On the Return to Venture Capital[1]

by

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ABSTRACT

We provide a model that links the high return to venture equity to the impatience of the VCs. VCs are scarce, and hence, they have market power and a high return on their investments. As a result, VCs are eager to terminate non-performing ventures so they can move on to new ones. The scarcity of VCs enables them to internalize their social value, and the competitive equilibrium is socially optimal. We estimate the model and back out the return of solo entrepreneurs which is always below that of the return of VCs.

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1 Introduction

This paper builds and estimates a model that explains several facts:

1. Venture investments yield returns several points above what their risk characteristics would warrant. Cochrane (2005) estimates Alpha at over 30 percent, Ljungqvist and Richardson find it to be at least five percent, Kaplan and Schoar (2005) argue that it may well be four or five percent, and Hall and Woodward (2007) estimate it at around two percent.[4]

2. Founders are more attached to and more patient with their projects than VCs who, according to Sahlman (1990), Wright and Robbie (1998) and Jones and Rhodes-Kropf (2004), behave as if their discount rates were as high as 30 to 50 percent,

3. VC-backed companies are worth more at IPO (Megginson and Weiss 1991, Hochberg 2004),

4. The survival of VC-backed companies is not any higher than that of solo ventures (Manigart, Baeyens, and Van Hyfte 2002, Goldfarb, Kirsch, and Miller 2006)

 Our model explains these facts in the following way:

1. VCs and, hence, operating VC funds are scarce compared to the number of proposals they see; this raises the rents that VC funds get from the projects that they finance, and raises the quality of projects that they accept. We do not explain why VCs are scarce, we only trace the consequences of that scarcity for the variables of interest.

2. Because the VC can always move on to a new company, looking after a non-performing company entails for him a foregone-earnings cost that a founder does not face, and this makes the VC less patient.

3. The impatient VC imposes a higher quality hurdle than the founder does, and this selection effect raises the IPO values of VC-backed firms above those of other firms.

4. The VC has a deeper pocket than the founder, but he is less patient. The net effect on terminations is ambiguous.

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[4]If the U.K. betas on venture are in the same ballpark as U.S. betas – say around the 1.7 percent that Hall and Woodward (2007) estimate – then the U.K. Alpha on Venture is even higher than the U.S. alpha. Writing in the early 90s (i.e, before the really high market returns of the late 90s which would, in the usual calculations imply a higher required rate of return) Bygrave (1994) says that "...VC returns have most often been in the teens, with occasional periods in the 20-30 per cent range and rare spikes above 30 per cent.

The main actors in the model are VCs[5] who have unlimited wealth, and entrepreneurs who are endowed with limited wealth and with projects. A project entails start-up and continuation costs. The start-up cost must be paid before any information about the project's quality can come in. After that, continuation costs must be paid until the project succeeds or is terminated. Start-up costs entail only capital, but continuation costs entail capital and effort: funds must be supplied and the entrepreneur must exert effort without interruption until the project yields fruit.

Project quality has two dimensions: Payoff size, and the waiting time until the payoff is realized. Neither dimension is known before a contract between a VC and entrepreneur is signed, after which the payoff is learned, but not the waiting time. Either party can, at any time, terminate the project. The entrepreneur can do so by withholding effort, the VC by withholding capital. The optimal contract is set up so that when a project is terminated, both are better off: The entrepreneur no longer wishes to exert effort, and the VC no longer wishes to lend. The VC is in the model a supplier of capital and is also able to recognize project quality in a way that banks cannot (in reality a good VC must have a hybrid of skills; he must be able to evaluate technology, demand, and managerial talent). Thus the VC does not add value to the project directly. We shall show that our model is consistent with Sørensen (forthcoming) who finds that the bulk of the observed positive association between VC quality and project quality is due not to direct VC influence on the payoff, but to sorting.

Because entrepreneurs have some wealth, some will opt to run their projects alone, "solo". Bank lending is unprofitable because banks lack the expertise of the VCs in assessing project quality. A solo entrepreneur may run out of money before the optimal termination date, which forces a poor entrepreneur to use VC funding or abandon her project.

The outcome is efficient for a wide range of parameter values. Efficiency pertains to both (*i*) contracting between VCs and entrepreneurs and the resulting termination rules, and to (*ii*) entrepreneurs' choices over whether to seek VC backing, whether to develop the project on their own, or whether to abandon the project altogether.

We fit a seven-parameter version of the model to panel data on 1355 firms from the 1980s and '90s, and display the results in a series of graphs. The model matches reality along several dimensions, such as the project-success hazard, the project-termination hazard, equity shares, and up-front investment costs. The fit is remarkable given the number of parameters is less than the number of variables fitted. The model emphasizes the behavior of real variables: Project duration, project value and age-related rate of return, rate of termination, and the equity share of the VC and entrepreneur. We only predict the equity shares, and not the finer features of contracts such as the use of convertible debt. In this stylized setting, the focus is on whether we see

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[5]We do not analyze how the rents that the VC funds earn are divided of rents among the general partners and the limited partners of a VC fund. Our paper is not about the division of these rents. In fact, the VC in our model should be thought of as the general and limited partners together.

rather debt or rather equity contracts. The result is that only equity can give the right incentives to both parties at the margin.

Venture capital is now exported to other countries, but there are significant barriers to its flow, even within the U.S.. The model therefore has meaningful cross-economy implications. The model says that places where venture capital is scarcer should not have appreciably different contractual forms as long as the nature of the projects is the same. For a wide range of distributions of project quality and of wealth (the latter affects an entrepreneur's ability to self finance and to bypass VCs) and for a wide range of VC supply, the observed contracts should be the same. Any differences in contracts that did arise should show up in just two dimensions: The share of up-front costs that the VC finances, and the VC's share in the equity of the project.

Notes on the literature.—Bergemann and Hege (2005), deal with a single VC and a single entrepreneur, with their outside options taken as given. Holmes and Schmitz (1990) analyze a market equilibrium and determine the rewards of founders relative to managers of firms, but do not model venture capital. Inderst and Muller (2004) model the market for venture capital as do Michelacci and Suarez (2004) who, in addition, analyze the termination decision and link it to the equilibrium value of a VC, but who do not look at data. Ueda (2004) analyzes the mode-of-financing decision in a model where one cost of VC financing is the threat that the VC use the information to set up a competing business. Cochrane (2005) deals with the valuing the income streams to VC portfolios but does not derive optimal termination rules. We discuss some of these papers and others further in the body of the paper.

Plan of the paper.—The next section describes the model, and Section 3 derives the equilibrium contract and shows that the competitive outcome is efficient. Section 4 derives several empirical implications of the model and discusses evidence. Section 5 solves an example by hand and fits it to longitudinal data on VC investments, spanning 1989-2000, and their performance outcomes. Section 6 concludes the paper and the Appendix describes the data and the estimation procedure.

2 Model

There is a measure x of infinitely lived VCs, each able to raise a *sufficient* amount of money at the rate r. There is also an inflow at the rate λ of potential projects, each in the possession of a different entrepreneur. The entrepreneurs cannot borrow, and have initial wealth w which is distributed according to the C.D.F. Ψ. An entrepreneur can have at most one project, ever.

A Project
For a project to succeed, it requires an immediate payment of a cost C, and after that it also requires k units of investment and a units of effort by E at every instant up until the project yields a payoff. A project can be undertaken by an entrepreneur

(E) alone, in which case she must rely on her own wealth only, or together with a VC. The project yields a payoff π at time τ, where π and τ are random variables independent of each other. Let G be the C.D.F. of π, and g its density. Let F be the C.D.F. of τ, and f its density. Both G and F are supported on $\mathbb{R}_+$. Assume that the hazard rate $f/(1-F) = h$ is bell shaped: It first increases up to the modal age, τ_{m},, then decreases. In other words, as time passes without realization of τ, the agents first become more optimistic about a quick realization of τ, then increasingly pessimistic.[6]

If the project is either not invested into or effort is not exerted, the project cannot yield a positive payoff, ever. Neither party knows π and τ, but their distributions are common knowledge. In a VC-backed firm, after the contract is signed and after E incurs C, π becomes known to both parties. However, no information about τ is received. In a solo venture, E alone incurs C at the outset, and thereby she learns π.

The expected social value to developing projects is assumed to be positive. That is, if financing decisions regarding the project are made optimally, it yields a positive payoff in expectation.

Preferences

All agents are risk neutral and discount the future at the rate r. Utility is additively separable in consumption and leisure. The VC maximizes the expected discounted present value of his net income. The entrepreneur maximizes the expected discounted present value of her income minus the stream of her exerted efforts.

Market Structure

When E gets an idea, she has to decide whether to abandon her project, to seek VC-backing, or to go solo, i.e., to implement her project alone. This decision is irreversible.

Suppose that at time t there is a measure n of VCs who is not in a contractual relationship with entrepreneurs and a measure of m of entrepreneurs who wishes to be financed by a VC. Then the number $\min\{n, m\}$ of VCs and entrepreneurs are randomly matched and can enter into a contract.

Timing
Events occur in the following sequence:

1. Entrepreneur chooses whether to (i) abandon her project, (ii) develop her project on her own, or (iii) sign a contract with a VC,

2. Under option (ii) or (iii), C is incurred immediately,

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[6] Jeremy Stein suggested the name "waiting-at-the-altar" hazard: The bride is unlikely to show up early, and the chances of her showing up rise at first. But after a certain point it starts to look like the bride will not show up at all. Our results would follow more easily if the hazard were to decline monotonically throughout. The bell-shape assumption conforms better to facts.

3. π is then revealed.

Contracting

Feasible Contracts.—The contract the VC can offer consists of two positive numbers: (p, s). The number p is a lump sum the VC pays E right after signing a contract. In that case E bears only $C - p$ of the up-front cost. The number s specifies how to share the payoff if the project succeeds. If the project yields payoff π, E gets $s\pi$ and the VC gets $(1 - s)\pi$. Neither E's effort nor the VC's subsequent investment are contractible. On the other hand the payments p and the sharing rule s are enforceable.

After the transfer p, this is a pure equity contract. We could allow for more complicated contracts in which s would depend on τ and π. We shall show, however, that these simple contracts already induce socially efficient decisions. Moreover, the equilibrium outcome of a game with more complicated contracts would be identical to ours.

Timing of the Contractual Relationship —First, the VC offers a contract, (p, s), to E. If E refuses the contract the game between these two parties ends; E then has to abandon the project, whereas the VC seeks to be matched with another entrepreneur. If E signs the contract she receives p from the VC up front. We interpret p as the amount that the VC pays towards financing C. The entrepreneur finances the remaining part of C and both parties then immediately learn the value of π.

At each date, if the payoff has not yet been realized, E has to decide whether to exert effort and the VC has to decide whether to invest. One can assume that the parties can observe the history of investments and effort up to time t, when making these decisions.[7] If the VC decides not to invest into the project, he is free to devote his time to another project. If E decides not to exert effort anymore, she leaves the market.

If the project yields a payoff π at time t, E gets $s\pi$ the VC gets $(1 - s)\pi$ and the game ends between the two parties. Again, the VC seeks a new match, and E leaves the market.

Banks

In our model, banks guarantee a risk-free interest rate, but they do not finance projects. This is because VCs are assumed to have two advantages over banks. First, banks lack the expertise of the VCs and entrepreneurs; banks can learn π only at the date of success, τ, not before. Second, banks also lack the monitoring ability of the VCs which ensures that Es do not divert investment to private consumption. As a result, banks do not offer contracts to entrepreneurs, for otherwise anybody could

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[7]So as to avoid coordination problems, we assume that at time t the VC observes the history of efforts on the interval $[0, t)$ and that the entrepreneur observes the history of investments on the interval $[0, t)$. Our equilibrium remains an equilibrium even if neither party observes whether the other is supporting the project.

pretend to be an E and divert the borrowed funds for personal use, so that banks would make negative profits.

3 Analysis

First, we characterize the socially optimal outcome of our economy. Then we show that this outcome is the unique outcome in the competitive market conditional on some distributional assumption on the wealth of Es.

3.1 Socially Optimal Decisions

We proceed as follows. First, we analyze the optimal decision regarding how long a project should be supported. This decision depends on whether the project is VC backed or supported by a solo entrepreneur. Second, we characterize the socially optimal decision whether E should go solo, seek VC backing, or abandon her project.

