The Price of Advice*

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Abstract

We develop a model of consulting (advising) where the role of the consultant is that she can reveal signals to her client which refine the client’s original private estimate of the profitability of a project. Importantly, only the client can observe or evaluate these signals (“clues”), the consultant cannot. We characterize the optimal contract between the consultant and her client. It is a menu consisting of pairs of transfers specifying payments between the two parties (from the client to the consultant or vice versa) in case the project is undertaken by the client and in case it is not. The main result of the paper is that in the optimal mechanism, the consultant obtains the same profit as if she could observe the signals whose release she controls.

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1 Introduction

An important question in economic theory is how information is transmitted between strategic agents with differing goals, and what type of mechanisms govern (or induce optimal) information transmission between such parties. In particular, one fascinating topic is the relationship between advisors (consultants) and their clients: how consultants “create value” (as they call it), and what characterizes optimal contracts between them and their clients.

In this paper, we put forward a model where the role of the consultant is that she is able to reveal signals to her client (him) which refine the client’s original private estimate regarding the profitability of a project. The client’s action as to whether he undertakes the project is contractible, but the information disclosed by the consultant is not. Indeed, we assume that only the client can observe (evaluate) the additional signals disclosed by the consultant, so the consultant does not know a priori whether her advice made the project look more or less profitable to the client.

In this model, we characterize the optimal contract between the consultant and the client. Since only the client’s action is contractible, the mechanism can be represented by a menu consisting of pairs of transfers specifying payments between the two parties contingent on whether or not the project is undertaken by the client.\footnote{In general, the menu chosen by the consultant may contain offers where the client pays the consultant if he decides to undertake the project and the consultant pays him if he does not, and, at the same time, offers where payments are made in the exact opposite direction (that is, the consultant pays the client if he undertakes the project, and vice versa).}

The main and surprising result of the paper is that the consultant can design a contract (which will be the optimal one) in which she obtains the same profit she could observe the signals whose release she controls but cannot actually observe.

Our way of modeling “advising”—where the consultant may disclose information that only the client can interpret—is new, perhaps unusual, but we believe accurate in many instances. For a concrete example, think of the client as a potential buyer of a good (e.g., a car, or a firm’s shares) who is uncertain of various characteristics of the good (the features of the car, the covariance of the stock’s return with other assets’ returns). Then, the consultant can be thought of as an industry expert who can disclose information regarding the good’s characteristics. Naturally, the consultant does not know the buyer’s original valuation-estimate. Moreover, she
does not know whether (or by how much) her information increases or decreases the buyer’s willingness to pay for the good, because that is also part of the buyer’s private information (what features he values in a car, what his existing portfolio consists of). Our question is, what is the consultant’s optimal contract, if it can only depend on the buyer’s contractible action (whether or not he buys the good), but not on the information received from the consultant.

More broadly, our model of consultancy is motivated by the widely held belief that the role of strategy and management consultants is to help to bring to light their clients’ own ideas so that the clients can realize what they are capable of. Advisors often only talk about the correct criteria to be used in decision making (the trade-offs), not the desirability of the client’s feasible actions. By discussing general ideas, industry trends, or similar cases seen before, the consultant provides useful information to the client: she makes the client’s private knowledge regarding the project at hand more nuanced. Nevertheless, the consultant may never learn exactly what effect her advice has had on the client’s objective function—in most cases, it is conceivable that only the client’s actions are observable (or contractible). This is the type of potential information transmission that we attempt to model in the paper.

The literature on experts and advisors usually model the relationship between an advisor and her client as a cheap talk game. The advisor has unverifiable private information (a signal that she is aware of), and the question is how precisely she can reveal it if the interests of the parties are not perfectly aligned. We respectfully depart from this literature by assuming that the expert does not know what effect her signal will have on the client’s action.

In earlier work (Eső and Szentes (2002)) we analyze the auction design problem where the seller-designer can disclose, without observing, private signals to the buyers that refine their initial private valuation-estimates for the good being sold. In that paper we characterize the revenue-maximizing selling mechanism and show that in the optimal mechanism the seller discloses all available signals (which only the buyers can observe) and attains the same revenue as if she could directly observe the realizations of these signals. This result is similar to the one we obtain in the

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2“I am your idea,” the recent campaign of Accenture (a consultancy), intends to advertise the firm’s “ability to act as a catalyst” to “bring [clients’] ideas to life.” (The quotes are taken from various press releases of Accenture at www.accenture.com.)
present paper, where the consultant obtains the same profit as if she could observe the effect of her signal on the client’s valuation for the project.

The analysis of the optimal contract in our model, it turns out, is equivalent to that of a Principal-Agent model (with specific quasilinear utility functions for both parties) where the value of the Agent’s outside option depends on his type. Such models have been studied in the literature by Lewis and Sappington (1989), Klibanoff and Morduch (1995), Maggi and Rodriguez (1995), among others, and most generally by Jullien (2000), see also the references therein. None of the above treatments of Principal-Agent problems with type-dependent reservation utility applies directly in our framework, but the solutions exhibit certain common features. We will comment more on this connection as we expose the model.

The paper is structured as follows. In the next section, we outline the model and introduce the necessary notation. In Section 3, we derive the optimal contract for the consultant. In Section 4, we compare the results with those obtained in a “benchmark” case, where the consultant can observe the signal that she can release. We show that her payoff is the same in either optimal contract. Finally, in Section 5, we conclude.

2 The Model

2.1 The Environment

There are two strategic, risk neutral actors in the model: a consultant (she) and her client (he). The client can undertake a project at cost \( r \), which will generate an ex post monetary benefit \( V \). The cost, \( r \in \mathbb{R} \), is deterministic and commonly known. On the other hand, \( V \) is random whose expected value, \( v \), is the client’s private information. In other words, the client only has an imperfect but unbiased estimate of the project’s actual profitability, and this estimate is his private information.

From the viewpoint of other parties (including the modeler) \( v \) is drawn according

\[ \begin{align*}
3 & \text{The client can undertake the project without asking the consultant for advice. If his original value-estimate (type) is below the project’s cost then the client’s outside option is worth zero. If his estimate exceeds the project’s cost then the client’s outside option is the project’s net profit, which is increasing in his type.}
4 & \text{For example, our setup does not satisfy Jullien’s (2000) homogeneity condition, and in the optimal contract certain agent- (or client-)types are excluded.}
\end{align*} \]
to a distribution $F$ from the unit interval.\(^5\) We will assume that $F$ has a positive density on $[0, 1]$, denoted by $f$, which is twice differentiable and logconcave (i.e., $d^2 \ln f(v)/dv^2 \leq 0$). This is an important, though standard, assumption in the literature on contracting with incomplete information. It implies (among other things) that $F$ satisfies certain monotone hazard rate conditions, which we will rely on (and point out) throughout the paper. Most widely used density functions satisfy logconcavity (see Bagnoli and Bergstrom (1989)).

We assume that the consultant can disclose to the client, without the consultant actually observing, a signal $s = V - v$, the difference between the actual realization of $V$ and the client’s expectation. It is important to understand that this assumption—that the sender of $s$ does not observe $s$ while the receiver does—is just the way one can model the situation where the consultant is unaware of the effect of her information on the client’s value-estimate. We assume that $s$ and $v$ are statistically independent; that is, the noise in the client’s value estimate is unrelated to the estimate itself. In other words, the client’s original private signal, $v$, concerns his valuation only; the client has no private information regarding the precision of $s$. Let $G$ denote the cdf corresponding to the distribution of $s$, on which we impose no condition except that it has full support on $(-\infty, +\infty)$.\(^6\)

What is the “value” of the consultant’s services to the client? Clearly, the closer $v$ is to the cost of undertaking the project ($r$), the more valuable it is for the client to know $V$ precisely. Formally: If the consultant releases $s$ to the client then he will undertake the project if and only if $v + s \geq r$, and his profit becomes

$$
\int_{r-v}^{\infty} (v + s - r) dG(s).
$$

(1)

When $v \leq r$, the client would not undertake the project without knowing $s$, which would then yield zero profit; therefore, for him the value of knowing $s$ is exactly (1). When $v \geq r$, the client would undertake the project without the consultant, and

\(^5\)The normalization of the project’s expected gross profit, $v$, to fall between 0 and 1 is innocuous as $r$ can be either positive, negative, or zero. All that this assumption implies is that the project’s (random) net expected profit is bounded.

\(^6\)Intuitively, the full support assumption ensures that no realization of $V$ (small or large) can be excluded with probability 1 given a particular estimate $v$. This assumption is made solely for ease of exposition. All our results go through (with more cumbersome notation) if the support of the distribution of $s = V - v$ is not the whole real line (details are available upon request).
his expected profit would be \( v - r \). By contracting with the consultant, his payoff becomes again (1), therefore his gain from knowing \( s \) is \( \int_{r-v}^{\infty} (v + s - r) dG(s) - (v - r) \). The client’s willingness to pay for the consultant’s information is,

\[
w(v) = \begin{cases} 
\int_{r-v}^{\infty} (v + s - r) dG(s) & \text{if } v \leq r \\
\int_{r-v}^{\infty} (v + s - r) dG(s) - (v - r) & \text{if } v \geq r
\end{cases}
\]

(2)

This function is strictly increasing for \( v < r \) and strictly decreasing for \( v > r \).

We assume that the release of information costs the consultant \( K \geq 0 \) in monetary terms. However, we will also assume that \( K \) is less than the “value” of the consultant’s information to any client type, that is, \( w(0) \geq K \) and \( w(1) \geq K \). This assumption means that it is always socially desirable that the consultant release her information to the client. We will see in Section 3 that in the optimal (second-best) contract, information transmission is socially inefficient unless \( K = 0 \) (in which case the consultant serves all types of the client).

