with which player $j$ ($j = 1, 2, ..., N$ and $j \neq i$) invests at date $t$, given $s_j$ and $h_t$. By assumption each player can only invest once. Therefore we impose the following restriction on $\Lambda(\cdot)$: if $a_1^j =$ invest, then $\Lambda(s_j, h_2) = 0$ (where $a_1^j$ denotes the action undertaken by player $j$ at time one). By assumption, no one can invest at time zero, therefore $\Lambda(s_j, h_0) = 0$. A behavioural strategy for the sender is a function $\Lambda_s : \{h, l\} \times H \to [0, 1]$. $\Lambda_s(\cdot, h_0)$ represents the probability that player $i$ sends $\hat{s}_i = h$. $\Lambda_s(\cdot, h_1)$ $(\Lambda_s(\cdot, h_2))$ represents the probability that player $i$ invests at date one (two). As before, $\Lambda_s(\cdot, h_2) = 0$ if $a_1^i =$invest.

In this model, players are solely interested in the relative number of optimists (as compared to the number of pessimists) in the population. Call $n$ the random number of optimists in our population. The higher $n$ (for any fixed $N$), the higher $P(H|n)$ and the higher the expected gain from investing. As expected payoffs are driven by $n$, instead of explicitly specifying beliefs concerning each players' type contingent on the history of the game, we use the probability assessments $\alpha(\cdot)$ and $\alpha_s(\cdot)$ when defining our relevant equilibrium concept. A probability assessment for the receivers is a function $\alpha : \{h, l\} \times H \times \{0, 1, 2, ..., N\} \to [0, 1]$ with the interpretation that $\alpha(s_j, h_t, n)$ represents the probability that $n$ players are optimistic given $h_t$ and $s_j$. Similarly, a probability assessment for the sender is a $\alpha_s : \{h, l\} \times H \times \{0, 1, ..., N\} \to [0, 1]$ where $\alpha_s(s_i, h_t, n)$ represents the probability that $n$ players are optimistic given $h_t$ and $s_i$. A perfect Bayesian equilibrium (PBE) is a $(\Lambda^*(\cdot), \Lambda_s^*(\cdot), \alpha^*(\cdot), \alpha_s^*(\cdot))$ such that:
(i) no player can gain by choosing a $\Lambda(\cdot)$ $(\Lambda_s(\cdot))$ different from $\Lambda^*(\cdot)$ $(\Lambda_s^*(\cdot))$ given the other players' strategies and given $\alpha^*(\cdot)$ $(\alpha_s^*(\cdot))$, and
(ii) $\alpha^*(\cdot)$ and $\alpha_s^*(\cdot)$ are computed via Bayes' law whenever possible.

As mentioned before, at time zero player $i$ influences the posteriors with which the receivers compute their equilibrium strategies in the waiting game. To simplify our notations from now on $x$ denotes the probability with which an optimistic sender sends message $\hat{s}_i = h$ (i.e. $x = \Lambda_s(h, h_0)$). Similarly, to simplify notations, from now on we replace $\Lambda_s(l, h_0)$ by $y$. Remind that $y$ represents the probability with which a pessimistic sender strategically sends a favourable message (even though she's a pessimist). Assume player $j$ is an optimist ($j = 1, 2, ..., N$ and $j \neq i$). At time one player $j$ revises her posterior by computing $q = P(H|s_j = h, \hat{s}_i, x, y)$. Assume $x = 1$ and $y = 0$. In this case an optimistic sender always sends a favourable message, while a pessimist always sends an unfavourable one. If player $i$ were to truthfully reveal her good signal at time zero, the remaining optimists would compute $P(H|h, \hat{s}_i = h, 1, 0) = \overline{q}$. If the contrary were to happen at time zero, optimists would compute $P(H|h, \hat{s}_i = l, 1, 0) = \underline{q}$. $q$ cannot lie outside of the range $[\underline{q}, \overline{q}]$, and - depending on $x$, $y$ and $\hat{s}_i$ - can take any value between $[\underline{q}, \overline{q}]$. In a similar fashion $q_\pi$ denotes a pessimist's posterior probability that $\Theta = H$ and $q_\pi \in [\underline{q}_\pi, \overline{q}_\pi]$.

I first characterise equilibrium behaviour in the waiting game for any possible posteriors, next I analyse player $i$'s incentives to truthfully report her private information given that she correctly anticipates how her message is going to affect posteriors. A more general version of this waiting game has already been analysed by Chamley and Gale (1994) and by Chamley (1997). My next section summarises their main results in an intuitive way.

# 3 Strategic waiting

To simplify notations we will mostly replace $\Lambda(h, h_1)$ by $\lambda$. Similarly, we use $\lambda_\pi$ instead of $\Lambda(l, h_1)$. As mentioned earlier, in this model the ex ante gain of investing is only determined by $n$, the number of optimists in the population. Unfortunately, by postponing one's investment decision, players observe $k$ instead of $n$. Hence, at time two all players who waited at time one face an inference problem: on the basis of $k$ they must try to get "as precise an idea" about $n$. The inference of $n$ out of $k$ depends on $\lambda$ and $\lambda_\pi$. This is logical: for instance observing $k = 0$ when $\lambda = \lambda_\pi = 0$ is less bad than observing $k = 0$ when $\lambda = 1$ and $\lambda_\pi = 0$ (in the former case you observe no investments when you never expected to see any, while in the latter case you learn for sure that no optimist is present in the economy).

In equilibrium, the informational gain of waiting must be offset by the discounting cost. To characterise equilibrium behaviour we thus need an expression for the gain of waiting. I first show the existence of a PBE in which $\lambda_\pi = 0$ and $\lambda^* \in [0,1)$. Assume player $j$ is an optimist. What is player $j$'s (undiscounted) gain of waiting given that $\lambda_\pi = 0$ and that $\lambda \in [0,1]$? If she waits, by assumption she observes the number of period-one investors. Upon observing $k$, she would then compute $P(H|q,k,\lambda)$[10] and would invest at time two iff $P(H|q,k,\lambda) \geq c$. Hence, for a given $k$ player $j$'s payoff equals $Max\{0, P(H|q,k,\lambda) - c\}$. Of course, player $j$ cannot a priori know how many players will invest at time one. Therefore, whenever $\lambda_\pi = 0$, player $j$'s ex ante gain of waiting (net of discounting costs), $W(q,\lambda)$, equals:

$$W(q,\lambda) = \sum_{k=0}^{N-1} Max\{0, P(H|q,k,\lambda) - c\} P(k|q,\lambda) \qquad (1)$$

To understand equation (1) assume first that $\lambda = 0$. Then $P(k=0|q,\lambda=0) = 1$. At time two, player $j$ would compute $P(H|q,0,0) = q$. This is logical, player $j$ would, independently of $n$, always observe zero period-one investments. Hence, if $\lambda = 0$, it's as if she doesn't receive any additional information concerning the realised state of the world. Therefore she has no reason to change her posterior and $P(H|q,0,0) = q$. Therefore $W(q,0) = q - c$.