3.1.1 The termination problem and the social value of a VC

The VC faces no budget constraint, that is, he is able to finance a project at any moment of time no matter what the payoffs of previously supported projects were. He can, however, handle only one project at a time. The social value of a VC is positive because he is able to support projects that otherwise would not be implemented.

The Social Value of a VC.—In what follows, we assume that VCs finance only those entrepreneurs who would otherwise abandon their projects. (Later this becomes a result.) Denote the social value of a free VC by W. A free VC immediately matches with an E, the cost C is incurred, and π is revealed. Then the planner decides how long to support the project if it does not yield fruit, $T(\pi)$. As soon as the project succeeds or is terminated, the VC becomes free and generates a value of W as of then. Hence, the social value of the VC can be expressed as the sum of the discounted expected payoff from a project and the discounted continuation value he generates after he is again free[8]:

$$W = -C + \max_{\substack{T(\pi) \\ \pi \in \mathbb{R}^+}} \int \int_0^{T(\pi)} \left(\pi + W - \frac{a+k}{h(t)} \right) e^{-rt} dF(t) + W e^{-rT(\pi)} \left(1 - F[T(\pi)]\right) dG(\pi).$$

(1)

Let us explain the RHS of (1). First the benefit: Suppose that π is revealed after C is sunk and that the planner supports the project up to time T. The expected benefit from this project is

$$\int_0^T (\pi + W) e^{-rt} f(t) dt + e^{-rT} \left(1 - F[T]\right) W,$$

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[8]This equation has a unique solution for W. This is because the RHS is positive at $W = 0$ since the social value of the project is positive, and its slope is less than one by the Envelope Theorem.
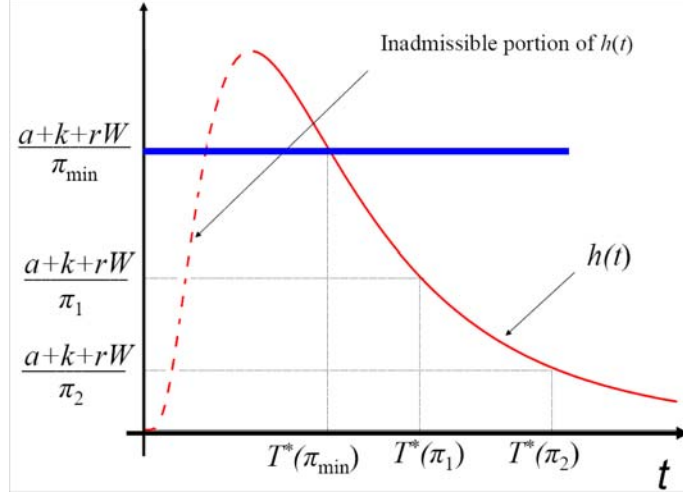
Figure 1: THE DETERMINATION OF $T^*(\pi)$

because it yields π at time t with likelihood $f(t)$ if $t \leq T$, in which case the benefit is $\pi + W$. If the project does not succeed until time T it is then terminated with probability $(1 - F[T])$ and its only benefit then is W. Second, the cost: What measure of projects will require investment at time t? If $t > T$ then the project surely exited already and no investment is made. If $t \leq T$ then a proportion $F(t)$ of the projects succeeded, and therefore only a measure of $1 - F(t)$ of them need investment. Total cost then is

$$\int_0^T (a+k) e^{-rt} (1 - F(t)) \, dt = \int_0^T (a+k) e^{-rt} \frac{1 - F(t)}{f(t)} dF(t) = \int_0^T e^{-rt} \frac{a+k}{h(t)} dF(t).$$

The Optimal Stopping Time of a VC-backed Project.—Once π is revealed, the planner solves

$$\max_T \left\{ \int_0^T \left(\pi + W - \frac{a+k}{h(t)} \right) e^{-rt} dF(t) + e^{-rT} (1 - F[T]) W \right\}. \qquad (2)$$

An interior solution for this problem, $T^*(\pi)$, must solve for t the first-order condition

$$a + k + rW = \pi h(t). \qquad (3)$$

The LHS of (3) is the cost of waiting another instance: The physical cost, $a+k$, and the opportunity cost, rW. The RHS is the expected benefit. The local second-order condition is $h'(T^*(\pi)) < 0$. Since h is bell shaped, this condition is also sufficient, as Figure 1 shows.

Notice however, that π may be so low that the project yields a negative expected payoff for all $T > 0$. Let $\pi_{\min}$ be the smallest value of π for which it is worth

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supporting the project for a strictly positive T. Then, as Figure 1 shows, the optimal termination age, $T^*(\pi)$, for a project of quality π, is

$$T^*(\pi) = \begin{cases} h^{-1}\left(\frac{a+k+rW}{\pi}\right) & \text{if } \pi > \pi_{\min}, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

3.1.2 The solo entrepreneur's termination problem

A solo entrepreneur may run out of money and have to terminate her project. Consider an E with wealth w. Since utility is linear in consumption, E will optimally defer all her consumption until the project is completed.[9] Let w_t denote E's wealth at time t given that her project did not succeed up to time t. Then w_t is defined by the differential equation $\dot{w}_t = rw_t - k$. The initial condition is $w_0 = w - C$ because C must be incurred immediately. In addition, at each instance of time, E has to invest k and receives interest on her remaining wealth. The solution for w_t is

$$w_t = \frac{k}{r} + \left(w_0 - C - \frac{k}{r}\right)e^{rt}.$$

Let $\tau(w)$ be the date at which this entrepreneur's wealth runs out. Then $\tau(w)$ solves for t the equation $w_t = 0$, that is

$$\tau(w) = \begin{cases} \frac{1}{r}\ln\left(\frac{k}{k-r(w-C)}\right) & \text{if } w < \frac{k}{r} + C \\ \infty & \text{if } w \geq \frac{k}{r} + C. \end{cases} \tag{5}$$

The unconstrained entrepreneur.—Suppose first that $w \geq k/r + C$, that is, E has enough money to finance her project forever. Since new ideas occur only to new entrepreneurs, if she terminates the project prior to success E gets zero as her terminal payoff. Therefore, she solves the following maximization problem:

$$\max_T \int_0^T \left(\pi - \frac{a+k}{h(t)}\right)e^{-rt}f(t)\,dt.$$

The solution, $T^E(\pi)$, is either equal to zero, or solves for t the first-order condition $a + k = \pi h(t)$ and the second-order condition $h'\left(T^E[\pi]\right) < 0$. Let $\pi^E_{\min}$ denote the smallest realization of π that the unconstrained entrepreneur should support. Then

$$T^E(\pi) = \begin{cases} h^{-1}\left(\frac{a+k}{\pi}\right) & \text{if } \pi > \pi^E_{\min}, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

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[9]In general entrepreneurs save more in order to get around liquidity constraints – see Buera (2004) and Basaluzzo (2004).

The wealth-constrained entrepreneur.—Suppose now that $w < k/r + C$. After paying the cost C and learning π, the expected value of the project is

$$q(\pi, w) \equiv \max_{T \in [0, \tau(w)]} \int_0^T \left(\pi - \frac{a+k}{h(t)} \right) e^{-rt} f(t) \, dt. \tag{7}$$

At an interior solution, the optimal stopping time, $T_w^E(\pi)$, solves for t the following first-order condition:

$$\pi h(t) \geq a + k$$

and it holds with equality whenever $T_w^E(\pi) \leq \tau[w]$. Therefore, if the project is worth pursuing further after incurring C, then the solution is $\min\left(\tau[w], T^E(\pi)\right)$, otherwise it is zero. Let $\pi_{\min}^E(w)$ be the smallest payoff for which it is worth starting to support the project. Any project with quality below $\pi_{\min}(w)$ would be terminated at once. Hence, T_w^E is defined as follows

$$T_w^E(\pi) = \begin{cases} \min\left\{\tau[w], h^{-1}\left(\frac{a+k}{\pi}\right)\right\} & \text{if } \pi > \pi_{\min}(w), \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

3.1.3 The Socially Optimal Financing Mode

The expected surplus, after incurring the cost C, of a solo entrepreneur with wealth w is

$$Q^E(w) = \int q(\pi, w) \, dG(\pi).$$

The next lemma characterizes some important features of the curve Q^E.

Lemma 1 *(i) For $w < k/r + C$, $\partial Q^E/\partial w > 0$. (ii) For $w \geq \frac{k}{r} + C$, $\partial Q^E/\partial w = 0$ and $Q^E(w) > C$.*

The intuition behind statement (i) of this lemma is the following. A budget-constrained entrepreneur can use an additional dollar to prolong the time of supporting her project, instead of using it to consume. Since sometimes it is socially efficient to finance the project longer than the budget-constrained entrepreneur can afford, she can generate a positive surplus.

An entrepreneur with $w > k/r + C$ can already support her project as long as it is socially optimal. She would invest her additional dollar at the rate r. This additional dollar does not generate excess return over the interest rate of a bank, explaining why the slope of Q^E is zero in this region. Since the expected social value of a project is positive by assumption, $Q^E(w) > C$ whenever $w > k/r + C$. This explains part (ii) of the lemma.

Going solo or abandoning the project. —Suppose, first, that VC backing was not an option. The payoff to going solo with wealth w is $w - C + Q^E(w)$. We now explain why this payoff must look as drawn in Figure 2. It is not defined for $w < C$
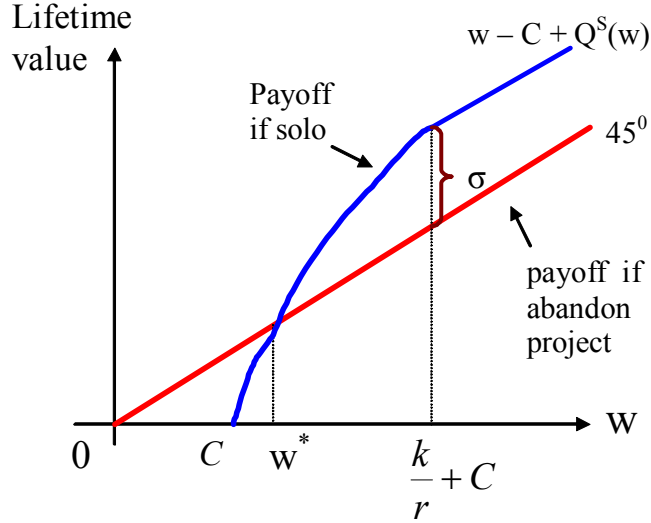
Figure 2: THE DETERMINATION OF w^*

because E cannot pay C. At $w = C$, it is zero because after paying C, E has no money left to invest in the project. Thereafter, by part (i) of Lemma 1, the slope exceeds unity. As w reaches $k/r + C$, $w - C + Q^E(w)$ reaches $w + \sigma$, where $\sigma > 0$ because $Q^E(k/r + C) > C$ by part (ii) of Lemma 1. The payoff is continuous in w, hence from the Intermediate Value Theorem it follows that there exists a value of wealth, denoted by w^*, were the payoff curve intersects with the 45^0 line, that is, $Q^E(w^*) = C$. Moreover, at any intersection, the slope of $w - C + Q^E(w)$ is strictly larger than unity, and therefore the intersection, w^*, is unique. The entrepreneur with wealth w^* is indifferent between going solo and abandoning the project.

Who should get VC-backing?—We shall assume that there are many poor entrepreneurs, with wealth below w^* who, in the absence of VCs, would abandon their projects. Then, it is socially optimal for VCs to back only entrepreneurs that have wealth less than w^*. The average duration of a VC-backed project is

$$\bar{t} = \int \int_0^\infty \min(t, T^*[\pi]) f(t) \, dt \, dG(\pi).$$

At any point in time, the rate at which free VCs flow in is $x/\bar{t}$. The inflow of entrepreneurs is λ and, of these, a measure $\lambda \Psi(w^*)$ would abandon their projects unless they could turn to a VC. When VCs are sufficiently scarce in the sense that

$$\frac{x}{\bar{t}} < \lambda \Psi(w^*), \tag{9}$$

we have the following result:

Proposition 1 *If (9) holds, the socially optimal outcome is described as follows:*

(i) An entrepreneur with initial wealth $w > w^*$ goes solo. A measure of $x/\bar{t}$ entrepreneurs gets VC-backing at every instance of time, each of them with wealth less than w^*. The rest abandon their projects.