The simplest situation that corresponds to the model’s assumptions is where the client has the opportunity to buy into a contract (or buy a good) at a reserve price \( r \). (This reserve price need not be positive.) The realized gross value of the contract or good will be \( V \), of which the client only has an estimate, \( v \). However, the consultant can help the client learn the value of the contract or good precisely without her actually learning anything about \( v \) or \( s = V - v \). Finally, we allow for the possibility that the consultant’s services are costly to provide, i.e., \( K \geq 0 \).

The willingness to pay function in (2), together with the distribution of \( v \), could be used to compute the monopoly price for the consultant’s services, that is, an optimal flat fee that she could charge for disclosing (without observing) her information. However, the purpose of this paper is to investigate what (how much more) can be done when the client’s action as to whether he undertakes the project is contractible. We will see that flat-fee contracts, in general, are not optimal. In the remaining part of this section, we turn to the description of contracts between the consultant and her client when the client’s choice of undertaking his project is contractible but the consultant’s information is not.
2.2 Feasible Contracts

In the interim stage (when the client already knows \( v \)) the consultant can offer a contract to the client. Naturally, the terms of the contract cannot depend on the realization of \( s \); however, the client’s decision whether to undertake the project is contractible. After a contract is offered by the consultant, the client may accept or reject it. As a simplifying assumption, we will require that when the contract is accepted, the consultant has to disclose (without observing) \( s \) to the client.\(^7\) This assumption highlights that there is no moral hazard in our model: the consultant’s effort regarding the release of information is perfectly contractible (in fact, when hired, she releases \( s \) with probability one). If the contract is rejected then the client has to decide whether to undertake the project solely on the basis of \( v \) and \( r \).

The consultant’s contract can be represented by a menu of pairs of monetary transfers (by convention, transfers are to be paid by the client to the consultant, but may be negative); the first transfer is carried out if the client undertakes the project, and the second transfer if he does not. Recall that if the client agrees to accept one of the menu items then the consultant reveals \( s \) to him.

Alternatively, we can represent a contract by a direct mechanism \( \{A, p, c\} \), where \( A \subseteq \[0,1\] \) and \( p, c : A \to \mathbb{R} \). In this type of mechanism, first the client reports his value-estimate to the consultant. If the reported \( v \) belongs to set \( A \) then the client pays a fee \( c(v) \) to the consultant (transfers may be negative) and the consultant reveals \( s \) to him; then, the client pays an additional premium \( p(v) \) in case he undertakes the project (this transfer may be negative as well). If \( v \) does not belong to \( A \) then no contract is signed, and the client decides whether to undertake the project without learning \( s \).

A contract \( \{A, p, c\} \) is incentive compatible if all \( v \in A \) are weakly better off reporting their own type than reporting \( v' \neq v \) (misreporting to a participating \( v' \in A \), or not participating by reporting \( v' \notin A \)), and all \( v \notin A \) are weakly better off not participating than reporting any type \( v' \in A \). For ease of exposition, we will not distinguish participation constraints (for \( v \in A \)) from incentive compatibility

\(^7\)If the consultant can costlessly release her signal, i.e. if \( K = 0 \), then there is no loss of generality from assuming that \( s \) is always disclosed when the contract is accepted. If \( K > 0 \) then this simplifying assumption is equivalent to forbidding the consultant to use mixed actions. Note that in any case, the consultant can choose to offer contracts that are not accepted by some types of the client, so disclosure of \( s \) need not always occur.
constraints, and include all these conditions in incentive compatibility.

In order to give a formal definition of incentive compatibility, introduce

\[
\pi(v, v') = \int_{r+p(v')-v}^{\infty} (v + s - p(v') - r) dG(s) - c(v'),
\]

where \( v \in [0, 1] \) and \( v' \in A \). That is, \( \pi(v, v') \) is the profit of the client with initial estimate (type) \( v \) pretending to have type \( v' \in A \). Denote the indirect profit function of type \( v \) in mechanism \( \{A, p, c\} \) by

\[
\Pi(v) = \begin{cases} 
\pi(v, v) & \text{if } v \in A \\
\max\{0, v - r\} & \text{if } v \notin A
\end{cases}.
\]

Then, a mechanism \( \{A, p, c\} \) is incentive compatible (and exactly all \( v \in A \) participate), if and only if,

\[
\Pi(v) \geq \max\{\pi(v, v'), 0\} \quad \text{for all } v \in [0, 1] \text{ and } v' \in A.
\]

In the next section, we derive the consultant’s optimal contract, that is, we find an incentive compatible mechanism \( \{A, p, c\} \) that maximizes the consultant’s ex-ante expected profits.

### 3 The Consultant’s Optimal Contract

We turn to the derivation of the consultant’s optimal contract. The consultant’s mechanism design problem is that of finding an incentive compatible mechanism \( \{A, p, c\} \) that maximizes the consultant’s ex-ante expected profit.\(^8\) We will first characterize all incentive compatible mechanisms, i.e., feasible contracts between the consultant and her client. Then we derive the solution to the consultant’s problem in special cases (Subsection 3.2). This will provide the foundation for the general case, \( r \in \mathbb{R} \), which we will do in Subsection 3.3. At the end of the section we illustrate our findings by numerical examples.

\(^8\)Note that the choice of the set \( A \) (the set of types that the consultant will contract with) is an interesting and non-trivial part of the consultant’s problem. The consultant may gain by excluding certain buyer types from receiving additional information.
3.1 Incentive Compatible Mechanisms

Let $X(v)$ denote the probability that client type $v$ (reporting his type truthfully) undertakes the project in mechanism $\{A, p, c\}$. Clearly, if $v \notin A$ then he undertakes the project if and only if $v \geq r$, so $X(v) = 1_{v \geq r}$, where $1$ is the indicator function. If $v \in A$ then the client undertakes the project if and only if $v + s \geq r + p(v)$, hence $X(v) = \Pr(v + s - r - p(v) \geq 0)$. We conclude that

$$X(v) = \begin{cases} 1 - G(r + p(v) - v) & \text{if } v \in A \\ 1_{v \geq r} & \text{if } v \notin A \end{cases}.$$  

(6)

In the following lemma we prove that the set $A$ must be an interval. (The proofs of lemmata are collected in the Appendix.)

**Lemma 1** If $\{A, p, c\}$ is incentive compatible then $A$ is an interval that contains $r$ whenever $r \in [0, 1]$.

In the interior of interval $A$, incentive compatibility implies that the first-order condition to the maximization problem $\max_{v' \in A} \pi(v, v')$ must hold at $v' = v$. If $c$ and $p$ are differentiable then this necessary condition becomes,

$$c'(v) = -p'(v)(1 - G(r + p(v) - v)).$$

(7)

It is easy to check that the necessary second-order condition is $p'(v) \leq 0$. Note, however, that these conditions rely on differentiability of $c$ and $p$, which would first have to be established. In the following lemma, we characterize all incentive compatible mechanisms without differentiability or other ad hoc assumptions.

**Lemma 2** A mechanism $\{A, p, c\}$ is incentive compatible if and only if $A$ is an interval with $r \in A$ whenever $r \in [0, 1]$, $p$ is weakly decreasing, and for all $v \in [0, 1],$

$$\Pi(v) = \Pi(0) + \int_0^v X(z)dz$$

(8)

and

$$\Pi(v) \geq \max\{0, v - r\},$$

(9)

where $\Pi$ is defined by (3)-(4), and $X$ by (6).
The characterization of incentive compatible mechanisms in Lemma 2 is useful because it expresses incentive compatibility in terms of constraints on \( p \) and \( \Pi \) (the premium that the client pays to the consultant when he undertakes the project and his indirect profit function, respectively). The premium function must be weakly decreasing in the client’s type, so higher types will face less distortion relative to the true cost of undertaking the project. The client’s profit function must start at \( v = 0 \) from a level \( \Pi(0) \geq 0 \), and must never go below his participation constraint: \( \Pi(v) \geq \max\{0, v - r\} \) for all \( v \in [0, 1] \). By incentive compatibility, the slope of \( \Pi \) at any point \( v \) must coincide with the probability that client type \( v \) undertakes the project, which is \( \Pr(v + s \geq r + p(v)) \equiv 1 - G(r + p(v) - v) \) for \( v \in A \) and \( 1_{v \geq r} \) for \( v \notin A \).

In the rest of the section, we will refer to incentive compatible mechanisms by the triplet \( \{A, p, \Pi\} \) such that the conditions of Lemma 2 are satisfied, that is, we replace \( c \) (the fee function) with \( \Pi \) (the client’s indirect profit function). Converting \( \{A, p, \Pi\} \) back into the form \( \{A, p, c\} \) is straightforward, and we will do that after deriving the optimal mechanism.

### 3.2 Preliminary Analysis of the Consultant’s Problem

We will now derive two formulae for the consultant’s expected payoff in any incentive compatible mechanism. These formulae enable us to find the consultant’s optimal contract in special cases (Lemma 3), which will be the basis of the derivation of the optimal contract in Subsection 3.3.

