Problem Set 1

Technology and Profit Maximization

1. Suppose that the technology of a firm is described by the following production function: $f(x_1, x_2) = Ax_1^{\alpha_1} x_2^{\alpha_2}$. For what values of the parameters A, α_1, and α_2 does the function f

a. exhibit increasing/decreasing/constant return to scale?

b. exhibit diminishing marginal product in both inputs?

c. exhibit diminishing technical rate of substitution?

2. Suppose that a firm needs to use two inputs (x_1, x_2) in order to produce an output (y). The disposal of input one is free, but disposing two units of input two requires giving up one unit of input one. If the firm uses x_1 units of input one and x_2 units of input two, it can produce $\min\{x_1, x_2\}$ units of output. However, the firm losses all of its output if it ends up with non-disposed inputs.

a. Is the production function of the firm monotonic?

b. Describe the production function of the firm!

c. Draw the map of the isoquant curves corresponding to the technology!

3. a. Suppose that a the set of isoquant curves of a production function, $f(x_1, x_2)$ is identical to the set of isoquant curves of the production function $x_1^{1/2} x_2^{2/3}$. Does it follow that the technology described by f is convex?

b. Give an example for a production function, $f(x_1, x_2)$, which exhibits diminishing technical rate of substitution, but does not exhibit diminishing marginal product!

c. Give an example for a production function, $f(x_1, x_2)$, which does not exhibit diminishing technical rate of substitution, but exhibits diminishing marginal product!

4. a. Suppose that a firm is expected to generate π profit every year. The expected interest rate on the market is 10% forever. What is the value of the firm?

b. Suppose that the firm's profit was π this year, and the firm is growing by a factor of g every year. (That is, the profit of the firm at year $t+1$ is g times its profit at year t.) The interest rate is still 10%. What is the value of the firm?

c. The value of Google in 2008 was around 40 times larger than its yearly profit. How fast does the market expect Google to grow if $r = 10\%$?

5. Suppose that the technology of a firm can be described by the following production function: $f(x_1, x_2) = 2x_1^{1/3} x_2^{1/3}$. Also suppose that the amount of $x_1 = 1$ but x_2 can be freely chosen in the short run. The price of the output, p, is 10 and the price of input 1, w_1, is 5.

a. Derive the short-run inverse factor demand curve of the firm for input two!

b. Suppose that the price of input two is also 5. What is the short-run profit of the firm?

c. What is the long-run profit of the firm?