(ii) The termination of a VC-backed project is determined by (4), and that of the solo project by (8).

3.2   The Competitive Outcome

We now derive conditions under which the social optimum is a competitive equilibrium.

For the VC to take socially optimal decisions, his market value must, as we shall show, coincide with his social value. But this can happen only if VCs can extract all the surplus from projects. They will have such market power if they support only projects that would otherwise be abandoned. The latter, in turn, arises if there are more poor entrepreneurs than there are free VCs.

But this is not the whole story. Since investment and effort are not contractible, the VCs must be able to provide contracts to Es that induce socially efficient termination by both parties while, at the same time, transferring all the surplus to the VC. We shall show that such contracts exist.

The Equilibrium Contract.—Recall that a contract consists of two numbers (p, s), where p is paid by the VC to E before π is realized, and s is the sharing rule upon the realization of π. We shall show that if the sharing rule is

$$s^* = \frac{a}{a + k + rW}, \tag{10}$$

the termination rules of both parties are indeed socially optimal.

But how can the VC extract the whole surplus from the entrepreneur? The VC enjoys all the rents if and only if the entrepreneur enjoys none. The entrepreneur's payoff from the contract (p, s^*) conditional on both parties supporting the project up to $T^*(\pi)$ is

$$p - C + \int \int_0^{T^*(\pi)} \left(s^* \pi - \frac{a}{h(t)} \right) e^{-rt} f(t)\, dt\, dG(\pi).$$

Hence, the VC extracts all the rents if

$$p^* = C - \int \int_0^{T^*(\pi)} \left(s^* \pi - \frac{a}{h(t)} \right) e^{-rt} f(t)\, dt\, dG(\pi). \tag{11}$$

Moreover, once terminated by a VC, the entrepreneur would not wish to continue the project alone (either through self finance or bank finance) because the VC retains his equity in the project even after ceasing to invest in it. Thus the entrepreneur's
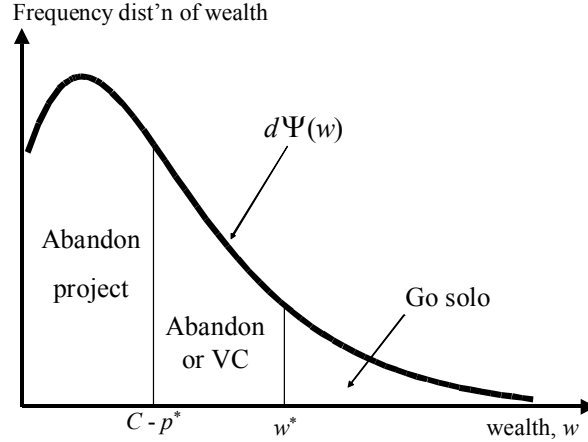
Figure 3: THE EQUILIBRIUM ALLOCATION OF ENTREPRENEURS TO ACTIVITIES

reward would not rise, but her costs would, and so she would strictly prefer to stop right away.

The selection of entrepreneurs into activities.—Entrepreneurs' choices of the mode of investment are described in Figure 3. If $p^* < C$, the VC does not pay the entire fixed cost. The entrepreneur must pay $C - p^*$ up front, and some potential entrepreneurs will have wealth insufficient to cover this amount. These are people with wealth below $C - p^*$ in Figure 3. The fraction of entrepreneurs that wish to get VC backing is $\Psi(w^*)$. But the fraction that can also afford to pay the cost of $C - p$ is just $\Psi(w^*) - \Psi(C - p)$. This is the area "Abandon or VC" in Figure 3. Hence the distributional assumption we need is

$$\Psi(w^*) - \Psi(C - p^*) > \frac{x}{\lambda \bar{t}}. \tag{12}$$

If the previous inequality holds, then (i) there are enough *poor* entrepreneurs who prefer to go with a VC and, (ii) among these there are enough who have enough wealth to finance their share of C. Notice that (12) requires w^* to be larger than $C - p^*$. In addition, in order to interpret p^* as the part of the cost C paid by the VC, we need $p^* > 0$. Both of these inequalities turn out to be true as we prove in the following

Lemma 2

$$\max\{0, C - w^*\} < p^*. \tag{13}$$

Proof. See Appendix A. ∎

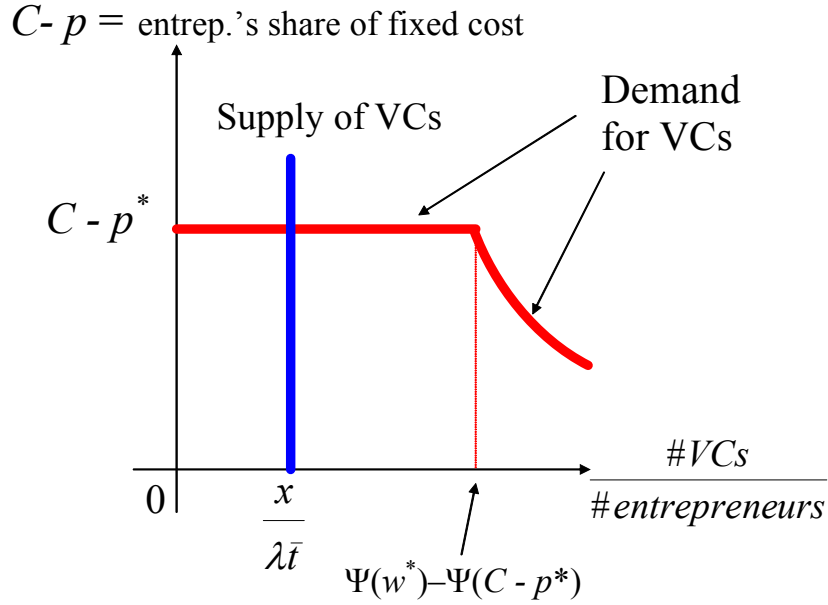Finally, we are ready to claim the following

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Figure 4: THE DETERMINATION OF p WHEN $s = s^*$.

Theorem 1 (The Welfare Theorem) *If (12) holds, the socially optimal outcome coincides with the unique competitive equilibrium outcome in which the allocations are supported by the following strategies:*

(i) A VC always offers the contract (p^*, s^*) given by (10) and (11). If the contract is accepted, he follows the socially optimal decisions, defined by (4).

(ii) An entrepreneur with wealth $w \geq w^*$ goes solo and follows the socially optimal termination rule defined by (8).

(iii) An entrepreneur with wealth $w \in (p^*, w^*)$ seeks VC-backing with probability $x / (\bar{t} \lambda [\Psi (w^*) - \Psi (C - p^*)])$ and abandons her project otherwise. Entrepreneurs seeking VC backing accept the contract offered by the VC, and follow the socially optimal decisions defined by (4).

(iv) An entrepreneur with wealth $w \leq C - p^*$ abandons her project.

The equilibrium is further described in Figure 4. In the figure, as p varies, s is held fixed at s^*. The vertical axis measures the up-front cost to E. The Figure shows the following:

1. If there were no VCs, $\Psi (w^*)$ entrepreneurs would abandon their projects, and the remaining $1 - \Psi (w^*)$ would go solo as shown in Figures 2 and 3.

2. Since abandoning her project offers E zero rents, Es' demand for VCs is infinitely elastic at $C - p^*$ up to the point $\Psi (w^*) - \Psi (C - p^*)$ (The poorest $\Psi (C - p^*)$ entrepreneurs cannot afford the up-front cost).

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3. As p rises above p^* some of Es that would otherwise go solo would start switching to VCs. In addition, some other Es, who had insufficient liquidity to pay their part of the fixed cost earlier, can now afford to contract with VCs instead of abandoning their projects.

3.3 Proof of the Welfare Theorem

First, we prove that given the decision about the financing mode, Es' as well as the VCs' decisions regarding the termination time of a project are indeed socially optimal. That is, we prove the second parts of claims (i), (ii), and (iii) of Theorem 1. If E decides to go solo, then she is the one who incurs all the costs related to the project, but she also enjoys all the potential benefits. In other words, her costs and benefits are identical to the social costs and benefits, hence she follows the socially optimal decision rules described by (8). Therefore, we only have to show these claims for VC-backed projects.

Second, we show that given the termination times, the decisions regarding the financing mode are as described in the first parts of claims (ii), (iii), and (iv) of the theorem.

Since the VC extracts all the surplus, he has no incentive to offer a different contract. For the same reason, Es with $w \in [C - p^*, w^*]$ are indifferent between the three options.

Incentive Compatibility and Optimality of the Contract (p^*, s^*)

We now analyze the incentives of the agents to support the project after a contract (p, s) is signed and both parties learn the value of π.

Entrepreneur.—If E trusts that the project is always financed by the VC, and that she will get $s^*\pi$ if the project is successful, she solves the following problem:

$$\max_T \int_0^T \left(s^*\pi - \frac{a}{h(t)} \right) e^{-rt} f(t) \, dt. \tag{14}$$

If the solution, $T^E(\pi)$, is interior it is defined by the corresponding first-order condition:

$$h\left(T^E(\pi)\right) = \frac{a}{s^*\pi}. \tag{15}$$

The local second-order condition, which is also is also the sufficient condition, is again $h'\left(T^E[\pi]\right) < 0$. Finally, if π is below a cutoff, denoted by π_c^E, she does not start to exert effort.

VC.—In equilibrium, the market value of a free VC is just W. If VC trusts that E will always support the project, and that he, VC, gets $(1 - s^*)\pi$ if the project succeeds, his maximization problem after signing the contract is

$$\max_T \int_0^T \left((1 - s^*)\pi + W - \frac{k}{h(t)} \right) e^{-rt} f(t) \, dt + e^{-rT}(1 - F[T]) W. \tag{16}$$

15

If the solution, $T^{VC}(\pi)$, is interior, it must solve the first-order condition

$$h\left(T^{VC}(\pi)\right) = \frac{k + rW}{(1 - s^*)\pi}. \tag{17}$$

The sufficient condition is again $h'\left(T^{VC}[\pi]\right) < 0$. If π is lower than a cutoff, denoted by π_c^{VC}, the VC does not invest in the project.

Incentive compatibility.—E and the VC stop supporting the project at the same moment if and only if $h\left(T^{VC}(\pi)\right) = h\left(T^E(\pi)\right)$ and $\pi_c^E = \pi_c^{VC}$. First, suppose that the solutions for both (14) and (16) are interior. Equations (15) and (17) define the same termination rules if and only if

$$\frac{a}{s^*\pi} = \frac{k + rW}{(1 - s^*)\pi}.$$

But this requires that

$$s^* = \frac{a}{(a + k + rW)}, \tag{18}$$

which is exactly the definition of s^*. It remains to show that the solution to (14) is interior if and only if the solution to (16) is interior. That is, the cutoffs, under which agents refuse to support the project, are the same for E and the VC. We show more: we prove in Appendix A that both cutoffs coincide with the socially optimal cutoff, $\pi_{\min}$.

Lemma 3 $\pi_c^E = \pi_c^{VC} = \pi_{\min}$.

Optimality.—We argue that the termination decisions described above are the socially optimal ones. First of all, by Lemma 3, those projects which are socially efficient to abandon right after paying C are also abandoned in equilibrium. Second, from (4) the socially optimal termination decision, $T^*(\pi)$, regarding a project which is worth starting to support, satisfies $h\left(T^*(\pi)\right) = (a + k + rW)/\pi$. Hence, in order to prove efficiency of the rule (15) (or equivalently (17)), we need that

$$\frac{a}{s^*\pi} = \frac{a + k + rW}{\pi},$$

which is satisfied by (18).

The Choice of Financing Mode of an Entrepreneur

We now show the first parts of claims (ii), (iii), and (iv) of Theorem 1.

Suppose first, that E has initial wealth $w \leq C - p^*$. Then, she cannot contract with a VC, because she cannot finance $C - p^*$ up-front. By Lemma 2 $w < w^*$, hence she is better off abandoning than going solo.

Suppose that $w \in (C - p^*, w^*)$. Since $w < w^*$, E is still better off abandoning than going solo. However, she has enough wealth to finance $C - p^*$ of the fixed

cost. Since the VCs extract all the surplus from the projects, these Es are indifferent between seeking VC-backing and abandoning. So they can randomize according to claim (iii) of Theorem 1.