Consider a mechanism \( \{A, p, \Pi\} \) that satisfies the conditions of Lemma 2 (incentive compatibility). Pick any \( v^* \in [0, 1] \). For all \( v \in [0, v^*] \), define

\[
F_L(v) = \frac{F(v)}{F(v^*)},
\]

(10)

and for all \( v \in [v^*, 1] \), define

\[
F_H(v) = \frac{F(v) - F(v^*)}{1 - F(v^*)}.
\]

(11)

That is, \( F_L \) and \( F_H \) are the conditional cdf’s of the client’s valuation conditional on \( v \) falling into intervals \( L = [0, v^*] \), and \( H = [v^*, 1] \), respectively. Also, let \( f_L = dF_L/dv \)
and \( f_H = dF_H/dv \) be the corresponding conditional densities on the respective domains, and note that these densities are logconcave because \( F \) is too.

We will now characterize the consultant’s expected payoff in mechanism \( \{A, p, \Pi\} \) conditional \( v \) belonging to \( L \), and \( H \), respectively. We will continue to require incentive compatibility on the whole unit interval (and use the results of Lemmata 1 and 2), but compute the consultant’s profit as if clients with types \( v \notin L \) (or \( v \notin H \), respectively) did not exist and were not served.\(^9\)

First, we derive a formula for the consultant’s expected payoff conditional on \( v \in L \). The ex-ante expected profit of the client conditional on \( v \in L \) is,

\[
\int_0^{v^*} \Pi(v) dF_L(v) = \Pi(0) + \int_0^{v^*} \int_0^v X(z) dz dF_L(v) \\
= \Pi(0) + \int_0^{v^*} \int_z^v X(z) dF_L(v) dz \\
= \Pi(0) + \int_0^{v^*} (1 - F_L(z)) X(z) dz.
\]

The consultant’s expected surplus can be calculated as the difference between the social surplus and the client’s profit. The social surplus is \( E_s[(v + s - r) \mathbf{1}_{v+s \geq r+p(v)} - K] \) for \( v \in A \) and \( (v - r) \mathbf{1}_{v \geq r} \) for \( v \notin A \). Therefore, the consultant’s ex-ante expected surplus conditional on \( v \in L \) in mechanism \( \{A, p, \Pi\} \) is

\[
U = \int_{v \in A} E_s \left[ (v + s - r) \mathbf{1}_{v+s \geq r+p(v)} - K \right] dF_L(v) + \int_{v \notin A} (v - r) \mathbf{1}_{v \geq r} dF_L(v) \\
- \int_0^{v^*} \frac{1 - F_L(v)}{f_L(v)} X(v) dF_L(v) - \Pi(0)
\]

Using \( X(v) = E_s \left[ \mathbf{1}_{v+s \geq r+p(v)} \right] \) for \( v \in A \) and \( X(v) = \mathbf{1}_{v \geq r} \) for \( v \notin A \), we get

\[
U = \int_{v \in A} E_s \left[ \left( v + s - r - \frac{1 - F_L(v)}{f_L(v)} \right) \mathbf{1}_{v+s \geq r+p(v)} - K \right] dF_L(v) \\
+ \int_{v \notin A} \left( v - r - \frac{1 - F_L(v)}{f_L(v)} \right) \mathbf{1}_{v \geq r} dF_L(v) - \Pi(0).
\]

\(^9\)For the purposes of this subsection, one may think of \( v^* \) as one of the endpoints of the unit interval. In the proof of Theorem 1 (Subsection 3.3), we will need general formulas for the consultant’s expected payoff conditional on \( v \in [0, v^*] \) and \( v \in [v^*, 1] \) for an appropriate \( v^* \).
A = [v, \bar{v}]$, and (by the arguments used in Lemma 1) $r \in A$ whenever $r \in [0, v^*]$. Therefore,

$$U = \int_v^{\bar{v}} \int_{r+p(v)-v}^{\infty} \left[ \left( v + s - r - \frac{1 - F_L(v)}{f_L(v)} \right) dG(s) - K \right] dF_L(v)$$

$$+ \int_v^{v^*} \left( v - r - \frac{1 - F_L(v)}{f_L(v)} \right) dF_L(v) - \Pi(0). \quad (12)$$

Based on this expression we can determine the consultant’s optimal mechanism conditional on $v \in L$ in certain cases. To do this, we will choose $p, \underline{v}, \bar{v}$ and $\Pi(0)$ “heuristically” to maximize (12), and then check whether the remaining conditions of incentive compatibility hold (as in Lemma 2). Set $\Pi(0) = 0$ and hence minimize the last (negative) term. For any given values $\underline{v}$ and $\bar{v}$, the first term of (12) is maximized by choosing $p(v) = (1 - F_L(v))/f_L(v)$ for all $v \in L$. Finally, set $\bar{v} = v^*$, and define $\underline{v}$ so that

$$\int_{r+1-F_L(v)}^{\infty} \left( \frac{v + s - r - \frac{1 - F_L(v)}{f_L(v)}}{1 - F_L(v)} \right) dG(s) \geq K, \quad \underline{v} \geq 0, \text{ with c.s.}, \quad (13)$$

where “c.s.” stands for “complementary slackness.” (If one of the two inequalities holds strictly, the other one holds as an equality.) We claim that for $v \in L$,

$$\int_{r+1-F_L(v)}^{\infty} \left( \frac{v + s - r - \frac{1 - F_L(v)}{f_L(v)}}{1 - F_L(v)} \right) dG(s) \geq K \quad (14)$$

if and only if $v \in [\underline{v}, v^*]$, so the consultant makes exactly those client types participate for which her gain is non-negative (according to the first term of equation 12). To see this, note that the left-hand side is continuous and increasing in $v$. Moreover, using (2), at $v = v^*$ it equals

$$\int_{r-v^*}^{\infty} (v^* + s - r) dG(s) = w(v^*) + 1_{v^*>r}(v^* - r) > w(v^*),$$

and $w(v^*) \geq K$ by assumption. Note that by logconcavity of $f_L$, the hazard rate of the distribution, $f_L/(1 - F_L)$, is weakly increasing,\(^\text{10}\) therefore $p(v)$ is weakly

\(^{10}\)See Prékopa (1973), cited in Fudenberg and Tirole (1991), chapter 7.
decreasing. The only remaining condition of incentive compatibility is that the
client’s profit in the mechanism, given by (8) or
\[ \Pi(v) = \int_v^0 \left[ 1 - G \left( r + \frac{1 - F_L(z)}{f_L(z)} - z \right) \right] dz, \]
has to meet or exceed the value of his outside option, \( \max\{0, v - r\} \). Since \( X(z) \in [0,1] \) for all \( z \in [0,1] \), it is sufficient to check whether \( \Pi(v^\ast) \geq \max\{0, v^\ast - r\} \). We conclude that if
\[ \int_v^{v^\ast} \left[ 1 - G \left( r + \frac{1 - F_L(v)}{f_L(v)} - v \right) \right] dv \geq \max\{0, v^\ast - r\} \quad (15) \]
then the mechanism characterized by \( p = (1 - F_L)/f_L \), \( A = [v, v^\ast] \), and (13) is optimal for the consultant conditional on \( v \in L \).

![Figure 1: Client’s profit in the optimal mechanism, \( p = (1 - F)/f \)](image)

The solution is illustrated in Figure 1, where the client’s profit and the value of his outside option are plotted against his type for some \( r \) such that \( v^\ast = 1 \) and (15) holds. Note that when \( v^\ast = 1 \) and \( r \geq 1 \), (15) holds. Therefore, we have found the (unconditionally) optimal mechanism for the special case when \( r \geq 1 \).

Now we derive another expression for the consultants expected payoff, this time we condition on \( v \in H = [v^*, 1] \). Suppose \( r < 1 \) throughout the derivation (as we already know the optimal mechanism for \( r \geq 1 \)). The client’s ex-ante expected
profit conditional on $v \in H$ is
\[
\int_{v^*}^{1} \Pi(v) dF_H(v) = \Pi(1) - \int_{v^*}^{1} \int_{v^*}^{1} X(z) dz dF_H(v) \\
= \Pi(1) - \int_{v^*}^{1} \int_{v^*}^{v} X(z) dF_H(v) dz \\
= \Pi(1) - \int_{v^*}^{1} F_H(z) X(z) dz.
\]

The consultant’s expected payoff conditional on $v \in H$ is the difference between social surplus and the client’s profit conditional on $v \in H$,
\[
U = \int_{v \in A} E_s \left[ (v + s - r) 1_{v + s \geq r + p(v)} - K \right] dF_H(v) + \int_{v \notin A} (v - r) 1_{v \geq r} dF_H(v) \\
+ \int_{v^*}^{1} F_H(v) X(v) dF_H(v) - \Pi(1),
\]
which can be rewritten, using $X(v) = E_s \left[ 1_{v + s \geq r + p(v)} \right]$ for $v \in A$ and $X(v) = 1_{v \geq r}$ for $v \notin A$, as
\[
U = \int_{v \in A} E_s \left[ \left( v + s - r + \frac{F_H(v)}{f_H(v)} \right) 1_{v + s \geq r + p(v)} - K \right] dF_H(v) \\
+ \int_{v \notin A} \left( v - r + \frac{F_H(v)}{f_H(v)} \right) 1_{v \geq r} dF_H(v) - \Pi(1).
\]

Then, by $A = [\underline{v}, \bar{v}]$ and $r \in A$ whenever $r \in [v^*, 1]$, we get
\[
U = \int_{\underline{v}}^{\bar{v}} \int_{r + p(v) - v}^{\infty} \left[ \left( v + s - r + \frac{F_H(v)}{f_H(v)} \right) dG(s) - K \right] dF_H(v) \\
+ \int_{\underline{v}}^{1} \left( v - r + \frac{F_H(v)}{f_H(v)} \right) dF_H(v) - \Pi(1). \quad (16)
\]

This expression can be used to solve the consultant’s contract design problem (conditional on $v \in H$) in certain cases. We will again choose $p$, $\underline{v}$, $\bar{v}$ and $\Pi(1)$ “heuristically” to maximize (16), and then check incentive compatibility of the resulting mechanism. First, set the profit of the highest client type equal to the value of this type’s outside option, $\Pi(1) = 1 - r$ (recall that $r < 1$). For any given values $\underline{v}$ and $\bar{v}$, the first term of (16) is maximized by choosing $p(v) = -F_H(v)/f_H(v)$. 