Next assume that $\lambda = 1$. Then, in the next period player $j$ learns how many optimists are present in the population (i.e. $k = n - 1$[11]). At time two player $j$ computes $P(H|n)$, and invests iff $P(H|n) \geq c$. As before, player $j$ cannot ex ante know how many optimists are present in the economy, and therefore:

$$W(q,1) = \sum_{n=1}^{N} Max\{0, P(H|n) - c\} P(n|q) \qquad (2)$$

Lemma 1 *$\forall N \geq 5$, $\forall q \in [\underline{q}, \overline{q}]$ and under A1, $W(q,1) > q - c$,*

Proof: See appendix.

The intuition behind lemma 1 goes as follows. We can rewrite the gain of investing as follows:

$$q - c = \sum_n P(H|n)P(n|q) - c$$

[10]From now on, whenever $\lambda_\pi$ does not appear as a conditioning variable in $P(\cdot|\cdot)$, this means that $\lambda_\pi$ is assumed to equal zero.

[11]By assumption player $j$ is an optimist who waited at time one when the other optimists invested with probability one. Therefore $k = n - 1$.

Suppose $\lambda = 1$ and assume that player $j$ decides to wait at time one and then to invest unconditionally (i.e. to invest at time two independently of $n$). The above equality merely states that investing at time one is payoff-equivalent (net of discounting costs) to *unconditionally* investing at time two. Equation (2) learns us that waiting (when $\lambda = 1$) is equivalent to making an optimal *conditional* second-period investment decision. Suppose player $i$ truthfully sent a favourable message at time zero. Then player $j$ knows that $n$ cannot take a value lower than two because player $j$ is also assumed to be an optimist. If $P(H|n = 2)$ is higher or equal than $c$, then the optimal conditional second-period investment decision always coincides with unconditionally investing at time two. This means that $\overline{q} - c$ is equal to $W(\overline{q}, 1)$. Hence, $W(\overline{q}, 1)$ will only be strictly greater than $\overline{q} - c$ iff $P(H|n = 2) < c$. In this model all players possess a signal of the same precision and $P(\Theta = H) = \frac{1}{2}$. Therefore, $\forall\ c \in (1 - p, p)$ it takes three pessimists to refrain an optimist, having received a favourable message from player $i$, from investing (and therefore $N$ must be greater or equal than five).

To focus on the interesting parameter range, I assume that:

A2: $\frac{\overline{q}-c}{W(\overline{q},1)} < \delta < 1$

Lemma 2 *Under A2, $q - c < \delta W(q, 1)$, $\forall q \in [\underline{q}, \overline{q}]$*

Proof: See appendix

The first inequality of A2 puts a lower bound on the discount factor $\delta$ such that player $j$, for all her possible posteriors, faces a positive option value of waiting (i.e. if player $j$ expects all the optimists to invest, then she rather waits). The first inequality ensures thus that $\lambda^* < 1$. The second inequality ensures that $\lambda^* > 0$ (whenever $q > c$).

Equation (1) is increasing in $\lambda$. To see this compare the following two "scenarios". In scenario one all optimists randomise with probability $\lambda'$, in scenario two all optimists randomise with probability $\lambda < \lambda'$. Call $k'$ ($k$) the number of players investing at time one when optimistic players invest with probability $\lambda'$ ($\lambda$). Now, having $n - 1$ players investing with probability $\lambda$ is ex ante equivalent to the following two-stage experiment: first let all $n - 1$ players invest with probability $\lambda'$. Next let all $k'$ investors re-randomize with probability $\frac{\lambda}{\lambda'}$. Therefore the statistic $k$ is generated by adding noise to the statistic $k'$. Therefore $k'$ is a sufficient statistic for

$k$. From Blackwell's value of information theorem (1951) we know that this implies that $W(q,\lambda') \geq W(q,\lambda)$. It can be shown that $\forall\lambda,\lambda' \in [0,\lambda^c], W(q,\lambda) = W(q,\lambda')$, while $\forall\lambda,\lambda' \in [\lambda^c,1], W(q,\lambda') > W(q,\lambda)$.

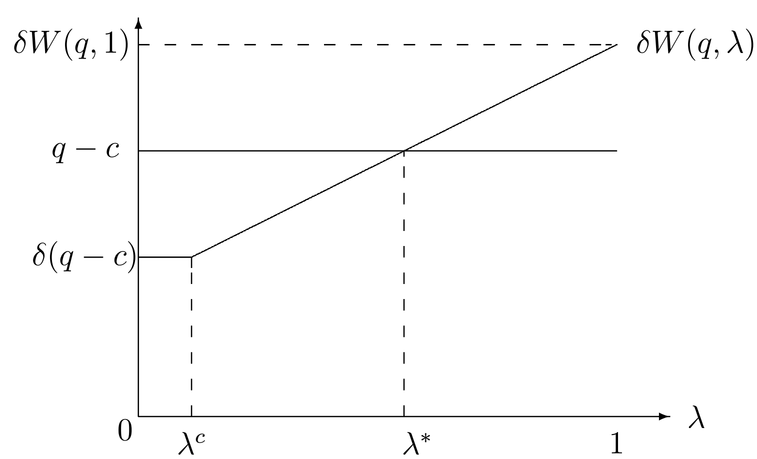
Intuitively, $\lambda$ captures the ex ante amount of information produced by the optimists. The higher $\lambda$, the easier one can infer $n$ out of $k$ (this can best be seen by comparing the two polar cases where $\lambda = 0$ and $\lambda = 1$ (see above)) and thus the higher the ex ante gain of waiting. We know enough to state:

PROPOSITION 1 *Chamley (1997)*
*1) $\forall q \in [c,\overline{q}]$, $\exists$ a PBE in which $\lambda^*_\pi = 0$ and $\lambda^* \in [0,1)$.*
*2) Whenever $\lambda^*_\pi = 0$, $\lambda^*$ is strictly increasing in $q$ in the range $[c,\overline{q}]$.*
*3) If $c \leq q_\pi$, there exist two stable equilibria. In the first one $\lambda^*_\pi = 0$ and $\lambda^* \in (0,1)$. In the second one $\lambda^*_\pi = \lambda^* = 1$.*

Proof: See Chamley (1997). Point 2 corresponds to lemma 4 in Chamley's paper.

The graph below illustrates the intuition behind point one.