If $w > w^*$, E goes solo, since her payoff, $w - C + Q^E(w)$, is larger than w, and her other options all provide her with a payoff of w.

Uniqueness of the competitive equilibrium actions.—The project-acceptance and termination decisions are uniquely determined, as is the equilibrium contract (p^*, s^*). The proof of uniqueness goes as follows. In any equilibrium, the VC-backed entrepreneurs do not enjoy any rent. Furthermore, at the equilibrium stopping time, both parties are indifferent between supporting and abandoning the project. These two observations imply that the Bellman equation defining the market value of a VC is identical to (1). Since (1) had a unique solution, the market value of a VC is also uniquely determined, and equals his social value. If the VC were to offer any other contract it would have to attain this value W and implement the actions $(\pi_{\min}, T^*)$.

3.4 Discussion of the Welfare Theorem

There are two reasons why here the validity of the First Welfare Theorem might seem surprising. First, agency problems could arise, since neither VC investment nor E's effort is contractible. Second, agents are not price-takers – VCs are strategic when offering contracts. Next, we explain why the competitive equilibrium is efficient despite of these problems.

Discussion of the Equilibrium Contract

We emphasize that the validity of the Welfare Theorem in our model is due to the existence of institutions and not a natural feature of the world. More precisely, it is the contractibility of the profit sharing rule that makes it possible for both E and the VC to make socially optimal decisions.

Intuition for the equilibrium contract.—The VC and E must be given the incentive to both support the project exactly up to $T^*(\pi)$. The equity s^* in (10) provides both the right incentives because the marginal cost of a VC-backed project is shared correctly between the VC and E. The VC invests k and incurs an opportunity cost of rW, while E exerts effort a. Then E cares only about her own cost, a, and not the social cost $a + k + rW$. However, if $s = a/(a + k + rW)$, then E's benefit is $[a/(a + k + rW)]\pi$, instead of the social benefit π. This means that the objective function of E is the objective function of the social planner down-scaled by the constant $a/(a + k + rW)$. Similarly, the objective function of the VC is down-scaled by $(k + rW)/(a + k + rW)$. Scaling does not affect decisions, however, and the VC and E both choose the same T as the planner would. Therefore, our model explains why we observe *equity* contracts between VCs and Es, and not, e.g., debt-contracts.

Debt contracts.—If the VC and E signed a debt contract, E would have to incur the total cost of a project on the margin. But if, as we assume, E would have to repay that debt from the project's payoff, π, her marginal benefit from a succeeding

project would be less than π, i.e., strictly less than the social marginal benefit. This would induce E to terminate the project too early. Thus, debt contracts would induce inefficient terminations.

Labor contracts.—If effort were observable, the VC could hire E as a worker and induce the efficient outcome. If E were to receive a wage of a at each instant, she would be exactly compensated for her effort, and would hence be willing to accept such a contract. The VC would then internalize the social costs and benefits of the project, and both parties would make socially optimal decisions. Such a contract, however, is not robust to the observability of E's effort. Had the VC not been able to observe E's effort, E would strictly prefer to shirk for two reasons. First, if she shirked, she would not incur the disutility of working. Second, by shirking she would prevent the project from succeeding, and would thereby prolong the time during which she received the wage. In our equilibrium, on the other hand, the observability of the effort (and even the observability of the project still being alive) plays absolutely no role because s^* gives E and the VC exactly the right incentive to support the project.

Discussion of the VC's market value

In order to achieve efficiency, the social value of a VC must coincide with his market value. Condition (12) contains two assumptions: (A) There must be fewer VCs than there are Es that seek VC-backing, and (B) Among these Es there must be sufficiently many that can afford to pay $C - p^*$ up-front. (A) is crucial to our result, for it provides the VCs with market power and with the ability to offer contracts that enable them to extract the full surplus from a project. That is why the market value and the social value of a free VC are the same. But (B) is less important and can be easily relaxed by introducing more complicated contracts. The VCs could extract surplus from more liquidity constrained entrepreneurs if the sharing rule s was increasing over time. Recall that we have restricted attention to time-independent sharing rules, and hence the entrepreneur was only indifferent between exerting effort and shirking at the time of termination, but strictly preferred to exert effort anytime before. If s was allowed to change over time, the entrepreneur could have been made indifferent between working and shirking at times before the termination of the project, and by such contracts surplus could have been extracted from *poorer* entrepreneurs without violating incentive constraints.

In the search-matching models of Inderst and Muller (2004) and Michelacci and Suarez (2004), Nash bargaining divides the rents between the VC and E. The "Hosios condition" (which states that factor shares in the constant-returns-to-scale matching function should equal the factors' relative bargaining strength) must hold for the equilibrium to be efficient. It is pure coincidence if that equality should obtain, and so generically these models imply inefficiency of equilibria – policies that change incentives for entry by one side or the other can generally improve the sum of the payoffs. In our model, by contrast, efficiency holds on an open set of all parameter values; since w^*, p^*, and $\bar{t}$ are continuous in the parameters of the model, condition (12) holds for a large range of parameters. It is true that our model takes the relative

numbers of Es and VCs as exogenous. Since VCs get the full social value of their capital, if we endogenized venture capital we would expect that an optimal amount of it would be created.

Bergemann and Hege (2005, henceforth BH) argue that dynamic contracts between Es and VCs are inefficient relative to first best. In their model the project succeeds with some probability in each period, and the payoff is proportional to the invested funds. As in our setup, as time passes without success, agents become more and more pessimistic but, in contrast to our model, E can divert the invested funds to private consumption (with or) without the VC observing it. The authors show that the project is supported for a time that is shorter than would be socially efficient, and it gets less funds. The explanation is that in order to provide E with proper incentives, the optimal contract must specify a decreasing stream of investment funds. While in BH incentives cause early and inefficient terminations of VC-backed projects, in our model the VC's high opportunity cost result in early, yet efficient, terminations. In any case, it would seem that the closeness of VC oversight makes it hard for E to divert funds to consumption.

4   Empirical implications

This section discusses some qualitative implications and assumptions of the model and compares them with evidence.

1. *The market power of the* VC.—(12) implies that in Figure 4 the supply of VCs is to the left of the kink in the demand curve. Small shifts in supply should leave the equilibrium return to venture capital unchanged. In support of this, Kaplan and Schoar (2005, Table 13) find that the entry of new funds does not significantly reduce returns on venture funds. But a small shift in the supply of VCs should also not affect the terms of the contracts, their duration, or the total amount invested, and here the evidence is less favorable: Gompers and Lerner (2000) find that outward shifts in the supply of venture capital act to raise the total amount that VCs pay into the companies they oversee, and Hochberg, Ljungqvist, and Lu (forthcoming) find that the likelihood of getting to IPO rises after a positive shock to the supply of funds.

2. *The excess rate of return to* VC*s and* E*s.*—Having estimated the model's parameters exclusively from the data on VC-backed projects, we can calculate the excess return that solo Es earn on their investments. This excess return depends on E's wealth. Figure 5 is based on our estimated model in which E's excess return which never reaches that of the VC. At w^*, E is indifferent between going solo and abandoning the project, hence there her excess return is zero. It then rises with E's level of wealth, becoming flat when $w = C + k/r$, i.e., the point where the solo E ceases to be liquidity constrained in any state of the world, i.e., for any realization of π. The formulas are in the Appendix. The second panel of Figure 5 is the estimated
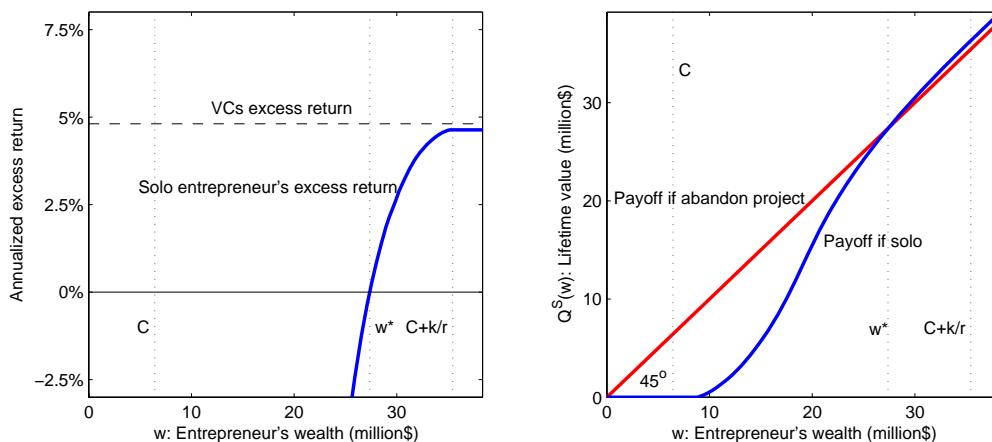
Figure 5: THE EXCESS RETURN OF SOLO ENTREPRENEURS

analog of Figure 2. At \$28 million, w^* is quite large. This estimate comes about because we estimate C to be \$6.4 million. In our model a solo E expects to earn at least the market rate, and therefore she cannot have a negative alpha. There is evidence of negative alphas for some Es (Moskowitz and Vissing-Jorgensen 2002) and independent inventors (Astebro 2003), and further work should reveal what may explain such patterns. Recently, however, Hopenhayn and Vereshchagina (2004) and Miao and Wang (2005) have argued that when one factors in the option value of the project, the alphas are nonnegative. This exercise should be qualified by noting that the types of ventures that VCs fund are high-tech and require more funding than the average small business.

3. *Terminations.*—VCs are less patient with their projects than rich Es, and therefore are more likely to terminate a non-performing venture. We cannot rank terminations of VC-backed firms and those of poor Es because on the one hand, the VC is less patient, but on the other, the poor entrepreneur may run out of money. This may explain why Manigart *et al.* (2002) and Goldfarb et al. (2006) find no significant difference between the failure hazards in the two populations. In contrast, Ber and Yafeh (2004) find that the probability of survival until the IPO stage is higher for VC-backed companies. Nevertheless, based on Figure 6, we know that VC-backed firms must have higher termination hazards *initially.*

4. *Value at IPO.*—Hochberg (2004) finds that the stock-market value of IPO is higher when a VC is present, and Megginson and Weiss (1991, Table III) find this to be true even when one controls for book values. In our model, book value presumably is $C + tk$, so that, given t, the book values of VC-backed and solo firms would be the same. The average market value of a VC-backed company that succeeds at age t is $E\left(\pi \mid T^{VC}\left(\pi\right) \geq t\right)$, and its counterpart for the value of a solo firm run by a rich E is
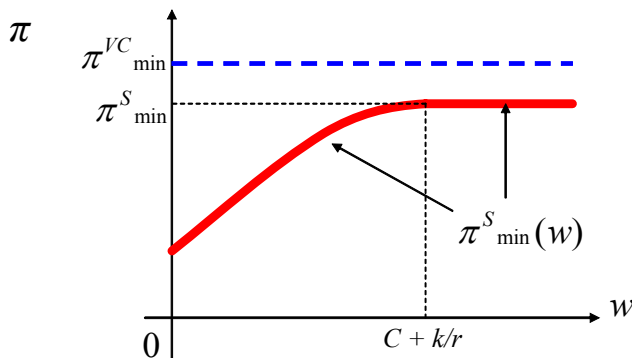
Figure 6: DATE-ZERO TERMINATIONS BY VC AND SOLO FIRMS

$E\left(\pi \mid T^{E}\left(\pi\right) \geq t\right)$. Since T^{VC} and T^{E} are both increasing in π, $E\left(\pi \mid T^{VC}\left(\pi\right) \geq t\right)$ will exceed $E\left(\pi \mid T^{E}\left(\pi\right) \geq t\right)$ given the following result, proved in Appendix A:

Proposition 2

$$T^{VC}\left(\pi\right) \leq T^{E}\left(\pi\right). \tag{19}$$

The proof first shows that the VC is more selective at age zero than E, that is, his cutoff for starting to support a project is higher than that of a wealthy solo E:

$$\pi_{\min}^{E} \leq \pi_{\min}^{VC}. \tag{20}$$

This is because the VC only starts supporting a project if it yields at least an expected payoff of W, while for a rich solo E it suffices that it yield a positive profit. The proof then shows that the VC is more selective at all ages than the rich E, because – while the marginal benefit of waiting is the same for a VC and E – for the VC the marginal cost is higher by the amount rW. Figure 6 illustrates (20) about age-zero terminations, and it portrays $\pi_{\min}^{E}\left(w\right)$ for all w, the shape of which is based on the following result, also proved in Appendix A:

Proposition 3 $T_{w}^{E}\left(\pi\right)$ *is increasing in w and constant on $[C + k/r, \infty)$.*

Therefore poor Es are also more selective than rich Es at age zero, and, according to (8), equally selective afterwards, as long as their funds last. Therefore expected market-book ratios of VC-backed firms exceed those for rich Es, but not necessarily those of poor Es.

