Problem Set 6

1. Consider the following two-player game:

	b_1	b_2	b_3	b_4
a_1	$2,3$	$4,2$	$3,0$	$0,4$
a_2	$3,4$	$5,1$	$4,3$	$1,1$
a_3	$3,2$	$1,1$	$3,1$	$1,2$
a_4	$2,3$	$2,1$	$3,1$	$3,1$

a. Does the row player have a strictly dominant strategy?
b. Does the column player have a strictly dominant strategy?
c. Does the row player have a strategy which is weakly dominated?
d. Does the column player have a strategy which is weakly dominated?
e. Find all the (pure-strategy) Nash Equilibria of this game.

2. Suppose that ten players play the following game. Each player announces an integer between one and hundred (simultaneously). The player who announces a number which is closest to the two third of the average of the announcements wins \$100. The others win nothing. In case of tie, the winners share the \$100 equally. More formally, let n_i denote the announcement of Player i. Then player i is a winner if

$$\left| n_i - (2/3) \sum_{j=1}^{10} n_j \right| \leq \left| n_k - (2/3) \sum_{j=1}^{10} n_j \right|$$

for all k.

a. What are the strategies of Player i which are weakly dominated by other strategies?
b. What are the Nash Equilibria of this game?

3. Suppose that Robinson lives in an island alone, where he can consume only coconuts. His production function is $f(l) = l^{1/2}$ where l denotes the amount of time spent on collecting coconuts. His utility is $u(c, l) = c^{1/2} - 2l$, where c is the amount of coconuts he eats. He sets up a firm and appoints himself to be the CEO. As a CEO he decides how much labor to employ and as a consumer he decides how much coconut to buy and how much labor to supply. He takes prices as given. Normalize the price of the coconut to be one and let w denote the wage.

a. Compute the labor demand and the coconut supply as a function of w.
b. Compute the labor supply and the coconut demand as a function of w and the dividend Robinson receives as the owner of the firm.
c. What is the equilibrium wage, labor, and coconut?
d. Show that the competitive outcome is Pareto optimal.