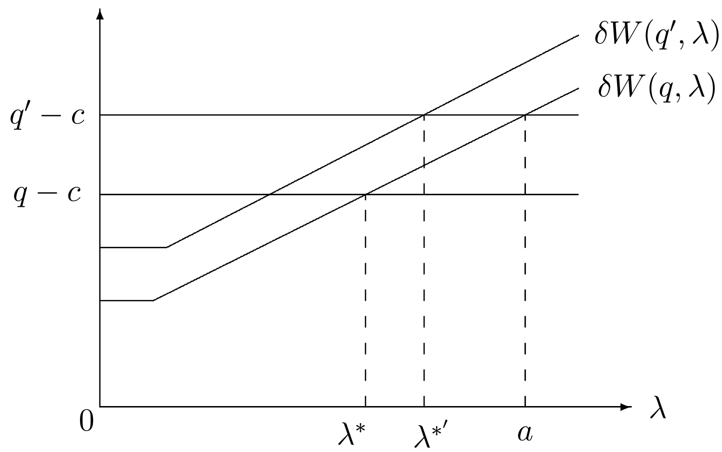
**Graph 1:** Existence of a PBE in which $\lambda^* \in [0,1)$.




In equilibrium the gain of waiting must be equal to the gain of investing, i.e. $q-c = \delta W(q,\lambda^*)$. If $\lambda = \lambda^c$, $\delta W(q,\lambda^c) = \delta[q-c] < q-c$. If $\lambda = 1$, by A2, $\delta W(q,1) > q-c$. By monotonicity, whenever $q > c$ there exists a unique $\lambda^*$ which makes the optimists indifferent between investing and waiting[12]. The intuition behind point 2 of the proposition above can best be explained on the basis of the following graph:

[12] $\lambda^*$ is computed under the assumption that players can only invest in two periods. As shown

**Graph 2:** The effect of a change in $q$ on $\lambda^*$.




Suppose player $j$ first anticipates that $\Theta = H$ with probability $q$. As before, graph two shows the existence of a unique $\lambda^*$ where the gain of investing equals the gain of waiting. Assume now that for some exogenous reason player $j$ becomes "more optimistic" in the sense that she now anticipates that $\Theta = H$ with probability $q' > q$. Graph two shows that the comparison between $\lambda^*$ and $\lambda^{*'}$ depends on the relative strength of two opposing effects. On the one hand, an increase in $q$ increases an optimist's gain of investing, which, were $W(q, \lambda)$ to remain unaffected by the change in $q$, would increase $\lambda$ from $\lambda^*$ to point $a$ in graph two. On the other hand, an increase in $q$ also leads to an increase in the gain of waiting. This second effect decreases $\lambda$ from point $a$ until the point $\lambda^{*'}$. The relative strength of both effects ultimately depends on how the shift of the gain of waiting compares to the one of the gain of investing. Chamley (1997) shows that the first effect always dominates the second one and thus that $\lambda^{*'} > \lambda^*$.

The intuition behind this result mainly lies in the presence of a discount factor in the model. An increase in $q$ increases $W(q, \lambda)$ for two different reasons: (i) it increases the likelihood that $P(H|q, k, \lambda) > c$ and thus that player $j$ will get a non-zero expected utility and (ii) it increases her expected gain of investing whenever player $j$ does so. However, the presence of $\delta$ in front of $W(q, \lambda)$ (and not in front of $q - c$) dampens this increase in $W(q, \lambda)$.

The intuition behind point 3 of proposition (1) goes as follows: assume $c \leq q_\pi < q$. Given these posteriors, $\lambda_\pi^* = \lambda^* = 1$ constitutes a PBE in the waiting game. This is easy to see. Suppose player $j$ decides to wait until period two. Then $\forall n$, player $j$

in CG this is without loss of generality. Indeed, they have shown that having the possibility to wait only one period or to wait an infinite number of periods leaves the equilibrium strategies unaffected. They coined this insight as the one-step property.

would always observe $N-1$ investments at time two. In other words, the investment actions become completely uninformative, there is no informational gain of waiting while its discounting cost (independently of $s_j$) is positive. Therefore, if $q_\pi \geq c$, it's a PBE for everyone to invest at time one. The intuition why another PBE exists in which $\lambda_\pi^* = 0$ and $\lambda^* \in (0,1)$ relies on points one and two of proposition (1). If $\lambda_\pi = 0$, then from point one we know that there exists a unique $\lambda^*$ such that optimists are indifferent between investing and waiting. Point two implies that $q_\pi - c < \delta W(q_\pi, \lambda^*)$, and therefore $\lambda_\pi^* = 0$.

# 4 Cheap Talk.

We now analyse player $i$'s incentives to truthfully reveal her private information at time zero. In equilibrium, the sender knows how her message affects all players' posteriors and consequent equilibrium behaviour. As usual, a separating equilibrium is a PBE in which $x^* = 1$ and $y^* = 0$ or $x^* = 0$ and $y^* = 1$. A pooling equilibrium is a PBE in which $x^* = y^*$.

How should the reader think about player $i$? In our opinion one may interpret her in two ways. First, one may interpret player $i$ as a "guru" whose opinion concerning investment matters is asked by the media. Second, given my assumptions one would want to introduce an opinion poll (instead of just interviewing one player) at time zero. Unfortunately, the game of cheap talk becomes analytically intractable when one introduces other players at time zero. Therefore one can also interpret my model as one explaining "the economics of opinion polls" under the simplifying assumption that the size of the opinion poll equals one. We first analyse a benchmark case in which players cannot postpone their investment decisions.

## 4.1 Benchmark Case: No Waiting

Assume we change the timing of the game in the following sense:

-1) The state of nature is realised and all players receive their signals,
0) One randomly drawn player (henceforth player $i$) is asked to report her signal. Her message, $\hat{s}_i \in \{h, l\}$, is made public to all the other players,
1) All players make their investment decisions,
2) All players learn the true state of the world. Payoffs are received and the game ends.

The proposition below summarises player $i$'s incentives to truthfully reveal her private information through cheap talk.

PROPOSITION 2 *If players cannot postpone their investment decisions, there exists an infinite number of PBE's in which $x^* \neq y^*$.*

Proof: Assume the sender is optimistic. By A1, independently of her message and of the other players' strategies, it will be optimal for her to invest at time one. This is a game of cheap talk. Therefore the sender's utility is only affected by her action and not by the message she sent. Therefore,

$$E(U_i|a_1^i = \text{invest}, \hat{s}_i = h) = p - c = E(U_i|a_1^i = \text{invest}, \hat{s}_i = l) \tag{3}$$

where $E(U_i|a_1^i = \text{invest},\ \hat{s}_i = \cdot)$ denotes player $i$'s expected utility given that she invests at time one and that she sent the message $\hat{s}_i = \cdot$. Therefore there exists a PBE in which the optimistic sender sends the message $\hat{s}_i = h$ with any arbitrary probability $x^*$. The same reasoning applies to a pessimistic sender: independently of her report and of the other player's strategies she will not invest at time one. Therefore, she always receives zero and there exists a PBE in which she sends the message $\hat{s}_i = h$ with any arbitrary probability $y^*$. Q.E.D.