   5. *Age at IPO.*—VC-backed IPOs should be stochastically younger, in the first-order sense, than the rich-solo IPOs, but they can be stochastically older than the IPOs of liquidity-constrained Es; the latter is true, roughly, if the liquidity constraint bites earlier than the VC's impatience constraint. The data on age of IPOs since firm
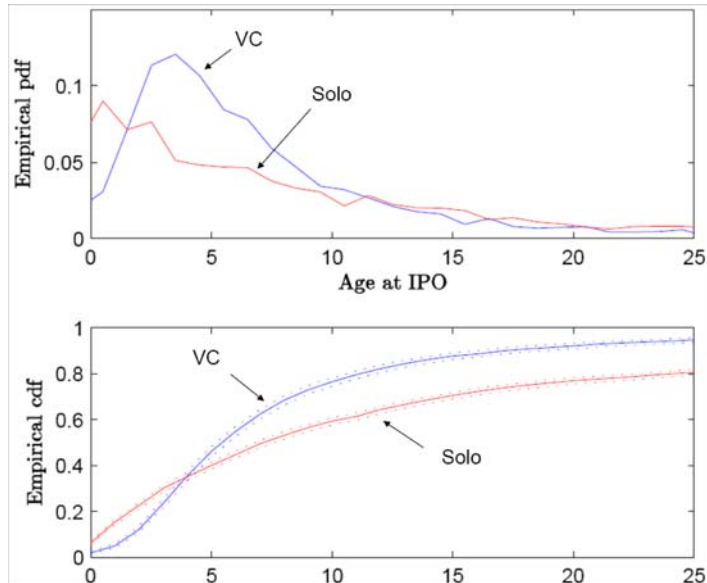
Figure 7: DISTRIBUTION OF AGE (SINCE FOUNDING) AT IPO

founding on the NYSE and Nasdaq shown in Figure 7 turn out to be somewhere in between – it excludes spinout and rollup (i.e., consolidation) IPOs in the plots, but including them would not change the results.

6. *Financing is staged and good projects receive more investment rounds.*—The VC infuses further capital at various points if and only if E has exerted effort – capital infusions are a part of an incentive contract to help elicit good behavior from E. Moreover, the duration of investment depends on π. Gompers (1995) finds that bad VC-backed projects are dropped and that good projects get more investment. This happens in our model; the duration of investment rises with π: Projects with $\pi \leq \pi_{\min}$ get no investment beyond C, and from (3),

$$\frac{dT^{*}(\pi)}{d\pi} = -\frac{h\left(T^{VC}[\pi]\right)}{h'\left(T^{VC}[\pi]\right)\pi} > 0,$$

because at the point of intersection $h' < 0$. Since $T^{*} = T^{VC}M$, this proves the claim for the VC-backed projects, and differentiation in (8) establishes the same property for T_{w}^{E}.

5 Adding random delay to the signal on π

The model says that all projects for which $\pi \leq \pi_{\min}$ are dropped at once. Our data show that while some terminations do occur early, they don't occur immediately. This could be because it takes time to learn π and so we shall now introduce a delay

in receiving the signal about π. This extension does not affect our theoretical results and our welfare theorem still holds, but the modified model will fit the data better.

Suppose that a perfect signal on π arrives at date x, and that x has distribution B and density b.

VC-backed projects.—If $x = 0$, the planner's problem is the same as that in Section 3.1.1, and conditional on W, the solution, T^*, is again (4), with $\pi^{VC}_{\min}$ still being the cutoff π. But for $x > 0$, the cutoff is different from $\pi^{VC}_{\min}$ for two reasons. First, the costs incurred for $t \in [0, x]$ are now sunk and, second, the waiting time to success is now $F\left(t \mid t > x\right)$. Therefore the cutoff, call it $\pi_{\min}\left(x\right)$, is the largest value of π that solves

$$W = \max_{T} \int_{x}^{T} \left(\pi + W - \frac{a+k}{h\left(t\right)}\right) e^{-r(t-x)} \frac{f\left(t\right)}{1 - F\left(x\right)} dt + e^{-r(T-x)}\left(1 - \frac{F\left(T\right) - F\left(x\right)}{1 - F\left(x\right)}\right) W. \tag{21}$$

The marginal condition for T is still (3), and so if $x < T^*\left(\pi\right)$ and if $\pi > \pi_{\min}\left(x\right)$, the project is still terminated at time $T^*\left(\pi\right)$. But if $x \geq T^*\left(\pi\right)$ or if $\pi \leq \pi_{\min}\left(x\right)$, the project is terminated at x, i.e., at once. To sum up: The optimal stopping time of a project with payoff π whose signal arrived at age x is

$$T^*\left(\pi, x\right) = \begin{cases} T^*\left(x\right) & \text{if } \pi > \pi_{\min}\left(x\right), \\ x & \text{otherwise.} \end{cases}$$

Let T_0 denote the age at which the planner terminates a project about which no signal has yet come in. If the payoff is π and if $x < T_0$, the social payoff of that project is

$$V_{T_0}\left(\pi, x\right) = -C + \int_{0}^{T^*\left(\pi, x\right)} \left(\pi + W - \frac{a+k}{h\left(t\right)}\right) e^{-rt} dF\left(t\right) + e^{-rT}\left(1 - F\left[T^*\left(\pi, x\right)\right]\right) W.$$

But if the payoff is π and if $x \geq T_0$, the social payoff of the project is

$$V_{T_0}\left(\pi, \infty\right) = -C + \int_{0}^{T_0} \left(\pi + W - \frac{a+k}{h\left(t\right)}\right) e^{-rt} dF\left(t\right) + e^{-rT_0}\left(1 - F(T_0)\right) W.$$

Hence, the Bellman equation defining W is

$$W = \max_{T_0} \left\{ \int_{x=0}^{T_0} \int V_{T_0}\left(\pi, x\right) dG\left(\pi\right) dB\left(x\right) + \left(1 - B\left(T_0\right)\right) \int V_{T_0}\left(\pi, \infty\right) dG\left(\pi\right) \right\}. \tag{22}$$

where T_0 denotes the termination time of a project with unknown π. Let T_0^* denote the solution to the maximization problem on the right-hand side of (22). Appendix A proves

Lemma 4 *The First-Order Condition corresponding to (22) is*

$$a + k + rW = h\left(T_0^*\right) \int \pi dG\left(\pi\right) + \delta\left(T_0^*\right) \int \frac{V_{T_0^*}\left(\pi, T_0^*\right) - V_{T_0^*}\left(\pi, \infty\right)}{1 - F\left(T_0^*\right)} dG\left(\pi\right), \tag{23}$$

where $\delta = b/\left(1 - B\right)$ denotes the hazard rate of the signal.

The LHS of (23) is the marginal cost of waiting. The RHS, the marginal benefit, decomposes into two terms: First, the project might succeed in the next instant; the likelihood of this event is $h(T_0^*)$, and the expected payoff is just $\int \pi dG(\pi)$, since no selection on π has yet occurred. Second, the signal on π may arrive in the next instant; the likelihood of this event is $\delta(T_0^*)$, and the project's value changes from $V_{T_0^*}(\pi, \infty)$ to $V_{T_0^*}(\pi, T_0^*)$.

Solo E*s.*—A rich solo E solves the same problem as the one above, except $W = 0$ in all formulas. An E facing budget constraint again terminates either at the same time as a rich solo E would, or when he runs out of money. This is true for x above the mode of h. To the left of the mode, however, $\pi_{\min}$s depend on both x and w, and they solve the analog of (21). We shall plot $\pi_{\min}(x, w)$ in Appendix B.

6   Fitting data

Data description.—The data comprise VC investments in U.S. portfolio companies founded between 1989 and 1993. Follow-on investments in these companies extend to 2001. The data are from the VentureXpert database provided by Thompson Venture Economics. They follow 1745 companies. After dropping inconsistent, irrelevant, and missing data, the final data cover \$27 billion of venture capital investment in 1355 portfolio companies, of which 25% exited through an IPO, 19% were acquired, 32% ceased to receive venture capital financing (terminated)[10]

DATA SUMMARY

age	ipo	acq	term	#evnts	#left	suc. hzd	fail hzd
0	12	8	0	20	1355	0.01	0.00
1	39	19	119	177	1335	0.04	0.09
2	54	49	103	206	1158	0.09	0.09
3	65	42	61	168	952	0.11	0.06
4	67	47	50	164	784	0.15	0.06
5	27	24	36	87	620	0.08	0.06
6	22	23	20	65	533	0.08	0.04
7	16	11	19	46	468	0.06	0.04
8	5	10	17	32	422	0.04	0.04
9	0	5	6	11	390	0.01	0.02
10	2	4	6	12	379	0.02	0.02
11	0	1	1	2	367	0.00	0.00
12	0	0	1	1	365	0.00	0.00

---

[10]We infer that a termination has occured 462 days after the last reported VC investment in a terminal period of inactivity. Thus a 'termination' annot occur in the first year of a firm's life and, hence, the 'zero' in the first row of column 4. This termination criterion was imposed on this sample in Guler (2003) and on its update in Guler (2007).

and 19% of companies in the sample were private companies still in the process of receiving venture capital financing, and for them the possibility of an IPO or acquisition was still deemed to exist as of 2001. The Table summarizes the data focusing on the relation between the firm's age in the first column, and the three events that may befall the firm.

Estimated example.—We shall fit the following example to the data:

$$G\left(\pi\right) \; = \; 1 - \left(\frac{\pi}{\pi_0}\right)^{-\lambda} \qquad\qquad\qquad \pi \geq \pi_0 \quad (24)$$

$$B\left(x\right) \; = \; 1 - e^{-\delta x} \qquad\qquad\qquad\qquad\qquad x \geq 0 \quad (25)$$

$$F\left(\tau\right) \; = \; \frac{\rho}{2+\rho}\left(\frac{\min\left(\tau,\tau_{\mathrm{m}}\right)}{\tau_{\mathrm{m}}}\right)^2 + I_{[\tau_{\mathrm{m}},\infty)}\frac{2}{2+\rho}\left[1-\left(\frac{\tau}{\tau_{\mathrm{m}}}\right)^{-\rho}\right] \qquad \tau \geq 0 \quad (26)$$

Thus π has a Pareto distribution, x has a constant hazard, δ, and τ has an increasing density below τ_{m} and a Pareto density above τ_{m}. Thus we have eight parameters: $\pi_0, \lambda, \delta, \rho, \tau_m, C, k$, and r. We set $\tau_m = 4.5$ since this is the peak of the empirical success-hazard rate. We set $r = 0.127$, which would be the rate of return required by the CAPM model given the β of VE returns and given the S&P 500 return over the period. See the Appendix for details. This leaves us with only six free parameters: $\pi_0, \lambda, \delta, \rho, C, k$. We fit the following eight features of the data:

1. the excess return,
2. the VC's rate of return conditional on age,
3. the cost profile,
4. the cumulative cash flow,
5. the equity share of E,
6. the survival function,
7. the success hazard,
8. the termination hazard.

We now discuss how each of these eight features are determined by the data, and how we constructed their theoretical counterparts in our example. We present the formulae for the case $\delta = \infty$, i.e., where the signal arrives immediately. Extending them to the case where the signal is delayed is done in the Appendix. Figure 8 represents the fit of our numerical example. We start with the first panel of Figure 8:

Panel 1: *Excess Return.*—We constrain the parameters to yield $\alpha = 4.8\%$ and we penalize highly any deviation from this value and so we end up with an $\hat{\alpha}$ very close to the desired value.