14
Note that by logconcavity of $f_H$, the ratio $f_H/F_H$ is weakly decreasing,\(^1^1\) therefore $p(v)$ is weakly decreasing. It is then optimal to set $v = v^*$ (i.e., as low as possible conditional on $v \in H$), because

$$\int_{r - \frac{F_H(v)}{f_H(v)}}^{\infty} \left( v + s - r + \frac{F_H(v)}{f_H(v)} \right) dG(s) - K$$

is strictly increasing in $v$ and non-negative. (Non-negativity follows because even at $v = 0$ the above expression is $\int_r^{\infty} (s - r) dG(s) - K \equiv w(0) \geq 0$.) Finally, in order to find the optimal $\bar{v}$, rewrite (16) as

$$U = \int_{v^*}^{\bar{v}} \int_{-\infty}^{r - \frac{F_H(v)}{f_H(v)}} \left[ - \left( v + s - r + \frac{F_H(v)}{f_H(v)} \right) dG(s) - K \right] dF_H(v)$$

$$+ \int_{v^*}^{1} \left( v - r + \frac{F_H(v)}{f_H(v)} \right) dF_H(v) - \Pi(1).$$

Now the double integral on the first line is strictly decreasing in $\bar{v}$ for $\bar{v} \geq r$ (which holds because $r < 1$), therefore it is maximized by choosing $\bar{v}$ such that

$$\int_{-\infty}^{\bar{v}} \frac{F_H(v)}{f_H(v)} \left( r - \frac{F_H(v)}{f_H(v)} - \bar{v} - s \right) dG(s) \geq K, \quad \bar{v} \leq 1, \text{ with c.s.} \quad (17)$$

The only condition of incentive compatibility that remains to be checked is that the client’s profit in the mechanism, given by $\Pi(v) = \bar{v} - r - \int_v^{\bar{v}} X(z) dz$ or

$$\Pi(v) = \bar{v} - r - \int_v^{\bar{v}} \left[ 1 - G \left( r - \frac{F_H(z)}{f_H(z)} - z \right) \right] dz,$$

must not fall below the value of the client’s outside option, $\max\{0, v - r\}$. We conclude that if

$$\bar{v} - r - \int_{v^*}^{\bar{v}} \left[ 1 - G \left( r - \frac{F_H(v)}{f_H(v)} - v \right) \right] dv \geq \max\{0, v^* - r\} \quad (18)$$

then the mechanism characterized by $p = -F_H/f_H$, $A = [v^*, \bar{v}]$, and (17), is optimal for the consultant. This solution is illustrated in Figure 2. Since $X(z) \in [0, 1]$ for

all \( z \in [v^*, 1] \), (18) automatically holds when \( r \leq 0 \) and \( v^* = 0 \), therefore we have found the (unconditionally) optimal mechanism for the special case \( r \leq 0 \).

![Figure 2: Client’s profit in the optimal mechanism, \( p = -F/f \)](image)

We summarize the solutions that we have found so far in the following Lemma. We will use these results in the proof of the general theorem in the next subsection.

**Lemma 3** If (15) holds (which is the case for \( r \geq 1 \) and \( v^* = 1 \)), then the consultant’s optimal contract conditional on \( v \in [0, v^*] \) is characterized by \( p = (1 - F)/f \), \( \Pi(0) = 0 \), and \( A = [v, v^*] \) where \( v \) satisfies (13).

If \( r < 1 \) and (18) hold (which is the case for \( r \leq 0 \) and \( v^* = 0 \)) then the consultant’s optimal contract conditional on \( v \in [v^*, 1] \) is characterized by \( p = -F/f \), \( \Pi(1) = 1 - r \), and \( A = [v^*, \bar{v}] \) where \( \bar{v} \) solves (17).

The solutions to the consultant’s contract design problem when \( r \geq 1 \) (with \( v^* = 1 \)) and when \( r \leq 0 \) (with \( v^* = 0 \)), respectively, are quite insightful and interesting on their own. When \( r \geq 1 \), the client is originally “pessimistic” in the sense that his estimate regarding the profitability of the project is always non-positive. He would never undertake the project without the consultant’s advice (i.e., without learning that \( s \) is sufficiently large and positive). Note that in this case, the client has to pay the consultant if he decides to undertake the project: \( p(v) > 0 \) for all \( v < 1 \). On the other hand, when \( r \leq 0 \), the client’s original profitability estimate is always “optimistic,” \( v \geq r \) for all \( v \), and without the consultant’s advice he would always undertake the project. In this case, it is the consultant who pays the client in case he undertakes the project: \( p(v) < 0 \) for all \( v > 0 \). In other words, when
$r \leq 0$, the client pays more to the consultant if he does not undertake the project than if he does.

In both cases, the client has to make a net payment to the consultant when the consultant’s advice makes the client change his mind: if he undertakes the project while $v \leq r$ for all $v$, or if he does not undertake the project while $v \geq r$ for all $v$. Of course, in general (when $r \in \mathbb{R}$) the consultant will not know a priori which of the two actions of the client signify that he has changed his mind. In the next subsection, we see how the contract is structured in this case.

### 3.3 The Optimal Contract in the General Case ($r \in \mathbb{R}$)

We now complete the analysis of the consultant’s problem by deriving the terms of the optimal contract for any $r \in \mathbb{R}$ in general.

We will prove that in the optimal mechanism $p$ is of the form

$$p(v) = \frac{b - F(v)}{f(v)}$$

for some $b \in [0, 1]$. Note that when $r \geq 1$, we have already established $b = 1$, while for $r \leq 0$, we have found $b = 0$ (see Lemma 3).

We will also show that the consultant offers a contract to types between certain thresholds $v_b$ and $\bar{v}_b$, such that $v_b < r < \bar{v}_b$ whenever $r \in (0, 1)$. The optimal contract is structured so that the client is indifferent between his offer and his outside option at either endpoint of $A$, except possibly at one of the endpoints when that point is on the boundary of $[0, 1]$. Finally, in the optimal contract, the consultant is indifferent between excluding and including the boundary types ($v_b$ and $\bar{v}_b$) in the mechanism, whenever these boundary points are inside the unit interval. We depict the client’s indirect profit function in the optimal mechanism in a situation where $0 < v_b < \bar{v}_b < 1$ in Figure 3.

For all $b \in [0, 1]$, define $v_b$ and $\bar{v}_b$ that solve

$$\int_{-\infty}^{\infty} \left( r + \frac{b - F(v_b)}{f(v_b)} - v_b - s \right) dG(s) \geq K, \quad \bar{v}_b \leq 1, \quad \text{with c.s. (20)}$$

$$\int_{-\infty}^{\infty} \left( v_b + s - r - \frac{b - F(v_b)}{f(v_b)} \right) dG(s) \geq K, \quad v_b \geq 0, \quad \text{with c.s., (19)}$$
Lemma 4 For all \( b \in [0, 1] \), there exist (unique) \( v_b < \bar{v}_b \) such that (19) and (20) hold. Moreover, \( F(v_b) \leq b \leq F(\bar{v}_b) \), and both \( v_b \) and \( \bar{v}_b \) are continuous and weakly increasing in \( b \) with \( v_0 = 0 \) and \( \bar{v}_1 = 1 \).

The particular value for \( b \) that will be used in the optimal mechanism is determined as follows:

\[
\text{If } \int_{v_b}^{1} \left[ 1 - G \left( r + \frac{1 - F(v)}{f(v)} - v \right) \right] dv \geq 1 - r \text{ then } b = 1; \text{ otherwise, } (21)
\]

\[
\text{if } \int_{0}^{v_b} \left[ 1 - G \left( r - \frac{F(v)}{f(v)} - v \right) \right] dv \leq \bar{v}_b - r \text{ then } b = 0; \text{ otherwise } (22)
\]

let \( b \) any solution to \( \bar{v}_b - r - \int_{v_b}^{v_b} \left[ 1 - G \left( r + \frac{b - F(v)}{f(v)} - v \right) \right] dv = 0. (23) \)

We claim that there exists \( b \in [0, 1] \) that satisfies (21)–(23). To see this, note that the right-hand side of (23) is continuous in \( b \). If (21) does not hold then this expression is positive at \( b = 1 \); if (22) does not hold then it is negative at \( b = 0 \). Therefore, if neither (21) nor (22) holds then there exists a \( b \in (0, 1) \), not necessarily unique, that satisfies (23).

Theorem 1 Define \( b \) by (21)–(23) and let \( v_b \) and \( \bar{v}_b \) solve (19)–(20). In the consultant’s optimal contract, client types \( v \in A = [v_b, \bar{v}_b] \) are offered a contract in which

\[
p(v) = \frac{b - F(v)}{f(v)}. \quad (24)
\]
The client’s profit function is, for all \( v \in [0, 1] \),
\[
\Pi(v) = \Pi(0) + \int_0^v X(z) \, dz,
\]  
(25)
where \( X \) is defined by (6). Furthermore, if \( v_b > 0 \) or \( \bar{v}_b = 1 \) then \( \Pi(v) = 0 \) for all \( v \in [0, v_b] \), while if \( v_b = 0 \) or \( \bar{v}_b < 1 \) then \( \Pi(v) = v - r \) for all \([\bar{v}_b, 1]\).