Proposition (2) is not a very surprising nor a very interesting result. However, it permits us to know which one's of my future results will be fundamentally driven by the assumption that players can postpone their investment plans.

## 4.2 Equilibrium behaviour when players can postpone their investment decisions

I first state and prove a negative result.

PROPOSITION 3 *If $c \in (\frac{1}{2}, p)$, there does not exist a PBE in which $x^* \neq y^*$.*

Proof: See appendix

Proposition (3) basically states that for "high-cost" projects, no information can be transmitted through cheap talk: as the message $\hat{s}_i = h$ is as likely to come from an optimistic sender as from a pessimistic one, posteriors are unaffected by the sender's message. The intuition for the inexistence of a PBE in which $x^* \neq y^*$ goes as follows: player $i$ only possesses a noisy signal concerning the realised state of the world and is primarily interested in knowing $n$ (and this is true for an optimistic as well as for a pessimistic sender). Point two of proposition (1) states that if player $i$ succeeds to increase $q$, this will enable her (whenever $\lambda_\pi$ remains equal to zero) to get a "better idea" of $n$ after observing $k$. Stated differently, the higher $q$, the higher player $i$'s gain of waiting (provided that $\lambda_\pi$ remains equal to zero). If $c > \frac{1}{2} = \overline{q}_\pi$, then $\lambda_\pi^*$ will -independently of $x$, $y$ and $\hat{s}_i$ - always be equal to zero. Therefore both sender's types want to send the message which yields the largest increase in $q$ and therefore the pessimist loses if she were to reveal her negative private information.

It is instructive to contrast proposition (3) with proposition (2). In this paper I assume that the returns of the investment project are unaffected by the number of investing players. Despite this absence in competition effects, no information can be transmitted through words because the sender wants to send a message to ease her second-period-inference problem. This insight should be robust to the introduction of positive network externalities (i.e. the returns of the investment project increase with the number of investors)[13]. Therefore, I believe this insight provides a rationale for the "ad hoc" private-information assumption present in the papers I mentioned in the first paragraph of this paper. As many of those papers consider static investment decisions (or, alternatively, assume that players cannot choose when to invest) one may believe that those models are fragile with respect to the introduction of cheap talk. However, as proposition (3) shows, one cannot think about information exchange without acknowledging that investors always have the possibility to wait and invest in the future.

Luckily, this paper also possesses a more "optimistic" result which is summarised below:

Proposition 4 *if $c \in (1-p, \frac{1}{2}]$, there exists a separating equilibrium. In the separating equilibrium, $\lambda_\pi^* = \lambda^* = 1$, whenever player $i$ sends a favourable message.*

Proof: For the sake of concreteness, assume players compute their posteriors under the assumption that $x^* = 1$ and $y^* = 0$. I first analyse how equilibrium behaviour in the ensuing waiting game is influenced by the message sent by player $i$.

Assume $\hat{s}_i = h$. If $s_j = l$, player $j$'s posterior $\overline{q}_\pi = \frac{1}{2}$. Note that by assumption $\frac{1}{2} \geq c$, so pessimists now also face a positive gain of investing. If $s_j = h$, player $j$

[13]See Melissas (2000) for the same model with a small amount of positive network externalities.

computes $\overline{q} > p$.

From point 3 of proposition (1), we know that in the continuation game there exists a PBE in which $\lambda_\pi^* = \lambda^* = 1$. Note that all receivers possess some public (i.e. the favourable message sent by player $i$) and some private information (i.e. their signals). All players, independently of their signals, rely on the public information by investing at time one. This behaviour is identical to the one followed by the players inside an informational cascade in BHW's and Banerjee's (1992) model. In those models all players also possess some public (i.e. the action(s) of the first mover(s)) and private information (i.e. their signals) and they, independently of their signals, all adopt the same action. Therefore, I call the PBE in the continuation game in which $\lambda_\pi^* = \lambda^* = 1$ an informational cascade.

Assume now that $\hat{s}_i = l$. In that case $P(H|s_j = l, \hat{s}_i = l, 1, 0) = \underline{q}_\pi < 1 - p < c$ and it's a dominant strategy for pessimists to wait at time one. Optimists compute $P(H|s_j = h, \hat{s}_i = l, 1, 0) = \underline{q} = \frac{1}{2}$. As $\frac{1}{2} \geq c$, optimists still face a positive gain of investing. From proposition (1) we know there exists a PBE in the continuation game in which $\lambda_\pi^* = 0$ and $\lambda^* \in [0, 1)$ ($\lambda^*$ only equals zero when $c = \frac{1}{2}$).

I now analyse player $i$'s incentives to reveal her private information, given that she correctly anticipates how her message influences equilibrium investment probabilities in the ensuing waiting game.

Assume first that player $i$ is a pessimist. If she sends a favourable message, $\lambda_\pi^* = \lambda^* = 1$, everyone invests and player $i$ computes $P(H|l, k = N-1, \lambda_\pi^* = \lambda^* = 1) = 1 - p$. This is logical: in an informational cascade one cannot learn a player's type by observing her action. Therefore, the sender's observation of the informational cascade should not affect her posterior probability that $\Theta = H$. As $1 - p < c$, she refrains from investing in both periods and gets zero. If she sends $\hat{s}_i = l$, $\lambda_\pi^* = 0$ and optimists randomise with probability[14] $\Lambda(s_j = h, \hat{s}_i = l) \geq 0$. In that case player $i$ will be able to "get a better idea" of $n$ upon observing $k$ and her payoff equals $\delta W(1 - p, \Lambda^*(h, l))$, where

$$W(1-p, \Lambda^*(h,l)) = \sum_{k=0}^{N-1} Max\{0, P(H|l, k, \Lambda^*(h,l)) - c\} P(k|l, \Lambda^*(h,l))$$

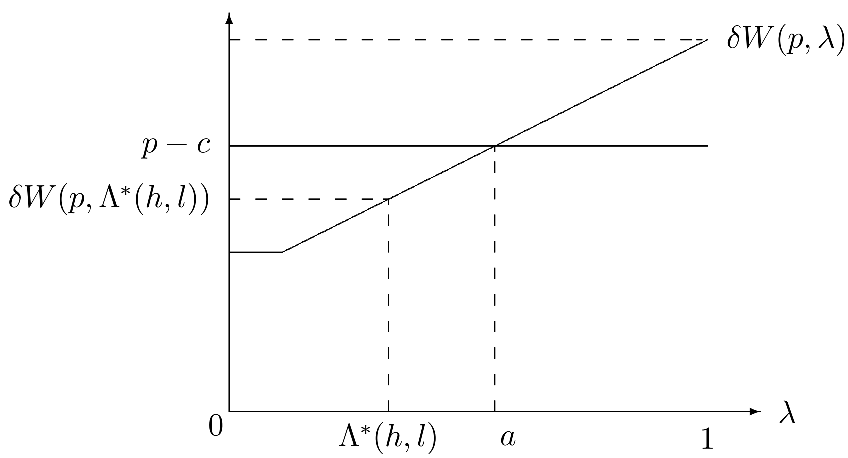
[14] I use here $\Lambda(\cdot)$ instead of $\lambda$ to denote investment probabilities in order to emphasise that $\lambda^*$ depends on the message sent by player $i$.