Panel 2: *The rate of return profile.*—Among projects that succeed at t, the VC's return, call it $R\left(t\right)$, solves the equation

$$\exp\left\{Rt\right\} = \left(1 - s^*\right)\frac{e^{-rt}\int \pi d\Gamma_t}{C + \frac{k}{r}\left(1 - e^{-rt}\right)}. \qquad (27)$$

where Γ is The distribution of π among projects that succeed at time t. This is an arithmetic return, because we first average the payoffs and then take the return. The RHS is the ratio of the present values of revenues (averaged over the whole portfolio) to those of costs. To derive it, note that projects with $\pi < \pi_{\min}$ are terminated at once. Between date zero and date $t = h^{-1}\left([a+k+rW]/\pi_{\min}\right)$, none are terminated, and successes come from the distribution $G(\pi \mid \pi \geq \pi_{\min})$. At $t = h^{-1}\left([a+k+rW]/\pi_{\min}\right)$, the truncation point, $(a+k+rW)/h(t)$ starts to move to the right. Thus the distribution of π among projects that bear fruit at date t is just

$$\Gamma_t(\pi) \equiv \begin{cases} G(\pi \mid \pi \geq \pi_{\min}) & \text{for } t < h^{-1}\left(\frac{a+k+rW}{\pi_{\min}}\right) \\ G\left(\pi \mid \pi \geq \frac{a+k+rW}{h(t)}\right) & \text{for } t \geq h^{-1}\left(\frac{a+k+rW}{\pi_{\min}}\right). \end{cases} \tag{28}$$

The vertical axis in Panel 2 measures $R(t)$ in (27). The horizontal axis measures the firm's age as of the date of the first VC investment. The highest returns are on projects that succeed early. We stop at the seventh year as data become sparse beyond that point and are heavily influenced by outliers.

Panel 3: The cost profile.—We shall fit C and k to the data on the investment profile, i.e., the sequences of investment rounds, but converted to flows of investment as a function of time. The predicted first-period investment relative to subsequent investment is

$$\frac{\text{VC's first-period investment}}{\text{investment in other periods}} = \frac{k+p^*}{k} = 1 + \frac{k+rW}{a+k+rW}.$$

We see some modest decline in the empirical series, but it comes several years later than the predicted, second year.

Panel 4: The cumulative cash flow.—Let $J(t)$ be the VC's cumulative net income. It obeys the ordinary differential equation

$$\frac{dJ}{dt} = \left(-k + h(t)(1-s^*)\int_{\frac{a+k+rW}{h(t)}}^{\infty} \pi d\Gamma_t(\pi)\right) S(t), \tag{29}$$

and the initial condition $J(0) = -p^*$, where p^* is given in (11). For both the model and the data in this case, we plot medians and not means so as to minimize the effect outliers at high ages.

Panel 5: E's equity.—Once E signs the contract with the VC, her share of the project drops from unity to s^*, where it remains until the end. We fit $s^* = \frac{a}{a+k+rW}$ to Kaplan and Strömberg's (2003, Table 2) numbers on cash flow rights, i.e., claims on equity shares. Pooled over rounds, the claim of founders is 31.1%, of VCs 46.7%, and of non-VC investors 22.2%. Since our model does not include non-VC investors, we constrain s^* to equal the share of founders in claims other than those of non-VC investors, i.e.,

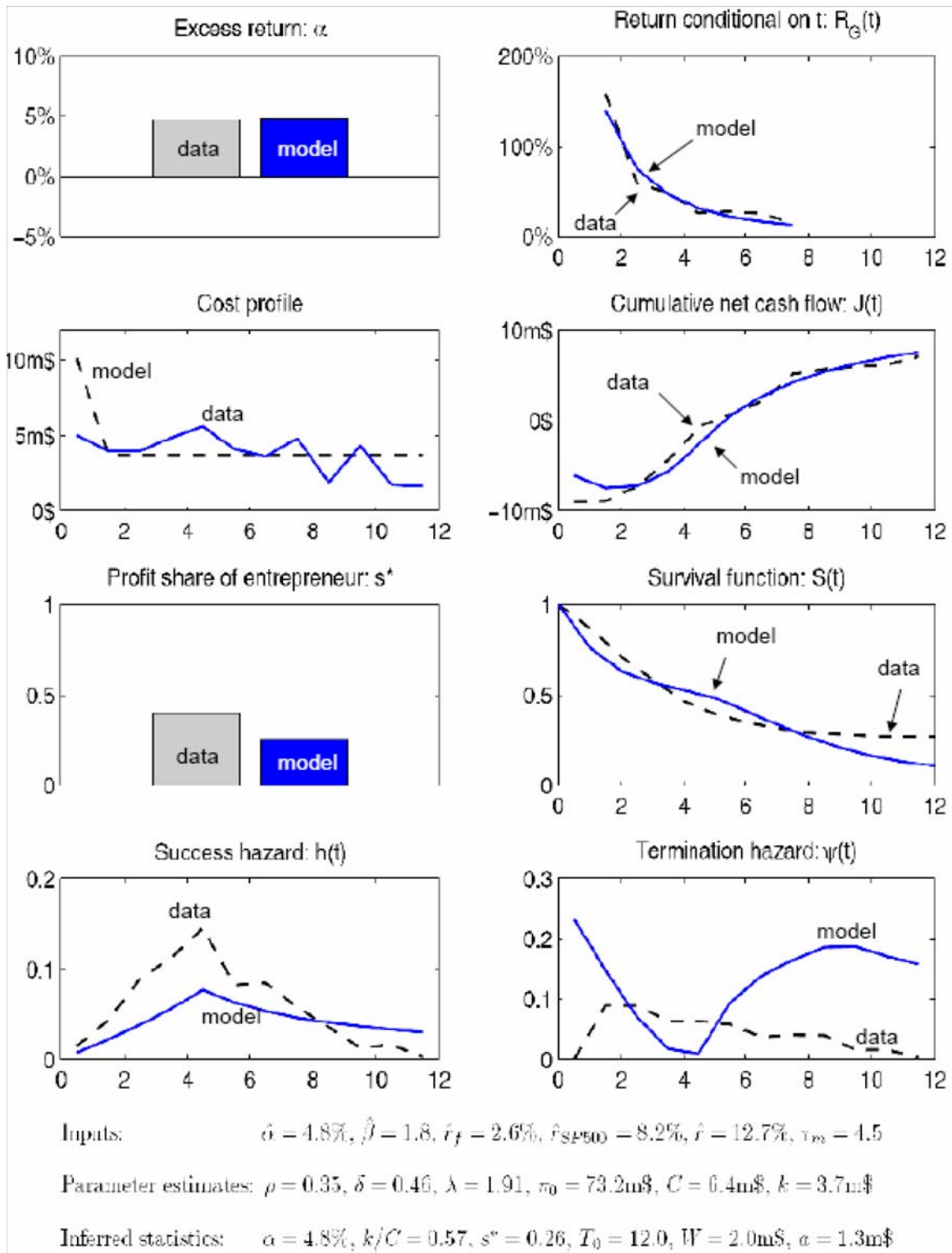$$s^* \approx \frac{31.1}{31.1 + 46.7} = 0.40. \tag{30}$$

Figure 8: FIT OF THE PARETO MODEL WHEN TARGETED $\alpha = 0.048$

The model underpredicts s^* because it needs a large k in order to lessen the predicted decline in Panel 3, and the final estimate is a compromise.

Panels 6, 7 and 8: Hazards and survivorship.—A project "survives" through date t if it has neither succeeded ($\tau > t$) nor been terminated ($T(\pi) > t$). We first need to define terminations. By (3), $\pi = (a + k + rW)/h(T)$, and so the fraction of projects terminated by date t *conditional on no success* until date t, i.e., $\Pr(T \leq t \mid \tau \geq t)$, is

$$\Phi(t) = \begin{cases} 0 & \text{for } t = 0 \\ G\left(\pi_{\min}^{VC}\right) & \text{for } t \in \left(0, h^{-1}\left[\frac{a+k+rW}{\pi_{\min}^{VC}}\right]\right) \\ G\left(\frac{a+k+rW}{h(t)}\right) & \text{for } t \geq h^{-1}\left(\frac{a+k+rW}{\pi_{\min}^{VC}}\right). \end{cases} \tag{31}$$

whence the termination hazard $\psi(t) \equiv \frac{\Phi'(t)}{1-\Phi(t)}$, is

$$\psi(t) = \begin{cases} \infty & \text{for } t = 0 \\ 0 & \text{for } t \in \left(0, h^{-1}\left[\frac{a+k+rW}{\pi_{\min}^{VC}}\right]\right) \\ (a + k + rW)\,\theta\left(\frac{a+k+rW}{h(t)}\right)\left(\frac{-h'(t)}{[h(t)]^2}\right) & \text{for } t \geq h^{-1}\left(\frac{a+k+rW}{\pi_{\min}^{VC}}\right) \end{cases}, \tag{32}$$

where $\theta(\pi) \equiv g(\pi)/(1 - G(\pi))$ is the hazard rate of G. Since τ and π are independent random variables, the CDF of t is $1 - S(t)$, where

$$S(t) = (1 - F[t])(1 - \Phi[t]) = \exp\left(-\int_0^t (h(s) + \psi(s))\,ds\right) \tag{33}$$

is the model 'Survivor function'. The last three panels of Figure are not independent: h and ψ imply S via (33). We measure $S(t)$ by the '#left.' This is the dashed line in panel 6 of the figure. Our measure of h, 'suc. hzd' is calculated as (`ipo` + `acq`)/(`#left`). Thus we treat IPOs and acquisitions as equivalent success realizations. Finally, 'fail hzd' is the ratio (`term`)/(`#left`). This is the empirical counterpart of $\psi(t)$. The Appendix explains in detail.

Estimation procedure.—The estimates of our free parameters are determined as the minimizers of the following loss function, RSS:

$$\text{RSS} = w_1[\ln\hat{\alpha} - \ln 0.048]^2 + w_2\left[\ln(\hat{k}/\hat{C}) - \ln(k/C)\right]^2 +$$

$$+ w_3\sum_{i=2}^{8}[\hat{R}(t) - R(t)]^2 + w_4\sum_{t=0}^{12}[\hat{J}_t - J_t]^2 + w_5[\ln\hat{s}^* - \ln s^*]^2 +$$

$$+ w_6\sum_{t=1}^{12}[\hat{h}_t - h_t]^2 + w_7\sum_{t=0}^{12}[\hat{S}_t - S_t]^2 \tag{34}$$

We only penalize two out of the three functions $h(t)$, $S(t)$ and $\psi(t)$ since knowledge of two pins down the third. We chose penalties on logarithms for some moments since

a simple quadratic scheme did not penalize absurd behavior sufficiently; for example, when the ratio k/C goes from $1/10$ to $1/100$, this is a big qualitative change in our eyes but would not be penalized much by a purely quadratic scheme. For the weighting scheme, we chose $w = (5, 1, 1, 0.1, 0.1, 0.5, 0.01)$. The high penalty on α reflects the importance we attach to this parameter. The other penalties are chosen to reflect the scale of the respective parameters. We used a *Matlab*-built-in line-search method to minimize RSS with respect to the parameters of the model. The minimization process proved to be very robust; the algorithm converged to the same solution for almost any of about 100 randomly chosen starting points. The estimates are reported together in Table 1, and some statistics of interest are reported in Table 2.

α	C	k	π_0	λ	τ_{m}	ρ	δ	RSS
0.048	\$ 6.4 mil.	\$ 3.7 mil.	\$ 73.2 mil.	1.91	4.5	0.35	0.46	0.5259

Table 1: PARAMETER ESTIMATES

Some implications of the parameter estimates.—Note that π_0, the smallest possible π in the support of G (see (24)), is estimated to be an order of magnitude higher than C and k. The implied returns are therefore high if they come early, but the expected waiting time until success is infinity because, since $\rho < 1$, the tail of the Pareto portion of F distribution is thick. According to (26), the probability that $\tau \leq \tau_{\mathrm{m}} = 4.5$ years (which is when the hazard peaks) is $\frac{\rho}{2+\rho} = \frac{0.35}{2.35} = 0.15$. Thus the returns are likely to be highly skewed regardless of the shape of G, simply because most of them are small, so that a large fraction is terminated. The early returns are the big winners. On top of this G is itself right skewed, with $E(\pi) = \frac{\lambda \pi_0}{\lambda - 1} = \$154$ mil. Finally, the mean signal-arrival time $1/\delta = 2.2$ years, i.e., the VC quickly learns the quality of the project.