Proof. If \( r \geq 1 \) or \( r \leq 0 \) then the validity of the theorem is established in Lemma 3. In the rest of the proof, suppose \( r \in (0, 1) \). We will consider three cases: \( b = 1 \), \( b = 0 \), and \( b \in (0, 1) \).

If \( b = 1 \) then \( v_1 \leq r \) (because the left-hand side of the inequality in (21) does not exceed \( 1 - v_1 \)), and \( \bar{v}_1 = 1 \) by Lemma 4. Let \( v^* = 1 \), hence \( F_L \equiv F \). Note that (13) is equivalent to (19) at \( b = 1 \), therefore \( v_1 = v \) as defined in (13). Condition (15) is satisfied because (21) holds, therefore the first part of Lemma 3 applies: in the optimal mechanism, \( p = (1 - F)/f \) as in (24), \( A = [v_1, 1] \), and \( \Pi(0) = 0 \), which implies \( \Pi(v) = 0 \) for all \( v \in [0, v_1] \). By Lemma 2, (25) must hold to ensure incentive compatibility. This completes the proof for the case \( b = 1 \).

If \( b = 0 \) then \( v_0 = 0 \) by Lemma 4, and \( \bar{v}_0 \geq r \) because the left-hand side in the inequality of (22) is non-negative. Let \( v^* = 0 \), hence \( F_H \equiv F \). Note that (17) is equivalent to (20) at \( b = 0 \), therefore \( \bar{v}_0 = \bar{v} \) as defined in (17). Condition (18) is satisfied because (22) holds, therefore the second part of Lemma 3 applies: in the optimal mechanism, \( p = -F/f \) as in (24), \( A = [0, v_0] \), and \( \Pi(1) = 1 - r \), which implies \( \Pi(v) = v - r \) for all \( v \in [\bar{v}_0, 1] \). By Lemma 2, (25) must hold to ensure incentive compatibility, and we are done with the case \( b = 0 \) as well.

Finally, suppose that \( b \in (0, 1) \). Note that \( v_b < r < \bar{v}_b \) because \( v_1 < r < \bar{v}_0 \) and both \( v_b \) and \( \bar{v}_b \) are weakly increasing in \( b \). If we set \( p = (b - F)/f \), as proposed in the statement of the theorem, then
\[
X(v) = \begin{cases} 
0 & \text{for } v < v_b \\
1 - G \left( r + \frac{b - F(b)}{f(b)} - v \right) & \text{for } v \in [v_b, \bar{v}_b] \\
1 & \text{for } v > \bar{v}_b
\end{cases}
\]
By setting \( \Pi(0) = 0 \), we have \( \Pi(v) = 0 \) for all \( v \in [0, v_b] \). By Lemma 2, equation (25) must hold for all \( v \) to ensure incentive compatibility. Then, by (23), we have
\[ \Pi(\bar{v}_b) = \bar{v}_b - r, \] which implies \( \Pi(v) = v - r \) for all \( v \in [\bar{v}_b, 1] \), moreover,

\[
\text{for all } v \in (\underline{v}_b, \bar{v}_b), \quad \Pi(v) \geq \max\{0, v - r\}. \tag{26}
\]

Therefore, the proposed mechanism \( \{A, p, \Pi\} \) is incentive compatible by Lemma 2. The only remaining question is whether it is optimal for the consultant.

Define \( v^* = F^{-1}(b) \), that is, \( F(v^*) = b \). Since \( F(\underline{v}_b) \leq b < F(\bar{v}_b) \), we have \( \underline{v}_b \leq v^* \leq \bar{v}_b \). Note that by (10)–(11) and \( b = F(v^*) \),

\[
\frac{b - F(v)}{f(v)} = \begin{cases} \frac{1 - F_L(v)}{f_L(v)} & \text{for all } v \in [0, v^*] \\ \frac{-F_H(v)}{f_H(v)} & \text{for all } v \in [v^*, 1] \end{cases}.
\]

Therefore, (13) is equivalent to (19), and similarly, (17) is equivalent to (20), hence \( \underline{v}_b = \underline{v} \) and \( \bar{v}_b = \bar{v} \). By \( \Pi(\underline{v}_b) = 0 \), \( \Pi(\bar{v}_b) = \bar{v}_b - r \) (25), and (23),

\[
\Pi(v^*) = \int_{\underline{v}}^{v^*} \left[ 1 - G \left( r + \frac{1 - F_L(v)}{f_L(v)} - v \right) \right] dv
= \bar{v} - r - \int_{v^*}^{\bar{v}} \left[ 1 - G \left( r - \frac{F_H(v)}{f_H(v)} - v \right) \right] dv. \tag{27}
\]

By (26),

\[
\Pi(v^*) \geq \max\{0, v^* - r\}. \tag{28}
\]

But then, by (27) and (28), (15) holds, and the first part of Lemma 3 applies: setting \( p = (1 - F_L)/f_L = (b - F)/f \) is optimal conditional on \( v \in [0, v^*] \). Similarly, by (27) and (28), (18) holds, and the second part of Lemma 3 applies: setting \( p = -F_H/f_H = (b - F)/f \) is optimal conditional on \( v \in [v^*, 1] \). Since the proposed mechanism is optimal conditional on \( v \in [0, v^*] \) and conditional on \( v \in [v^*, 1] \), it is unconditionally optimal. \( \blacksquare \)

**Remark 1.** When information disclosure is costless, \( K = 0 \), it is easy to see from (19)–(20) that the consultant will contract with all types of the client, that is, \( \underline{v}_b = 0 \) and \( \bar{v}_b = 1 \), for all \( b \in [0, 1] \).

The up-front fee-schedule, \( c \), can be easily computed based on (24) and (25). Note that optimal \( p \) is differentiable, hence the first-order condition (7) must hold for all
\( v \in A \). Therefore, \( c \) is determined by this differential equation with a boundary condition for either \( c(v_b) \) or \( c(\bar{v}_b) \) (whichever is more convenient). If \( \Pi(v_b) = 0 \), which is the case if \( v_b > 0 \) or \( \bar{v}_b = 1 \), then \( c(v_b) = w(v_b) \), while if \( \Pi(\bar{v}_b) = \bar{v}_b - r \), which is the case if \( \bar{v}_b < 1 \) or \( v_b = 0 \), then \( c(\bar{v}_b) = w(\bar{v}_b) \).

Since the resulting fee function, \( c \), is non-decreasing, while \( p \) is non-increasing, the implicit relation between \( c \) and \( p \) is inverse: a lower premium (paid in case the project is undertaken, chosen by better client types) requires the payment of a higher up-front fee, and vice versa. We quantify this relationship in numerical examples in the next subsection.

### 3.4 A Numerical Example

Assume that \( v \) is uniform on \([0, 1]\), that is, \( F(v) = v \) and \( f(v) = 1 \) on the domain. Let \( r \in (0, 1) \), and, for simplicity, set \( K = 0 \). Suppose that \( s \) is drawn from a uniform distribution on \([-\varepsilon, \varepsilon]\), with \( \varepsilon > 0 \), so

\[
G(s) = \begin{cases} 
0 & \text{if } s < -\varepsilon \\
(s + \varepsilon)/(2\varepsilon) & \text{if } -\varepsilon \leq s \leq \varepsilon \\
1 & \text{if } s > \varepsilon 
\end{cases} .
\] (29)

The density, \( g \), is zero outside \([-\varepsilon, \varepsilon] \), and equals \( 1/(2\varepsilon) \) on it.

According to Theorem 1, in the consultant’s optimal contract,

\[
p(v) = \frac{b - F(v)}{f(v)} = b - v,
\]

where \( b \) can be calculated by solving

\[
\int_0^1 G(r + b - 2v)dv = r,
\] (30)

and “snapping” the resulting value of \( b \) to 0, or 1, whenever it falls below 0, or above 1, respectively. In order to find the value of \( b \), first define \( \underline{\alpha} = (r + b - \varepsilon)/2 \) and \( \bar{\alpha} = (r + b + \varepsilon)/2 \), the two thresholds of \( v \) where \( r + b - 2v \) equals \( -\varepsilon \) and \( \varepsilon \), respectively.
respectively. Since \(G(r + b - 2v)\) is monotone decreasing in \(v\), we have

\[
G(r + b - 2v) = \begin{cases} 
1 & \text{if } v \in [0, \alpha) \\
(r + b + \varepsilon - 2v)/(2\varepsilon) & \text{if } v \in [\alpha, \bar{\alpha}] \\
0 & \text{if } v \in (\bar{\alpha}, 1]
\end{cases}.
\]

Case 1: \(r < \varepsilon/2\). Suppose \(r + b < \varepsilon\). Then \(\bar{\alpha} < 0 < \bar{\alpha} < 1\), and

\[
\int_0^1 G(r + b - 2v)dv = G(0)\bar{\alpha} = \frac{(r + b)^2}{2\varepsilon}.
\]

Equation (30) is satisfied by \(b = \sqrt{8\varepsilon} - \varepsilon - r\). (Note that if \(r < \varepsilon/2\) then \(b < \sqrt{4\varepsilon^2} - \varepsilon - r = \varepsilon - r\), so indeed, \(r + b < \varepsilon\), as assumed.) It can be checked that \(b \geq 0\) if and only if \(r \geq (\sqrt{2} - 1)^2\varepsilon\).