As $\delta W(1-p, \Lambda^*(\cdot)) \geq 0$ a pessimist cannot gain by deviating.

Suppose now that player $i$ is optimistic. If she sends a favourable message, everybody invests in the first period and she doesn't learn anything. As player $i$ is optimistic she also faces a positive gain of investing. If she invests at time one, she gets $p-c$. If she waits, she gets $\delta(p-c)$. As $\delta < 1$, it's optimal for her to invest at time one and her gain of sending message $h$ equals $p-c$.

However, an optimist's gain of sending an unfavourable message also equals $p-c$. This can best be illustrated on the basis of the following graph:

**Graph 3:** An optimist's optimal time 1 action after a deviation.




To understand graph 3, first note that an optimist's payoff of sending an unfavourable message equals $\max\{p-c, \delta W(p, \Lambda^*(s_j = h, \hat{s}_i = l))\}$. $p-c$ denotes her payoff of investing at time one, given that she sent an unfavourable message. $\delta W(p, \Lambda^*(s_j = h, \hat{s}_i = l))$ denotes her payoff of waiting given that she sent an unfavourable message. Call $a$ the probability with which optimists must invest to make player $i$ indifferent between investing and waiting. If our game would not possess a communication stage then all optimists would anticipate that $\Theta = H$ with probability $p$ and they would then invest at time one with probability $a$. As the optimists received an unfavourable message, they anticipate that $\Theta = H$ with a probability equal to $\underline{q} = \frac{1}{2} < p$. From point 2 of proposition (1), we know that $\Lambda^*(h, l) < a$ (because $\frac{1}{2} < p$). From Blackwell's Theorem follows that $\delta W(p, \Lambda^*(\cdot)) < p - c$. Therefore, if an optimist were to deviate and sent message $l$, it would still be optimal for her to invest at time one. Thus an optimist, independently of her report, always gets $p - c$ and she cannot gain by deviating. Therefore, if $c \in (1-p, \frac{1}{2}]$, a separating equilibrium exists. Q.E.D.

Proposition (4) is not trivial. Both sender's types share the same preferences over the receivers' actions in the sense that both of them want the optimists to invest with a higher probability and the pessimists to remain quiet. We would therefore not expect to find the existence of a separating equilibrium.

In proposition (4) a separating equilibrium is fundamentally driven because: (i) both sender's types face different opportunity costs of waiting and (ii) sending a favourable message creates an informational cascade. An optimist believes the investment project is good. For her "time is money" and she is only willing to postpone her investment plans (with probability one) if pessimists don't invest *and* if optimists invest with a probability higher than $a$. Unfortunately these two aims cannot be simultaneously achieved by none of the two messages. Therefore, she is indifferent between reporting $\hat{s}_i = l$ and $\hat{s}_i = h$ and she always invests at time one. A pessimist believes the investment project is bad. She is unwilling to invest unless she observes "relatively many" optimists investing at time one. If the pessimist were to deviate and sent a favourable message, an informational cascade would occur, she wouldn't receive any payoff-relevant information and she would get zero. Hence, it is the informational cascade which ultimately induces a pessimist to send an unfavourable message. If $\lambda_\pi^*$ would always be equal to zero (as is the case in CG where pessimists do not have an investment option and can therefore not invest[15]), a pessimist would never want to send a negative message because - if this message were to be believed - this would reduce $\lambda^*$.

## 4.3 How important is the informational cascade to elicit private information?

Proposition (4) highlights a PBE in which an informational cascade induces a pessimist to truthfully reveal her private information. A natural question arises: does there exist another PBE in which $x^* \neq y^*$ and in which $(\lambda_\pi^*, \lambda^*) \neq (1,1)$ whenever player $i$ sends a favourable message? If this were the case one may object that the informational cascade only acts as a truthtelling device in *a* PBE of my game, but that there exist another PBE (or possibly many other PBE's) in which $x^* \neq y^*$ and in which revelation (or possibly partial revelation) of private information is not

[15]Or as is the case for "high-cost" investment projects.

driven by the fact that pessimists "fear" the occurence of an informational cascade. My next proposition deals with this objection.

PROPOSITION 5 *$x^* \neq y^*$ if and only if $\lambda^*_\pi = \lambda^* = 1$ whenever the sender sends a favourable message.*

Proof: First, I show that if $x^* \neq y^* \Rightarrow \lambda^*_\pi = \lambda^* = 1$ whenever player $i$ sends a favourable message.[16] By contradiction, assume there exists a PBE in which $x^* > y^*$ and in which $(\lambda^*, \lambda^*_\pi) \neq (1,1)$ whenever $\hat{s}_i = h$. If player $i$ sends a favourable message, there are two possibilities:
(i) $P(H|l, \hat{s}_i = h, x^* > y^*) < c$
(ii) $P(H|l, \hat{s}_i = h, x^* > y^*) \geq c$

In case (i) it's a dominant strategy for the pessimists to wait at time one and therefore $\lambda^*_\pi = 0$. However, from the proof of proposition (3) we know that this implies that $x^*$ cannot be different from $y^*$.

In case (ii) from proposition (1) we know that the ensuing waiting game is characterised by two symmetric, stable equilibria. In the first equilibrium, $\lambda^*_\pi = 0$ and $\Lambda^*(h,h) \in (0,1)$. In the second equilibrium $\lambda^*_\pi = \lambda^* = 1$. Assume player $i$ anticipates that if she were to send a favourable message, in the waiting game players would invest with probabilities $\lambda^*_\pi = 0$ and $\Lambda^*(h,h) \in (0,1)$. As before, from the proof of proposition (3) we know that, as $\lambda^*_\pi = 0$, this implies that $x^*$ cannot be different from $y^*$ because a pessimist never wants to reveal her unfavourable information.