In our model, venture capital is homogeneous, but some differences could easily be introduced. Suppose, e.g., that some VCs had better signals about a project's likely success, or could see them sooner. Such VCs would face a more favorable distribution of waiting times, F, and of payoffs, G. Panel 1 of the Figure 8 shows that returns drop off quickly as the waiting time increases so that an ability to bring successes forward seems to have a very high return. If VCs differed in quality because some could see a better prior signal, then the high-quality VCs would have a W higher than other VCs. The high-W VCs would then be more selective, having higher $\pi_{\min}$s and lower $T(\pi)$s. Therefore *ex-post* project qualities would be positively correlated with the qualities of the VCs that backed them. This would be consistent with Sørensen (forthcoming) who finds that the bulk of the observed positive association between VC quality and project quality is due not to direct VC influence on the payoff but to sorting. The following additional statistics were obtained from a Monte-Carlo simulation of 3 million projects using the estimated parameters:

Success rate	22.1%	Screened	94.3%
Succeeded unscreened	5.4%	$E[x]$	2.19
Success after screening	16.7%	$E[\pi]$	154m$
Termination rate	77.9%	$E[\pi|\text{success}]$	247m$
Terminated unscreened	0.2%	$E[\tau|\text{success}]$	6.37
Terminated on revelation of π	42.4%	$E[\text{term. time}|\text{termination}]$	6.21
Terminated at $T^*(\pi)$	35.2%		

Table 2: ADDITIONAL STATISTICS

7   Conclusion

We estimated a model of the market for venture capital in which VCs were scarce relative to the number of potential projects. The estimates imply a high equilibrium return on VC capital which makes the VCs impatient to start new funds and to terminate existing non-performing projects. This leads to a selection effect that gives rise to a tendency for VC-backed companies to reach IPOs earlier, and to be worth more at IPO than other start-ups. Our results show that many of the differences between the VC-backed and solo companies can be explained without assuming that VCs add value to projects on aside from providing liquidity, or that the VC-backed projects are ex-ante different from solo ones.

We used the estimated model to infer the rate of return on venture capital and on entrepreneurship, the latter rising with the entrepreneur's wealth. The VC earned a higher excess return than even the wealthiest entrepreneurs, but not by much. The equilibrium was socially optimal for a range of parameter values.

The features of the contract were also determined by a sharing rule and an up-front payment. For a wide range of distributions of project quality and wealth on the one hand, and for a wide range of VC supply on the other, the model says that contracts should be the same. To the extent that they should arise at all, cross-economy differences in contracts should be expressible in terms of just two statistics: The share of up-front costs that the VC finances, and the VC's share in the equity of the project.

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8 Appendix A: Omitted Proofs

Proof of Lemma 1. (i) Suppose that $w < k/r + C$. From (5), $\tau'(w) = 1/[k - r(w - C)]$. Suppose now, that $h^{-1}((a+k)/\pi) > \tau(w)$. That is, the project with payoff π is worth supporting longer than time $\tau(w)$. Since the support of π is $\mathbb{R}_+$, the measure of these $\pi's$ is positive. Then, by (7) and (8)

$$q(\pi, w) = \int_0^{\tau(w)} \left(\pi - \frac{a+k}{h(t)}\right) e^{-rt} f(t) \, dt,$$

and therefore

$$\frac{\partial q(\pi, w)}{\partial w} = \tau'(w) \left(\pi - \frac{a+k}{h(\tau(w))}\right) e^{-rt} f(\tau(w))$$

$$= \frac{1}{k - r(w - C)} \left(\pi - \frac{a+k}{h(\tau(w))}\right) e^{-rt} f(\tau(w)).$$

The first term is positive since $w < k/r + C$. The second term is also positive since $h^{-1}((a+k)/\pi) \geq \tau(w)$. Suppose now that $h^{-1}((a+k)/\pi) < \tau(w)$, that is E can support the project with payoff π as long as it is socially optimal. The measure of such $\pi's$ is positive because $\pi_{\min}$ is positive. Then, by (7) and (8)

$$q(\pi, w) = \int_0^{T^E(\pi)} \left(\pi - \frac{a+k}{h(t)}\right) e^{-rt} f(t) \, dt,$$

and

$$\frac{\partial q(\pi, w)}{\partial w} = 0.$$

We can conclude that

$$\frac{\partial q(\pi, w)}{\partial w} = \begin{cases} \frac{h(\tau(w))\pi - a + k}{h(\tau(w))[k - r(w - C)]} e^{-rt} f(\tau(w)) & \text{if } h^{-1}((a+k)/\pi) \geq \tau(w) \\ 0 & \text{if } h^{-1}((a+k)/\pi) < \tau(w) \end{cases}. \quad (35)$$

Notice that

$$\frac{dQ^E}{dw} = \int \frac{\partial q(\pi, w)}{\partial w} dG(\pi) = \int_{h^{-1}((a+k)/\pi) \geq \tau(w)} \frac{\partial q(\pi, w)}{\partial w} dG(\pi)$$

$$= \int_{h^{-1}((a+k)/\pi) \geq \tau(w)} \frac{h(\tau(w))\pi - a + k}{h(\tau(w))[k - r(w - C)]} e^{-rt} f(\tau(w)) > 0.$$

(ii) Suppose now that $w < k/r + C$, that is, E never runs out of money when she is supporting a project. Then, by (7) and (8)

$$q(\pi, w) = \int_0^{T^E(\pi)} \left(\pi - \frac{a+k}{h(t)}\right) e^{-rt} f(t) \, dt,$$

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and hence $dQ^E/dw = 0$. By assumption, the social value of the project is positive. This implies that the unconstrained entrepreneur generates a larger expected payoff by optimally supporting a project than abandoning. But the payoff of supporting a project in excess to that yielded by abandoning is exactly $Q^E(w) - C$, and hence $Q^E(w) > C$. ∎

Proof of Lemma 2. Recall that $Q^E(w^*) = C$ and $Q^E(C) = 0$ (see Figure 2). Since Q^E is strictly increasing in w on $[0, k/r + C]$ by Lemma 1, it follows that $w^* > C$. Hence, in order to prove (13) we only have to show that $p^* > 0$. We show more, we prove that

$$p^* = \frac{k + rW}{a + k + rW}C. \tag{36}$$

Recall from (11) that

$$
\begin{aligned}
C - p^* &= \int \int_0^{T^*(\pi)} \left(s^* \pi - \frac{a}{h(t)} \right) e^{-rt} f(t) \, dt \, dG(\pi) \tag{37} \\
&= \int \int_0^{T^*(\pi)} \left(\frac{a}{a + k + rW} \pi - \frac{a}{h(t)} \right) e^{-rt} f(t) \, dt \, dG(\pi) \\
&= \frac{a}{a + k + rW} \int \int_0^{T^*(\pi)} \left(\pi - \frac{a + k + rW}{h(t)} \right) e^{-rt} f(t) \, dt \, dG(\pi), \tag{38}
\end{aligned}
$$

where the second equality used the definition of s^*. Notice, that

$$\int_0^{T^*(\pi)} -\frac{rW}{h(t)} e^{-rt} f(t) \, dt = -\int_0^{T^*(\pi)} (1 - F(t)) rW e^{-rt} dt$$

and integrating by parts

$$-\int_0^{T^*(\pi)} (1 - F(t)) rW e^{-rt} dt = \left[e^{-rT} (1 - F[T^*(\pi)]) W \right] - W - \int_0^{T^*(\pi)} e^{-rt} f(t) W.$$

Plugging this back to (37) we get that $C - p^*$ can be written as

$$
\begin{aligned}
&\frac{a}{a + k + rW} \int \int_0^{T^*(\pi)} \left(\pi + W - \frac{a + k}{h(t)} \right) e^{-rt} f(t) \, dt + \left[e^{-rT} (1 - F[T]) W \right] - W \, dG(\pi) \\
&= \frac{a}{a + k + rW} \left[\int \int_0^{T^*(\pi)} \left(\pi + W - \frac{a + k}{h(t)} \right) e^{-rt} f(t) \, dt + \left[e^{-rT} (1 - F[T]) W \right] dG(\pi) - W \right] \\
&= \frac{a}{a + k + rW} [W + C - W] = \frac{aC}{a + k + rW},
\end{aligned}
$$

where the third equality follows from (1). Therefore

$$C - p^* = \frac{aC}{a + k + rW},$$

35

which is equivalent to (36). ∎

Proof of Lemma 3. Let us introduce the following notations

$$V\left(\pi, T\right) = \int_0^T \pi - \frac{a+k}{h\left(t\right)} e^{-rt} f\left(t\right) dt,$$

$$V^{VC}\left(\pi, T\right) = \int_0^T \left(1 - s^*\right)\pi - \frac{k}{h\left(t\right)} e^{-rt} f\left(t\right) dt,$$

$$V^E\left(\pi, T\right) = \int_0^T s^*\pi - \frac{a}{h\left(t\right)} e^{-rt} f\left(t\right) dt.$$

That is, $V\left(\pi, T\right)$ $(V^{VC}\left(\pi, T\right)$, $V^E\left(\pi, T\right))$ is the social (VC's, entrepreneur's) payoff generated from a project with payoff π if it is supported up-to time T. Recall that

$$\max_T V\left(\pi, T\right) = W \Longleftrightarrow \pi \leq \pi_{\min}, \tag{39}$$

$$\max_T V^{VC}\left(\pi, T\right) = W \Longleftrightarrow \pi \leq \pi_{\min}^{VC}, \text{ and}$$

$$\max_T V^E\left(\pi, T\right) = 0 \Longleftrightarrow \pi \leq \pi_{\min}^E.$$

First, we show that $\pi_{\min} = \pi_{\min}^{VC}$. By (39), it is enough to show that

$$V\left(\pi, T\right) = V^{VC}\left(\pi, T\right) \Longleftrightarrow V\left(\pi, T\right) = W. \tag{40}$$

Notice that, given that a project is supported up to time T, $V\left(\pi, T\right) - V^{VC}\left(\pi, T\right)$ is

$$\int_0^T \left(\pi - \frac{a+k}{h\left(t\right)} - \left[\left(1 - s^*\right)\pi - \frac{k}{h\left(t\right)}\right]\right) e^{-rt} f\left(t\right) dt$$

$$= \int_0^T \left(\pi - \frac{a+k}{h\left(t\right)} - \left[\frac{k+rW}{a+k+rW}\pi - \frac{k}{h\left(t\right)}\right]\right) e^{-rt} f\left(t\right) dt$$

$$= a \int_0^T \left(\frac{1}{a+k+rW}\pi - \frac{1}{h\left(t\right)}\right) e^{-rt} f\left(t\right) dt.$$

Multiplying through by $\left(a+k+rW\right)/a$,

$$0 = V\left(\pi, T\right) - V^{VC}\left(\pi, T\right) \Longleftrightarrow$$

$$0 = \int_0^T \left(\pi - \frac{a+k+rW}{h\left(t\right)}\right) e^{-rt} f\left(t\right) dt$$

$$= \int_0^T \left(\pi - \frac{a+k}{h\left(t\right)}\right) e^{-rt} f\left(t\right) dt + W \int_0^T -re^{-rt}\left(1 - F\left[t\right]\right) dt. \tag{41}$$

Now, integrating by parts, one can rewrite the last expression in the previous equality chain as

$$\int_0^T -re^{-rt}\left(1 - F\left[t\right]\right) dt = \left. e^{-rt}\left(1 - F\left[t\right]\right)\right|_0^T + \int_0^T e^{-rt} f\left(t\right) dt$$

$$= e^{-rT}\left(1 - F\left[T\left(\pi\right)\right]\right) - 1 + \int_0^T e^{-rt} f\left(t\right) dt \tag{42}$$

Substituting from (42) into (41) we see, that (41) reads

$$
\begin{aligned}
0 &= \int_0^T \left(\pi - \frac{a+k}{h(t)}\right) e^{-rt} f(t)\, dt + W\left(e^{-rT}\left(1 - F[T]\right) - 1 + \int_0^T e^{-rt} f(t)\, dt\right) \\
&= \int_0^T \left(\pi - \frac{a+k}{h(t)}\right) e^{-rt} f(t)\, dt + W\left(e^{-rT}\left(1 - F[T]\right) + \int_0^T e^{-rt} f(t)\, dt\right) - W \\
&= \int_0^T \left(\pi + W - \frac{a+k}{h(t)}\right) e^{-rt} f(t)\, dt + W e^{-rT(\pi)}\left(1 - F[T(\pi)]\right) - W \\
&= V(\pi, T) - W.
\end{aligned}
$$

Therefore (41) and (42) imply (40).