Case 2: \(r \in [\varepsilon/2, 1 - \varepsilon/2]\). Suppose \(r + b \in [\varepsilon, 2 - \varepsilon]\). Then \(0 \leq \alpha < \bar{\alpha} \leq 1\), and

\[
\int_0^1 G(r + b - 2v)dv = \alpha + \frac{\varepsilon}{2} = \frac{r + b}{2}.
\]

Equation (30) is satisfied by \(b = r\), and indeed, \(r + b \in [\varepsilon, 2 - \varepsilon]\) as assumed.

Case 3: \(r > 1 - \varepsilon/2\). Suppose \(r + b > 2 - \varepsilon, r > \varepsilon/2\). Then \(0 < \alpha < 1 < \bar{\alpha}\), and

\[
\int_0^1 G(r + b - 2v)dv = 1 - \frac{(1 - \alpha)(1 - G(1))}{2} = 1 - \frac{(2 + \varepsilon - r - b)^2}{8\varepsilon}.
\]

Equation (30) is satisfied by \(b = 2 + \varepsilon - r - \sqrt{8(1 - r)\varepsilon}\). (Note that by \(r > 1 - \varepsilon/2\), i.e., \((1 - r) < \varepsilon/2\), we have \(b > 2 + \varepsilon - r - \sqrt{4\varepsilon^2} = 2 - \varepsilon - r\).) It can be checked that \(b \leq 1\) if and only if \(r \leq 1 - (\sqrt{2} - 1)^2\varepsilon\).

To summarize: if \(r < (\sqrt{2} - 1)^2\varepsilon\) then \(b = 0\); if \(r > 1 - (\sqrt{2} - 1)^2\varepsilon\) then \(b = 1\); otherwise

\[
b = \begin{cases} 
\sqrt{8\varepsilon} - \varepsilon - r & \text{if } r \in [(\sqrt{2} - 1)^2\varepsilon, \varepsilon/2] \\
r & \text{if } r \in [\varepsilon/2, 1 - \varepsilon/2] \\
2 + \varepsilon - r - \sqrt{8(1 - r)\varepsilon} & \text{if } r \in [1 - \varepsilon/2, 1 - (\sqrt{2} - 1)^2\varepsilon]
\end{cases}.
\]

From now on, let us focus on the case where \(r\) is inside the unit interval and \(\varepsilon\) is small relative to \(r\), specifically \(r \in [\varepsilon, 1 - \varepsilon]\), so in the optimal contract of our example \(p(v) = r - v\). Then, we can easily determine the upfront fee in the optimal
contract, $c(v)$. For $v = 0$, we need $\Pi(0) = 0 \iff c(0) = \int_r^\infty (s - b - r)g(s)ds$. Since $r \geq \varepsilon$, the integral is empty, so $c(0) = 0$. For $v > 0$, we use (7), which can be rewritten as $c'(v) = 1 - G(2r - 2v)$. Straightforward integration of this equation, with $G$ defined in (29), yields,

$$c(v) = \begin{cases} 0 & \text{if } v < r - \varepsilon/2 \\ (v - r + \varepsilon/2)^2/(2\varepsilon) & \text{if } r - \varepsilon/2 \leq v \leq r + \varepsilon/2 \\ v - r & \text{if } r + \varepsilon/2 < v \end{cases}.$$

Note that client types $v < r - \varepsilon/2$ or $v > r + \varepsilon/2$ are essentially not served by the consultant, but all other types are.

Our example with uniformly distributed $v$ and small, uniformly distributed $\varepsilon$ (so that $r \in [\varepsilon, 1 - \varepsilon]$) illustrates the interesting relationship between the upfront fee, $c$, and the premium, $p$, in the optimal menu offered by the consultant. The upfront fee that client type $v$ is supposed to choose is strictly increasing in $v$. All client types that may benefit from knowing $s = V - v$ pay their fee and learn the realization of the shock. In exchange, the client agrees to paying a premium, which is decreasing in his type, in case he undertakes the project. Furthermore, this premium is positive for $v < r$ and negative for $v > r$. Therefore, in this particular example with $r \in [\varepsilon, 1 - \varepsilon]$, when the undecided client undertakes the project, he pays the consultant “extra” whenever learning the value of the shock changes his mind, and conversely, the consultant pays him back whenever it does not.\footnote{We made the same observation in the general model for $r > 1$, and $r < 0$, when we found that $p = -F/f < 0$, and $p = (1 - F)/f > 0$, in these two cases respectively.} This happens in the optimal mechanism despite the fact that the consultant cannot observe directly whether her release of information actually changed the clients mind.

4 Main Result: Comparison with a Benchmark

In this section we derive the optimal contract when the consultant can in fact verify the realization of the shock, $s = V - v$, as she releases it (i.e., in case she decides to contract with the client). We will show that her utility is the \textit{same} as in the optimal contract of the previous section, where she could not observe $V - v$ but could only condition on the client’s action. This result (Theorem 2) is the main, surprising
result of the paper.

When the consultant can verify the value of $s$ (in case $s$ is released by her), we can represent a contract by an incentive compatible revelation mechanism consisting of a set $A^* \subseteq [0, 1]$, and functions $d^*: A \to \mathbb{R}$ and $x^*: A \times \mathbb{R} \to \{0, 1\}$. Here $A^*$ is the set of client types that the consultant is contracting with (i.e., releases $s$ to), $d(v)$ is a transfer from the client to the consultant, and $x^*(v, s)$ is the indicator of whether or not the client has to undertake the project.\textsuperscript{13} Incentive compatibility of $\{A^*, d^*, x^*\}$ means that no client type $v$ has an incentive to report $v' \neq v$ before learning $s$.

Given a mechanism $\{A^*, d^*, x^*\}$, introduce $X^*(v) = \int x^*(v, s)dG(s)$ for $v \in A^*$, and $X^*(v) = 1_{v \geq r}$ for $v \notin A^*$. Define

$$\pi^*(v, v') = \begin{cases} \int x^*(v', s)(v + s - r)dG(s) - d^*(v') & \text{if } v' \in A^* \\ \max\{v - r, 0\} & \text{if } v' \notin A^* \end{cases}$$

the deviation payoff to the client when he has type $v$ and reports $v'$. Let $\Pi^*$ the client’s indirect profit function in mechanism $\{A^*, d^*, x^*\}$, that is, $\Pi^*(v) = \pi^*(v, v)$, or

$$\Pi^*(v) = \begin{cases} \int x^*(v, s)(v + s - r)dG(s) - d^*(v) & \text{for } v \in A^* \\ \max\{0, v - r\} & \text{for } v \notin A^* \end{cases}$$

Incentive compatibility (and participation) immediately implies $\Pi^*(v) \geq \max\{0, v - r\}$ for all $v \in [0, 1]$. Using arguments similar to those in the proof of Lemma 1, it is easy to show that $A^*$ must be an interval that contains $r$ whenever $r \in [0, 1]$. In fact, one can prove the following counterpart to Lemma 2 for the case when the consultant can verify the information that she may release (the proof is similar to but simpler than that of Lemma 2).

**Lemma 5** Assume the consultant can verify $V - v$. A mechanism $\{A^*, d^*, x^*\}$ is incentive compatible if and only if $A^*$ is an interval with $r \in A^*$ whenever $r \in [0, 1]$, $X^*$ is weakly increasing, and for all $v \in [0, 1]$,

$$\Pi^*(v) = \Pi^*(0) + \int_0^v X^*(z)dz,$$

\textsuperscript{13}We will use “starred” symbols throughout this section to avoid confusion with notation used in the previous section.
\[ \Pi^*(v) \geq \max\{0, v - r\}. \quad (32) \]

The client’s ex-ante expected profit in an incentive compatible mechanism \( \{A^*, d^*, x^*\} \) can be written as

\[
\int_0^1 \Pi^*(v) dF(v) = \Pi^*(0) + \int_0^1 \int_0^v X^*(z) dz dF(v) = \Pi^*(0) + \int_0^1 X^*(z) dF(v) dz = \Pi^*(0) + \int_0^1 X^*(z)(1 - F(z)) dz.
\]

The consultant’s expected utility, which is the difference between the social surplus and the client’s profit, equals

\[
U^* = \int_{v \in A^*} \int [(v + s - r)x^*(v, s) - K] dG(s) dF(v) + \int_{v \notin A^*} (v - r) \mathbf{1}_{v \geq r} dF(v) - \int_0^1 \frac{1 - F(v)}{f(v)} X^*(v) dF(v) - \Pi^*(0),
\]

which can be rewritten as

\[
\int_{v \in A^*} \int \left[ \left( v + s - r - \frac{1 - F(v)}{f(v)} \right) x^*(v, s) - K \right] dG(s) dF(v)
+ \int_{v \notin A^*} \left( v - r - \frac{1 - F(v)}{f(v)} \right) \mathbf{1}_{v \geq r} dF(v) - \Pi^*(0). \quad (33)
\]

The final step before characterizing the optimal mechanism in the benchmark is to prove

**Lemma 6** Assume the consultant can verify \( V - v \). In the optimal mechanism, if \( x^*(v, s) = 1 \) then for all \( s' > s \), \( x^*(v, s') = 1 \) as well.

The interesting consequence of the last lemma is the following. For all \( v \in A \) there exists a \( p^*(v) \) such that

\[
x^*(v, s) = 1 \text{ if and only if } s \geq r + p^*(v) - v.
\]
Observe that the probability that client type $v \in A$ undertakes the project can be written as 

$$X^*(v) = 1 - G(r + p^*(v) - v),$$

and so if $v - p^*(v)$ is weakly increasing then $X^*$ is weakly increasing. The expected payoff of client type $v$ if he reported $v' \in A$ is 

$$\pi^*(v, v') = \int_{r + p^*(v') - v} (v + s - r)dG(s) - d^*(v').$$

(34)

Now we are ready to prove the result of this section. We show that the consultant cannot be made better off (relative to her optimal contract seen in Section 3) even if the information that she can reveal is verifiable and contractible.