I now show that if $\lambda^*_\pi = \lambda^* = 1$ whenever the sender sends a favourable message $\Rightarrow$ $x^* \neq y^*$. Assume there exists a PBE in which $\lambda^*_\pi = \lambda^* = 1$ whenever $\hat{s}_i = h$ and in which $x^* = y^*$. As $x^* = y^*$, this implies that $P(H|l, \hat{s}_i = \cdot, x^* = y^*) = 1 - p$ (a pessimist's posterior is not affected by the message sent by player $i$ in a pooling equilibrium). Under A1, it's then a dominant strategy for each pessimist to wait at time one. Therefore $\lambda^*_\pi$ cannot be different from zero, a contradiction. Q.E.D.

[16] From now on I will assume that "a favourable message" equals "message $h$". Of course this is without loss of generality.

## 4.4 Some normative and positive implications of proposition (4)

In my previous two propositions I highlighted the importance of an informational cascade to induce truthful revelation of private information. In this section I argue that this insight bears some positive and normative consequences. I first analyse the consequence of an investment subsidy on the incentives for truthful revelation. Denote by $b > 0$ a subsidy granted to each investor. My next proposition shows that subsidising investments encourages truthful revelation of private information.

PROPOSITION 6 *$\forall c \in (1-p, \frac{1}{2}+b]$, $\exists$ a separating equilibrium. The range of parameter values in which a separating equilibrium exists is increasing in the subsidy.*

Proof: Define $c'$ as $c' = c - b$. Obviously, our players now base their investment decisions on $c'$ instead of $c$. A PBE in the continuation game in which $\lambda_\pi^* = \lambda^* = 1$ exists if and only if $P(H|l, \hat{s}_i = h, 1, 0) \geq c'$. All players possess a signal of the same precision and $P(\Theta = H) = P(\Theta = L) = \frac{1}{2}$. Therefore, $P(H|s_j = l, \hat{s}_i = h, 1, 0) = \frac{1}{2}$. Therefore, a separating equilibrium exists as soon as $c' \leq \frac{1}{2}$ or as soon as $c \leq \frac{1}{2} + b$. Q.E.D.

The intuition behind proposition (6) is simple. Proposition (4) teaches us that a separating equilibrium exists if the cost of the investment project is "low enough". If $c$ is "too high" then - independently of the message sent - pessimists always face a negative gain of investing (and the continuation game can then impossibly be characterised by a PBE in which $\lambda_\pi^* = 1$). Proposition (6) rests on the observation that a subsidy, by "artificially" reducing the investment cost, may induce everyone to invest at time one (after having received favourable news from the sender), thereby discouraging a pessimist to send a favourable message.

I now address a different question: "How does the sender's ability influence her incentives for truthful revelation?" So far I assumed that the sender was "as able" as the receivers in the sense that all players possess a signal of the same precision. One may find it more natural to endow player $i$ with a more precise signal. After all, in my model she can be interpreted as a guru and people typically think of them as being better informed investors (that's the reason why they appear in the media). There is a straightforward way to allow for a better informed sender. Let's assume that player $i$'s signal is drawn from the distribution: $P(h|H) = P(l|L) = r$ and $P(l|H) = P(h|L) = 1 - r$ (where $1 > r > p$). The higher $r$, the "smarter" or the better informed the sender. My main result is summarised below:

Proposition 7 *$\forall c \in (1-p, \min\{p, \frac{(1-p)r}{(1-p)r+p(1-r)}\})$, $\exists$ a separating equilibrium. This range of parameter values cannot decrease in the precision of the sender's signal.*

Proof: A PBE in the continuation game in which $\lambda^*_\pi = \lambda^* = 1$ exists if and only if $P(H|l, \hat{s}_i = h, 1, 0) \geq c$. This posterior probability is now computed as:

$$P(H|l, \hat{s}_i = h, 1, 0) = \frac{P(H, \hat{s}_i = h|l, 1, 0)}{P(\hat{s}_i = h|l, 1, 0)} = \frac{(1-p)r}{(1-p)r + p(1-r)}$$

If $c \in (1-p, \frac{(1-p)r}{(1-p)r+p(1-r)})$, then there exists a PBE in the continuation game in which $\lambda^*_\pi = \lambda^* = 1$. The reader can now easily see that there exists a separating equilibrium if $c \in (1-p, \min\{p, \frac{(1-p)r}{(1-p)r+p(1-r)}\})$ (the proof is identical to the one I outlined when proving proposition (4)). Q.E.D.

The intuition behind proposition (7) is also simple. As I showed in proposition (4) and (5), a separating equilibrium only exists if $\lambda^*_\pi = 1$ after the arrival of some positive information. In other words, a separating equilibrium only exists if the sender can make the pessimists change their minds. Proposition (7) therefore rests on the intuitive idea that the "smarter" the sender (or the more precise her private information), the "easier" it will be for her to make the pessimists change their minds. If the sender cannot convince the remaining pessimists to invest at time one (either because the sender is commonly perceived to be "stupid" or because of a high investment cost) then she doesn't want to reveal any unfavourable information because this will worsen her second-period inference problem.

# 5 How robust is proposition (4)?

I first check how proposition (4) is altered if the player's types were drawn out of a richer distribution. Next, I explain how my results are affected if one were to add a small amount of network externalities in the model.

## 5.1 Multi-type setting

This subsection is divided in two parts. First, I analyse the case where only the sender's type is drawn from a richer distribution. Next, I discuss the more general case where all players' types are drawn from a richer distribution.

#### 5.1.1 The case of an uninformed sender

Let's assume that with some probability $\epsilon$ player $i$ does not possess any private information.[17] More specifically, assume that $P(s_i = h|H) = P(s_i = l|L) = p - \frac{\epsilon}{2}$; $P(s_i = \phi|H) = P(s_i = \phi|L) = \epsilon$ and $P(s_i = l|H) = P(s_i = h|L) = 1 - p - \frac{\epsilon}{2}$ (where $\frac{\epsilon}{2} \in (0, 1-p])$. Player $i$'s message is now $\in \{l, \phi, h\}$. Throughout this subsection, I assume that $c \in (1 - p, \frac{1}{2}]$.

In this set-up there exists a semi separating-pooling equilibrium in which the $l$-type and the $\phi$-type both send the same message (say, message $\hat{s}_i = \phi$) and the $h$-type sends a different message (say, message $\hat{s}_i = h$). To prove this, I first explain how in equilibrium player $j$ computes her posteriors given the different sender's strategies. First, assume player $j$ is an optimist. Upon receiving the message $\hat{s}_i = \phi$, she computes:

$$P(H|s_j = h, \hat{s}_i = \phi, \text{only } l\text{-type and } \phi\text{-type send message } \phi) > \frac{1}{2}$$

Next, assume player $j$ is a pessimist, she computes:

$$P(H|s_j = l, \hat{s}_i = \phi, \text{only } l\text{-type and } \phi\text{-type send message } \phi) < 1 - p \qquad (4)$$

Similarly, if player $i$ sends $\hat{s}_i = h$, a pessimist computes:

$$P(H|s_j = l, \hat{s}_i = h, \text{only } h\text{-type sends message } h) > \frac{1}{2}$$

From the previous section we know that a pessimistic sender strictly prefers to send message $\hat{s}_i = \phi$ rather than message $\hat{s}_i = h$. Consider now a sender who doesn't possess any information. What is her expected gain of sending message $\hat{s}_i = h$? In that case from the previous section we know that it's a PBE (in the continuation game) for everyone to invest at time one. As player $i$ faces a positive gain of investing, she gets $\frac{1}{2} - c \geq c$. What is her expected gain of sending message $\hat{s}_i = \phi$? Upon receiving message $\hat{s}_i = \phi$, from (4) follows that it's a dominant strategy for pessimists to wait at time one. Optimists compute $P(H|s_j = h, \hat{s}_i = \phi, \cdot)$ and invest with probability $\Lambda^*(h, \phi)$. If player $i$ invests she gets $\frac{1}{2} - c$. If she waits, she gets $\delta W(\frac{1}{2}, \Lambda^*(h, \phi))$. From my previous section we know that the following equalities

[17] Note that, for simplifying reasons, I still assume that $s_j \in \{h, l\}$ $(j \neq i)$, i.e. only the sender may be uninformed.

and inequality are satisfied:[18]

$$\text{gain send } h = \frac{1}{2} - c = \delta W(\frac{1}{2}, a) < \delta W(\frac{1}{2}, \Lambda^*(h,\phi)) = \text{gain send } \phi$$

Therefore it's optimal for her to wait at time one and she strictly prefers to send message $\hat{s}_i = \phi$.

Finally, from the previous section we also know that an optimistic sender cannot gain by deviating neither. The proposition below summarises the insight present in this subsection:

PROPOSITION 8 *If there exists a probability $\epsilon \in (0, 2(1-p)]$ of player $i$ being uninformed, then $\forall c \in (1-p, \frac{1}{2}]$, there exists a semi separating-pooling equilibrium. In that equilibrium $\lambda^* = \lambda^*_\pi = 1$ whenever player $i$ sends a favourable message.*

Two conclusions can be drawn out of my last proposition :
(i) the separating equilibrium highlighted in proposition (4) is driven by the assumption that the sender can either be an optimist or a pessimist,
(ii) however this does not mean that the insight present in proposition (4) is worthless. After all, the occurence of an informational cascade along the equilibrium path is also stressed in proposition (8). My last proposition shows that one should not interpret proposition (4) as follows: "Informational cascades induce all possible types of players to truthfully reveal their private information". Instead, proposition (4) should be interpreted as: "Informational cascades put an upper limit above which some types of players don't want to misrepresent their information".

### 5.1.2 All players' types are drawn from a richer distribution

Proposition (1) highlights the existence of an equilibrium entailing a large aggregate investment activity with a low amount of learning. Chamley (1997) and Chamley (2000) shows that one needs to impose very little assumptions to recover this kind of equilibrium (which he calls the "high activity equilibrium") in a more general model. Unfortunately analysing a model where players' types are drawn from a continuous distribution is a complex task and I doubt whether any analytical results can be

[18] In the equation above $a \in (0,1)$ denotes the probability with which optimists must invest at time one to make an uninformed sender indifferent between investing and waiting. As $\epsilon > 0$, $P(H|s_j = h, \hat{s}_i = \phi, \cdot) > \frac{1}{2}$. By point 2 of proposition (1) we know that this implies that $a < \Lambda^*(h,\phi)$. By Blackwell's value of information theorem we know that this implies the inequality present in the equation above.

obtained along these lines. However, a priori it's not clear why the insight present in proposition (8) shouldn't be robust in a more general model. If player $i$' s signal were above a certain threshold value[19] she would - irrespective of her message - invest at time one and could not gain by misrepresenting her private information. If player $i$' s signal were below a certain threshold value, she wouldn't want to send too favourable a message to the remaining players out of fear that this would ignite an equilibrium (in the continuation game) with a large investment activity and low learning. It is also not clear why the insights present in propositions (6) and (7) should not be robust in a more general model. As shown by Chamley, the high activity equilibrium only exists for low cost projects (and a subsidy still artificially lowers the investment cost even if signals were drawn from a continuous distribution). Similarly, it should be easier for a "smart" sender to ignite the high activity equilibrium by sending a favourable message.

## 5.2 Positive network externalities

In the model I analysed in section four, an optimist is indifferent between sending $\hat{s}_i = h$ or $\hat{s}_i = l$. In both cases she invests at time one and gets $p - c$. Therefore my game of cheap talk is also characterised by an infinite number of semi separating-pooling equilibria in which $x^* \in (0, 1]$ and $y^* = 0$ (as the optimistic sender is indifferent between sending $\hat{s}_i = h$ or $\hat{s}_i = l$, she may as well randomise over these two messages). The pessimist, however, strictly prefers to send an unfavourable message (if $c < \frac{1}{2}$). In Melissas (2000), I analysed the same model as the one outlined in sections 3 and 4 except that I worked under the following technological assumption:

A3: The investment generates a revenue equal to 1 if $\Theta = H$ and if at least two players invested. If only one player invests then the investment generates a revenue equal to $\gamma < 1$ (where $\gamma$ is close to 1) if $\Theta = H$.

Unfortunately, the computational complexity of the model increases considerably when working under A3. However under A3, I showed that an optimist then strictly prefers to send a favourable message because by doing so she minimises the probability of getting $\gamma$ instead of 1. Intuitively, assumption A3 introduces some positive network externalities in the model. As the optimist then wants to be imitated, her

[19] If players' types are drawn from a continuous distribution, players would adopt cut-off strategies (see Chamley (2000)): a player possessing a signal below a certain threshold value $s^*$ waits (with probability one), while all players possessing a signal above $s^*$ invest.

expected gain of sending $\hat{s}_i = h$ is strictly greater than the one of sending $\hat{s}_i = l$. Therefore under A3, my game of cheap talk is only characterised by separating and pooling equilibria.