**Theorem 2** If $p$ defined in Theorem 1 is strictly decreasing then in the optimal contract then the consultant cannot increase her expected utility even if the information that she can release is contractible.

**Proof.** The proof of this theorem consists of showing that mechanisms $\{A, p, c\}$ under non-contractible $s$ and $\{A^*, d^*, x^*\}$ under contractible $s$ are equivalent (in terms of incentive compatibility and consultant’s utility) when $p$ strictly decreasing, $A = A^*$, $p = p^*$ (as induced by $x^*$), and for all $v \in A$,

$$c(v) = d^*(v) - (1 - G(r + p(v) - v))p(v).$$

(35)

First, note that the first-order condition of incentive compatibility of a mechanism under contractible $s$, based on the expression for $\pi^*$ in (34), is

$$\frac{\partial \pi^*(v, v')}{\partial v'} \bigg|_{v' = v} = p^*(v)g(r + p^*(v) - v)(1 - p''(v)) - d''(v) = 0$$

for all $v \in A^*$. Using (35), this condition becomes

$$c'(v) + p'(v)(1 - G(r + p(v) - v)) = 0,$$

which is exactly (7), the first-order condition of incentive compatibility of a mechanism under non-contractible $s$. The second-order conditions of incentive compat-
ibility of both mechanisms hold when \( p \) is strictly decreasing.\(^{14}\) We conclude that whenever \( p \) is strictly decreasing, the two mechanisms defined above are either both incentive compatible or neither is.

Second, the two mechanisms defined above generate the same expected utility for the consultant. To see this, compare (12), where \( v^* = 1 \) and \( F_L = F \), with (33). This completes the proof. ■

5 Conclusions

We analyzed a model of the advisor–client relationship where the role of the advisor (or consultant) is that she can disclose “clues” to the client that only he, not even the advisor, can understand. These clues, or signals, refine the client’s original private estimate regarding the profitability of the client’s project. We assumed that the consultant has the power to design the contract governing the relationship, and that she can contract on the client’s action (whether or not he undertakes the project). In other words, the consultant could offer a deal where the client pays her differently depending on whether he undertakes the project upon evaluating her advice (a pair of transfers since the client’s action is binary). We derived the consultant’s optimal contract, which can be thought of as a menu of such transfer-pairs.

The most interesting finding, we believe, is that the consultant’s payoff in the optimal contract is the same as if she could in fact “decipher her clues,” that is, as if she precisely understood how the client’s value-estimate was changed by her advice. To put it differently: even if the consultant is ignorant regarding how her advice affects her client, as long as she has the power to design their contract and can condition it on the (binary) decision of the client, she can do just as well as if she understood the precise effect of her advice.

Appendix

Proof of Lemma 1. First, suppose that \( v < r, v \in A \). Then, in any incentive compatible mechanism (with participation), \( v \) prefers accepting his contract, \( c(v) \)

\(^{14}\)Note, however, that the two necessary second-order conditions differ: under non-contractible \( s \), \( p \) must be weakly decreasing, while under contractible \( s \), \( v - p(v) \) must be weakly increasing.
and \( p(v) \), to not accepting it (and, by \( v < r \), not undertaking the project), hence \( \Pi(v) \geq 0 \). For any client type \( v' > v \),

\[
\pi(v', v) = \int_{r+p(v)-v}^{\infty} (v + s - p(v) - r)dG(s) - c(v) \\
+ (v' - v)(1 - G(r + p(v) - v)) + \int_{r+p(v)-v'}^{r+p(v)-v} (v' + s - p(v) - r)dG(s) \\
> \Pi(v) + (v' - v)X(v),
\]

where the inequality follows because the integral term in the second line is positive. By \( \Pi(0) \geq 0, v_0 > v \), and \( X(v) \geq 0 \), we have

\[
\pi(v', v) > 0.
\]

By incentive compatibility, \( \Pi(v') \geq \pi(v', v) \), therefore \( \Pi(v') > 0 \) as well. But this means that client type \( v' \in (v, r] \cap [0, 1] \) has to learn \( s \) in the mechanism, because otherwise, by \( v' \leq r \), he does not undertake the project and his profit is zero.

Second, suppose that \( v > r, v \in A \). For all \( v' < v \),

\[
\pi(v', v) = \int_{r+p(v)-v}^{\infty} (v + s - p(v) - r)dG(s) - c(v) \\
- (v - v')(1 - G(r + p(v) - v)) - \int_{r+p(v)-v'}^{r+p(v)-v} (v' + s - p(v) - r)dG(s) \\
> \Pi(v) - (v - v')X(v),
\]

where the inequality follows because the integral term on the second line is negative. By \( X(v) \leq 1 \) and \( \Pi(v) \geq v - r \), we have

\[
\pi(v', v) > v' - r.
\]

By incentive compatibility, \( \Pi(v') \geq \pi(v', v) \), therefore \( \Pi(v') > v' - r \) as well. But this means that client type \( v' \in [r, v) \cap [0, 1] \) has to learn \( s \) in the mechanism because otherwise, by \( v' \geq r \), he undertakes the project and his profit is \( v' - r \). We conclude that \( A \) must be an interval that contains \( r \) whenever \( r \in [0, 1] \). 

**Proof of Lemma 2.** (Necessity.) Consider an incentive compatible mechanism, \( \{A, p, c\} \). (9) is the participation constraint, which is necessary for all \( v \). By Lemma
1, \([0,1]\). \(A\) consists of two disjoint intervals (either of which may be empty), and on the lower interval \(X \equiv 0\) and \(\Pi \equiv 0\), while on the upper interval \(X \equiv 1\) and \(\Pi(v) = v - r\).

Suppose that \(v, v' \in A, v < v'\). Rewriting the definition of \(\pi(v,v')\) just as we did in the proof of Lemma 1,

\[
\pi(v,v') = \int_{r+p(v')-v}^{\infty} (v' + s - p(v') - r)dG(s) - c(v') \\
+ (v - v')(1 - G(r + p(v') - v')) - \int_{r+p(v')-v'}^{r+p(v)-v} (v + s - p(v') - r)dG(s) \\
> \Pi(v') - (v' - v)X(v').
\]

Similarly,

\[
\pi(v',v) = \int_{r+p(v)-v}^{\infty} (v + s - p(v) - r)dG(s) - c(v) \\
+ (v' - v)(1 - G(r + p(v) - v)) + \int_{r+p(v)-v'}^{r+p(v')-v} (v' + s - p(v) - r)dG(s) \\
> \Pi(v) + (v' - v)X(v).
\]

Incentive compatibility requires \(\Pi(v) \geq \pi(v,v')\) and \(\Pi(v') \geq \pi(v',v)\), therefore

\[
(v' - v)X(v) \leq \Pi(v') - \Pi(v) < (v' - v)X(v').
\]

Cross-dividing by \((v' - v) > 0\), we have for all \(v < v'\) (such that \(v, v' \in A\)),

\[
X(v) < \frac{\Pi(v') - \Pi(v)}{v' - v} < X(v').
\]

From this, \(X\) is strictly increasing on \(A\) and \(\Pi\) is continuous on \(A\)’s interior. (It is easy to see that \(\Pi\) must be continuous at \(\underline{v}\) and \(\bar{v}\) as well.) This, together what we know about \(X\) and \(\Pi\) outside \(A\), implies that \(d\Pi(v)/dv = X(v)\) almost everywhere. Since \(X\) is monotonic, it is integrable, and (8) follows.

Now we show that \(p\) weakly decreasing is also necessary. Suppose towards con-
Therefore $X > v$ and $p(v') > p(v)$. Decompose $\pi(v, v')$ as

$$
\pi(v, v') = \int_{r+p(v')-v}^{\infty} (v + s - p(v) - r)dG(s) - c(v')
$$

$$
+ (p(v) - p(v'))(1 - G(r + p(v) - v)) - \int_{r+p(v')-v}^{\infty} (v + s - p(v') - r)dG(s)
$$

$$
> \Pi(v) + c(v) - c(v') - (p(v') - p(v))X(v),
$$

where the inequality holds because the integrand in the second line is negative (as $s$ has full support). Similarly,

$$
\pi(v', v) = \int_{r+p(v')-v}^{\infty} (v' + s - p(v') - r)dG(s) - c(v)
$$

$$
+ (p(v') - p(v))(1 - G(r + p(v') - v')) + \int_{r+p(v')-v}^{\infty} (v' + s - p(v) - r)dG(s)
$$

$$
> \Pi(v') + c(v') - c(v) + (p(v') - p(v))X(v'),
$$

where the inequality follows because the integrand in the second line is positive. By incentive compatibility $\pi(v, v') \leq \Pi(v)$ and $\pi(v', v) \leq \Pi(v')$, hence

$$(p(v') - p(v))X(v') < c(v) - c(v') < (p(v') - p(v))X(v).$$

Therefore $X(v') < X(v)$, contradiction. We conclude that $p$ is weakly decreasing.

(Sufficiency.) Assume that (8)–(9) hold and $p$ is weakly decreasing on $A$ (which is an interval). Note that no type $v$ would prefer to deviate to $v' \notin A$ because (9). Also note that from $p$ weakly decreasing $X$ weakly increasing follows.