# 6 Conclusions

In this paper I introduced cheap talk in an investment model with information externalities. I first showed how information transmission via words may be impeded when players can postpone their investment decisions and I argued that this may justify why some models (which I listed in the first paragraph of this paper) assume away any efficient preplay communication. From propositions (3), (4) and (5) we know that the problem in my game comes from the pessimist (an optimist always invests in the first period and therefore cannot gain by misrepresenting her private information). A pessimist is reluctant to divulge her bad information because this worsens her inference problem. Therefore she only truthfully reveals her bad information if she learns more (or does not learn less) by doing so. As she doesn't learn anything upon observing an informational cascade (which occurs whenever the sender sends a favourable message) revelation of bad information is compatible with maximising behaviour. Hence in this paper informational cascades induce players to transmit payoff-relevant information through cheap talk. Finally, I argued that in my context subsidising investments encourages truthtelling and that "smart" people have more incentives to truthfully reveal their private information than "stupid" ones.

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## Appendix

### Proof of lemma (1):

As explained in the paper $q - c < W(q, 1)$ as soon as there exists an $n$ such that: $P(H|n) < c$. Assume player $j$ anticipates that $\Theta = H$ with probability $q = \overline{q}$. This posterior can only be generated if (i) $s_j = h$, (ii) player $i$ send a favourable message and (iii) $x = 1$ and $y = 0$ (or $x = 0$ and $y = 1$). Therefore if $q = \overline{q}$, $n$ cannot take a value lower than two. Now:

$$P(H|n = 2) = \frac{C_N^2 p^2 (1-p)^{N-2}}{C_N^2 p^2 (1-p)^{N-2} + C_N^2 (1-p)^2 p^{N-2}}$$

where $C_N^2$ represents the number of possible combinations of two players out of a population of $N$ players. It can easily be shown that $\forall N_1 > N_2 \geq 2$:

$$\frac{p^2(1-p)^{N_1-2}}{p^2(1-p)^{N_1-2} + (1-p)^2 p^{N_1-2}} < \frac{p^2(1-p)^{N_2-2}}{p^2(1-p)^{N_2-2} + (1-p)^2 p^{N_2-2}}$$

From statistical textbooks (see e.g. De Groot (1970)) we know that in my set-up $P(H|n)$ is driven by the difference between the good and the bad signals in the population.[20] Therefore if $N \geq 5$, $P(H|n = 2) \leq 1 - p$ which is strictly lower than $c$ by A1. Assume now that $q < \overline{q}$, meaning that player $j$ puts a strictly positive probability on either one of the following two events: (i) player $i$ is an optimist and (ii) player $i$ is a pessimist. In case (i), $n$ cannot be lower than two and the analysis above goes through. In case (ii) $n$ cannot be lower than one and if $N \geq 5$, $P(H|n = 1) < P(H|n = 2) \leq 1 - p$. Q.E.D.

### Proof of lemma (2):

Call $\lambda^*(q)$ ($\lambda^*(\overline{q})$) the probability with which optimists must invest such as to make a player who anticipates that $\Theta = H$ with probability $q$ ($\overline{q}$) indifferent between investing and waiting. From proposition (1) and under A2, we know that: $\lambda^*(q) < \lambda^*(\overline{q}) < 1$. Therefore $\forall q < \overline{q}$:

$$q - c = \delta W(q, \lambda^*(q)) < \delta W(q, 1)$$

Q.E.D.

[20] For example, $P(H|n = 1, N = 3) = P(H|n = 2, N = 5) = 1 - p$. In both cases: #pessimists $-$# optimists $= N - n - n = 1$.

**Proof of proposition (3):**

First note that $\overline{q}_\pi = P(H|l, \hat{s}_i = h, 1, 0) = \frac{1}{2}$. Therefore, independently of $x$, $y$ and $\hat{s}_i$, $q_\pi$ is always strictly lower than $c$. Hence it's a dominant strategy for pessimists to wait at time one. By contradiction, assume there exists a PBE in which $x^* \neq y^*$. Without loss of generality assume that $x^* > y^*$. Assume player $i$ is a pessimist. Is her gain of sending message $l$ higher than (or equal to) her gain of sending message $h$? Upon receiving message $\hat{s}_i = h$, an optimist computes:

$$P(H|h, \hat{s}_i = h, x^* > y^*) > p > P(H|h, \hat{s}_i = l, x^* > y^*)$$

From point 2 of proposition (1), this implies that[21]:

$$\Lambda^*(s_j = h, h_1 = \hat{s}_i = h) > \Lambda^*(s_j = h, h_1 = \hat{s}_i = l)$$

Player $i$'s gain of sending message $l$, $E(U|\hat{s}_i = l)$, equals:

$$E(U|\hat{s}_i = l) = \max\{\delta W(1-p, \Lambda^*(s_j = h, h_1 = \hat{s}_i = l)), 0\}$$

where:

$$W(1-p, \Lambda^*(\cdot)) = \sum_{k=0}^{N-1} \max\{0, P(H|1-p, k, \Lambda^*(\cdot)) - c\} P(k|1-p, \Lambda^*(\cdot))$$

Player $i$'s gain of wrongfully sending message $h$ equals:

$$E(U|\hat{s}_i = h) = \max\{\delta W(1-p, \Lambda^*(s_j = h, h_1 = \hat{s}_i = h)), 0\}$$

As $P(H|h, \hat{s}_i = h, x^* > y^*) > p$, it follows that optimists face a strictly positive expected gain of investing at time one. Therefore, $\Lambda^*(s_j = h, h_1 = \hat{s}_i = h) > 0$. Therefore

$$\delta W(1-p, \Lambda^*(s_j = h, h_1 = \hat{s}_i = h)) > 0 \tag{5}$$

This is easy to see: as $\Lambda^*(\cdot) > 0$, there exists a (very small but nonetheless) strictly positive probability that all the $N-1$ remaining players are optimists and will invest at time one. As $N \geq 5$, this implies that:

$$P(H|s_i = l, k = N-1, \Lambda^*(\cdot)) > c$$

and player $i$ then faces a strictly positive gain of investing.

[21] Below, I replaced $\lambda^*$ by its more general counterpart $\Lambda^*(\cdot)$. This was done to explicitly show that the period-one strategies depend on the message sent.

There are now two possibilities:
(i) $\delta W(1-p, \Lambda^*(s_j = h, h_1 = \hat{s}_i = l)) = 0$, and
(ii) $\delta W(1-p, \Lambda^*(s_j = h, h_1 = \hat{s}_i = l)) > 0$
In case (i), from inequality (5) follows that $E(U|s_i = l, \hat{s}_i = h) > E(U|s_i = l, \hat{s}_i = l)$.
In case (ii) from Blackwell's theorem follows that:

$$\delta W(1-p, \Lambda^*(h, \hat{s}_i = h)) > \delta W(1-p, \Lambda^*(h, \hat{s}_i = l))$$

Hence, in both cases a pessimist strictly prefers to send a favourable message. Q.E.D.