Let $v' > v$, $v' \in A$. Then, using yet another decomposition of $\pi(v, v')$, we have

$$
\pi(v, v') = \int_{r+p(v')-v}^{\infty} (v' + s - p(v'))dG(s) - c(v')
$$

$$
- \int_{1_{v'+s-r-p(v') \geq 0}(v' + s - r - p(v'))dG(s)}
$$

$$
+ \int_{1_{v'+s+v'-r-p(v') \geq 0}(v' + s + v - v' - r - p(v'))dG(s)}
$$

$$
= \Pi(v') - \int_{s+v-v'}^{a} 1_{v'+\sigma-r-p(v') \geq 0}d\sigma dG(s),
$$

30
where, on the last line, we used the identity (true for all \( v' \) and \( s' \leq s \))

\[
1_{v'+s-r-p(v') \geq 0}(v' + s - r - p(v')) - 1_{v'+s-r-p(v') \geq 0}(v' + s' - r - p(v')) \\
= \int_{s'}^{s} 1_{v'+s-r-p(v') \geq 0} d\sigma.
\]

However,

\[
\int \int_{s+v-v'}^{s} 1_{v'+s-r-p(v') \geq 0} d\sigma dG(s) = \int \int_{v-v'}^{0} 1_{v'+s+x-r-p(v') \geq 0} dx dG(s) \\
= \int_{v-v'}^{0} \int 1_{v'+s+x-r-p(v') \geq 0} dG(s) dx \\
= \int_{v-v'}^{0} (1 - G(r + p(v') - v' - x)) dx,
\]

therefore

\[
\pi(v, v') = \Pi(v') - \int_{v-v'}^{0} (1 - G(r + p(v') - v' - x)) dx.
\]

On the other hand, by (8),

\[
\Pi(v) = \Pi(v') - \int_{v}^{v'} (1 - G(r + p(v) - \nu)) d\nu \\
= \Pi(v') - \int_{v-v'}^{0} (1 - G(r + p(v' + x) - v' - x)) dx \\
\geq \Pi(v') - \int_{v-v'}^{0} (1 - G(r + p(v') - v' - x)) dx \\
= \pi(v, v'),
\]

where the inequality holds because \( p(v' + x) \geq p(v') \) for all \( x \leq 0 \) by monotonicity of \( p \). Therefore, client type \( v \) has no incentive to imitate a type \( v' > v \), \( v' \in A \).
Now assume $v \in A$, and take any $v' > v$. Then,

\[
\pi(v', v) = \int_r^{\infty} (v + s - r - p(v)) dG(s) - c(v)
- \int 1_{v+s-r-p(v) \geq 0} (v + s - r - p(v)) dG(s)
+ \int 1_{v+s+v'-v-r-p(v) \geq 0} (v + s + v' - v - r - p(v)) dG(s)
= \Pi(v) + \int \int_{s+v'-v} 1_{v+\sigma-r-p(v) \geq 0} d\sigma dG(s).
\]

Furthermore,

\[
\int \int_{s+v'-v} 1_{v+\sigma-r-p(v) \geq 0} d\sigma dG(s) = \int \int_{0}^{v'-v} 1_{v+s+x-r-p(v) \geq 0} dx dG(s)
= \int_{0}^{v'-v} \int 1_{v+s+x-r-p(v) \geq 0} dG(s) dx
= \int_{0}^{v'-v} (1 - G(r + p(v) - v - x)) dx,
\]

therefore

\[
\pi(v', v) = \Pi(v) + \int_{0}^{v'-v} (1 - G(r + p(v) - v - x)) dx.
\]

On the other hand, by (8),

\[
\Pi(v') = \Pi(v) + \int_{v}^{v'} (1 - G(r + p(v) - \nu)) d\nu
= \Pi(v) + \int_{0}^{v'-v} (1 - G(r + p(v + x) - v - x)) dx
\geq \Pi(v) + \int_{0}^{v'-v} (1 - G(r + p(v) - v - x)) dx
= \pi(v', v),
\]

where the inequality holds because $p(v + x) \leq p(v)$ for all $x \geq 0$ by monotonicity of $p$. Therefore, client type $v'$ has no incentive to imitate a type $v < v'$, $v \in A$. We conclude that the mechanism is indeed incentive compatible. ■

**Proof of Lemma 4.** To see existence, let $v^* = F^{-1}(b)$, that is, $F(v^*) = b$. The left-hand side of the first inequality in (19) is continuous and strictly increasing
in $v_b$. At $v_b = v^*$, the expression becomes $\int_{v^*-\infty}^{\infty} (v^* + s - r) dG(s)$, which equals $w(v^*) + 1_{v^* > r}(v^* - r)$ by (2). However, $w(v^*) > K$ by assumption, so indeed there exists $v_b$ such that (19) holds. Similarly, the left-hand side of the first inequality in (20) is continuous and strictly decreasing in $\bar{v}_b$. At $\bar{v}_b = v^*$, it becomes $\int_{-\infty}^{-v^*} (r - v^* - s) dG(s) = \int_{-\infty}^{\infty} (v^* + s - r) dG(s) + r - v^*$, which equals $w(v^*) + 1_{r>v^*}(r - v^*)$ by (2), and $w(v^*) > K$ by assumption. From this argument it is also clear that $F(v_b) \leq b \leq F(\bar{v}_b)$, which then implies $v_0 = 0$ and $\bar{v}_1 = 1$.

It is easy to see that $v_b$ and $\bar{v}_b$ are continuous in $b$ (no matter what the distribution of $s$ is). Since the integral in (19) is strictly decreasing in $b$, and the integral in (20) is strictly increasing in $b$, both $v_b$ and $\bar{v}_b$ are weakly increasing in $b$.

**Proof of Lemma 5.** Suppose $v, v' \in A^*$, $v < v'$. Then, by the definition of $\pi$,

$$\pi^*(v, v') = \Pi^*(v') - (v' - v)X^*(v') \leq \Pi^*(v)$$

and

$$\pi^*(v', v) = \Pi^*(v) + (v' - v)X^*(v) \leq \Pi^*(v'),$$

where the inequalities follow from incentive compatibility. Therefore,

$$(v' - v)X^*(v) \leq \Pi^*(v') - \Pi^*(v) \leq (v' - v)X^*(v'),$$

and hence $X^*$ is weakly increasing on $A^*$, moreover, $d\Pi^*/dv = X^*$ on $A^*$. Clearly, $X^*$ is also weakly increasing on $[0, 1]\setminus A^*$ and $d\Pi^*/dv = X^*$ there as well. Since $\Pi^*$ is continuous everywhere (which follows from continuity of $\pi^*(v, v')$ in $v$ and incentive compatibility), we get (31). Since the other conditions were established in the text, this concludes the proof of necessity.

Now assume that (31)–(32) hold and $X^*$ is weakly increasing. Note that no type $v$ has an incentive to imitate $v' \not\in A$ because (32) holds. If $v' \in A$ and $v' < v$ then

$$\Pi^*(v) = \Pi^*(v') + \int_{v'}^{v} X^*(z)dz$$

$$\geq \Pi^*(v') + \int_{v'}^{v} X^*(v')dz$$

$$= \Pi^*(v') + (v - v')X^*(v') = \pi^*(v, v'),$$

where the inequality follows because $X$ is weakly increasing. If $v' \in A$ and $v' > v$
then similarly,
\[
\Pi^*(v) = \Pi^*(v') - \int_v^{v'} X^*(z)dz \\
\geq \Pi^*(v') - \int_{v'}^v X^*(v')dz \\
= \Pi^*(v') + (v - v')X^*(v') = \pi^*(v, v').
\]

Therefore, the mechanism is indeed incentive compatible. ■

**Proof of Lemma 6.** Let \(\mu_G\) denote the measure generated by \(G\) on \(\mathbb{R}\). Notice that if the claim of the lemma is not true then there is a type \(v \in A^*\) and there exist subsets of \(\mathbb{R}\), \(B\) and \(C\), such that \(B \leq C\), \(\mu_G(B) = \mu_G(C)\) and

\[
x^*(v, s) = 1 \forall s \in B \quad \text{and} \quad x^*(v, s) = 0 \forall s \in C.
\]

We now show that the consultant can do weakly better by defining a new allocation rule \(\hat{x}\) as follows.

\[
\begin{align*}
\hat{x}^*(v, s) &= 0 \text{ if } s \in B, \\
\hat{x}^*(v, s) &= 1 \text{ if } s \in C, \quad \text{and} \\
\hat{x}^*(v', s) &= x^*(v', s) \text{ if } v' \neq v \text{ but } s' \notin B \cup C.
\end{align*}
\]

Also, define \(\hat{\Pi}^*\) according to (31) using \(\hat{X}^* = \int \hat{x}^*(., s)dG(s)\), and so that \(\hat{\Pi}^*(0) = \Pi^*(0)\). Note that since \(\hat{X}^*(v) = X^*(v)\) for all \(v\), the indirect profit function did not change either, \(\hat{\Pi}^* = \Pi^*\). Hence the mechanism \(\{A^*, d^*, \hat{x}^*\}\) is incentive compatible. However, in the consultant’s objective function, the term

\[
\int \left[ \left( v + s - r - \frac{1 - F(v)}{f(v)} \right) x^*(v, s) - K \right] dG(s)f(v)
\]

\[
\leq \int \left[ \left( v + s - r - \frac{1 - F(v)}{f(v)} \right) \hat{x}^*(v, s) - K \right] dG(s)f(v)
\]

Hence \(x^*\) can be replaced by \(\hat{x}^*\) without decreasing the consultant’s objective function. ■
References