Now we show that $\pi_{\min} = \pi^E_{\min}$. From (39), it is enough to show that

$$
V^E(\pi, T) = 0 \iff V(\pi, T) = W.
$$

Since $s^* = a/(a+k+rW)$

$$
\begin{aligned}
V^E(\pi, T) &= \int_0^T \left(\frac{a}{a+k+rW}\pi - \frac{a}{h(t)}\right) e^{-rt} f(t)\, dt \\
&= \frac{a}{a+k+rW} \int_0^T \left(\pi - \frac{a+k+rW}{h(t)}\right) e^{-rt} f(t)\, dt.
\end{aligned}
$$

But this is exactly $V(\pi, T) - V^{VC}(\pi, T)$ (see (41)). Hence,

$$
V^E(\pi, T) = 0 \Leftrightarrow V(\pi, T) = V^{VC}(\pi, T) \Leftrightarrow V(\pi, T) = W,
$$

where the second equivalence follows from (40). ∎

Proof of Proposition 2. First, we show that (20) holds. This would imply that if $\pi \leq \pi_{\min}$, then (19) is valid. Recall that for $w \geq C + k/r$, $\pi^E_{\min}$ solves

$$
\int_0^{T^E(\pi)} \left(\pi - \frac{a+k}{h(t)}\right) e^{-rt} f(t)\, dt = 0. \tag{43}
$$

Using the notations of the previous lemma, it is enough to show that

$$
\max_T V\left(\pi^E_{\min}, T\right) \leq W. \tag{44}
$$

Notice that

$$
\begin{aligned}
V\left(\pi^E_{\min}, T\right) &= \int_0^T \left(\pi^E_{\min} + W - \frac{a+k}{h(t)}\right) e^{-rt} f(t)\, dt + e^{-rT}\left(1 - F[T]\right) W \\
&= \int_0^T \left(\pi^E_{\min} - \frac{a+k}{h(t)}\right) e^{-rt} f(t)\, dt + \int_0^\infty W e^{-r\min\{t, T^{VC}(\pi^E_{\min})\}} f(t)\, dt \\
&\leq \int_0^{T^E(\pi^E_{\min})} \left(\pi^E_{\min} - \frac{a+k}{h(t)}\right) e^{-rt} f(t)\, dt + \int_0^\infty W e^{-r\min\{t, T^{VC}(\pi^E_{\min})\}} f(t)\, dt \\
&= \int_0^\infty W e^{-r\min\{t, T^{VC}(\pi^E_{\min})\}} f(t)\, dt \leq W.
\end{aligned}
$$

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The inequality follows from T^E being the solo's optimal termination rule (and not T^{VC}), and the last equality from (43). Since T was arbitrary in the previous inequality chain, (44) follows.

It remained to show that (19) even if $\pi > \pi_{\min}$. If $\pi > \pi_{\min}$ then $T^{VC}(\pi) > 0$ and by (20) $T^E(\pi) > 0$. Since h is decreasing at $T^{VC}(\pi)$ and $T^E(\pi)$, a comparison of (6) and (4) implies $T^E(\pi) > T^{VC}(\pi)$. ∎

Proof of Proposition 3. First, we show $\pi_{\min}(w)$ is decreasing in w and constant, $\pi_{\min}^E$, on $[k/r + C, \infty)$. Recall that $\pi_{\min}(w)$ can be defined such that

$$q(\pi, w) = 0 \Leftrightarrow \pi \leq \pi_{\min}(w).$$

Hence, in order to prove that $\pi_{\min}(w)$ is decreasing it is enough to show that $q(\pi, w)$ is increasing in w. Suppose that $w_1 > w_2$. Recall from (7) that

$$
\begin{aligned}
q(w_2, \pi) &= \max_{T \in [0, \tau(w_2)]} \int_0^T \left(\pi - \frac{a+k}{h(t)} \right) e^{-rt} f(t) \, dt \\
&\leq \max_{T \in [0, \tau(w_1)]} \int_0^T \left(\pi - \frac{a+k}{h(t)} \right) e^{-rt} f(t) \, dt = q(w_1, \pi),
\end{aligned}
$$

where the inequality follows because τ is increasing (see (5)). Also notice from (5) that the inequality is weak whenever $w_1, w_2 \geq k/r + C$. This shows that $\pi_{\min}(w) = \pi_{\min}^s$ whenever $w \geq k/r + C$.

Finally, since $\pi_{\min}(w)$ is decreasing in w and τ is increasing and constant on $[k/r + C, \infty)$, the statement of the proposition directly follows from (8). ∎

Proof of Lemma 4. First, notice that

$$\frac{\partial \int_0^{T_0} \int V(\pi, x) \, dG(\pi) \, dB(x)}{\partial T_0} = \frac{\partial \int_0^{T_0} \int V(\pi, x) \, dG(\pi) \, b(x) \, dx}{\partial T_0} = b(T_0) \int V(\pi, T_0) \, dG(\pi). \tag{45}$$

Also notice that

$$\frac{\partial (1 - B(T_0)) \int V(\pi, \infty) \, dG(\pi)}{\partial T_0} = \int \frac{\partial (1 - B(T_0)) V(\pi, \infty)}{\partial T_0} \, dG(\pi).$$

But

$$\frac{\partial (1 - B(T_0)) V(\pi, \infty)}{\partial T_0} = -b(T_0) V(\pi, \infty) + (1 - B(T_0)) \frac{\partial V(\pi, \infty)}{\partial T_0},$$

and

$$\frac{\partial V(\pi, \infty)}{\partial T_0} = e^{-rT_0} f(T_0) \left(\pi - \frac{k + a + rW}{h(T_0)} \right). \tag{46}$$

This latter equality follows from exactly the same argument that led to (3). Hence:

$$
\frac{\partial \left(1 - B\left(T_0\right)\right) \int V\left(\pi, \infty\right) dG\left(\pi\right)}{\partial T_0} \tag{47}
$$

$$
= \int -b\left(T_0\right) V\left(\pi, \infty\right) + \left(1 - B\left(T_0\right)\right) e^{-rT_0} f\left(T_0\right) \left(\pi - \frac{k + a + rW}{h\left(T_0\right)}\right) dG\left(\pi\right)
$$

$$
= -b\left(T_0\right) \int V\left(\pi, \infty\right) dG\left(\pi\right) + \left(1 - B\left(T_0\right)\right) e^{-rT_0} f\left(T_0\right) \left(\int \pi dG\left(\pi\right) - \frac{k + a + rW}{h\left(T_0\right)}\right) \tag{48}
$$

The first-order condition is the sum of (45) and (47) equated with zero, that is

$$
\begin{aligned}
0 &= \left(1 - B\left(T_0^*\right)\right) e^{-rT_0} f\left(T_0^*\right) \left[\int \pi dG\left(\pi\right) - \frac{a + k + rW}{h\left(T_0^*\right)}\right] \\
&\quad + b\left(T_0^*\right) \int \left[V\left(\pi, T_0\right) - V\left(\pi, \infty\right)\right] dG\left(\pi\right).
\end{aligned}
$$

Rearranging,

$$
\begin{aligned}
\left(1 - B\left(T_0^*\right)\right)\left(1 - F\left(T_0^*\right)\right) e^{-rT_0} \left(a + k + rW\right) &= \left(1 - B\left(T_0^*\right)\right) e^{-rT_0} f\left(T_0^*\right) \int \pi dG\left(\pi\right) - \\
&\quad + b\left(T_0^*\right) \int \left[V\left(\pi, T_0\right) - V\left(\pi, \infty\right)\right] dG\left(\pi\right).
\end{aligned}
$$

After dividing through by $\left(1 - B\left(T_0^*\right)\right)\left(1 - F\left(T_0^*\right)\right)$,

$$
a + k + rW = h\left(T_0^*\right) \int \pi dG\left(\pi\right) + e^{rT_0} \delta\left(T_0^*\right) \int \frac{V\left(\pi, T_0\right) - V\left(\pi, \infty\right)}{1 - F\left(T_0\right)} dG\left(\pi\right). \tag{49}
$$

∎

9   Appendix B: Data and the Estimation

This Appendix was written by Matthias Kredler

9.1   Data

In general, we will denote the quantities implied by the model by plain letters (x, e.g.), whereas the moments from the data are denoted by the same letter carrying a hat ($\hat{x}$).

9.1.1   The data on α and r

The value $\hat{\alpha} = 0.048$ that we use is inferred from the quarterly value $\alpha_q = 0.0117$ that Jones and Rhodes-Kropf (2004) report on the bottom of panel A in their table 2. In order to compute the discount rate $r = 0.127$ that we use in the computations, we create the typical return for an investment with the characteristics of VC projects following the capital asset pricing model (CAPM):

$$r = r_f + \beta_{VC}(r_m - r_f) = 0.127$$

We compute $r_f = 0.026$ as the mean of the return on 3-month treasury bills from 1980 to 1999, which is the sample period for Jones and Rhodes-Kropf (2004)'s data. Similarly, we compute $r_m = 0.082$ as the mean of the return on a value-weighted portfolio of stocks listed on the NYSE, AMEX and NASDAQ provided by the Center for Research in Security Prices (CRSP) over the same period. The value $\beta_{VC} = 1.80$ is calculated as the sum of the 5 quarterly β-coefficients that Jones and Rhodes-Kropf (2004) provide in their table 1, panel B.

9.1.2   The data on $R(t)$

We calculate the following statistic over all projects that succeed during year t:

$$R(t) = \frac{1}{t} \log \left[\frac{\frac{1}{N} \sum_{i=1}^{N} e^{-rt} \pi_i}{\frac{1}{N} \sum_{N} \left(\sum_{s=1}^{t} e^{-rs} I_{i,s} \right)} \right],$$

where time is counted from zero on for each project when it receives its first investment round. The term in brackets can be seen as the excess return over r of a big portfolio of VC projects that succeeded after t years. Taking logs and dividing by t annualizes this quantity. We take the logs after averaging over all projects since VC returns are driven by outliers; there is a strong Jensen's-inequality effect that understates the influences of big winners when logs are taken before averaging.

For r, we use the same number as derived in paragraph 9.1.1. In the denominator, we have investment cash flows for project i in year s (after the first investment). The data are taken again from Guler (2003). They are annualized log returns for the projects in Guler's sample and we reproduce them in Table A1:

Year	1-2y	2-3y	3-4y	4-5y	5-6y	6-7y	7-8y
$R(t)$	1.59	0.59	0.49	0.26	0.28	0.26	0.16

Table A1

The data are also plotted as the dashed line in Panel 2 of Figure 8. We restrict it to the years 2 to 8 for the following reasons: For projects succeeding during the first year, it is impossible to distinguish between investment and revenue since there is only one net cash flow in the data. For the years after the 8th year, the data become very sparse – we have less than 10 data points in each of these years. Therefore these are not used in the estimation.

9.1.3 The data on k/C

As a typical value for C, we take $\hat{C} = \$2.6$m. This number is reported by Kaplan, Sensoy and Strömberg (2002) in table 2 as the book value of a VC portfolio company at the business plan in the median. In order to get an estimate $\hat{k}$ for the typical monetary investment k that a portfolio company requires each year, we employ again the data from Guler (2003, Table 6, column 2):

Investment round	1	2	3	4	5	6	7	8	9	10	11	12
Amount (\$ millions)	6.5	5.3	5.5	7	8.4	6.4	5.9	8.2	3.4	8.1	3.4	3.4

Table A2

Because the model has k paid per unit of time and C at the outset, we need to convert these data for *spending by investment round* into *spending per year*. By comparing the speed of terminations we arrived at the conversion factor for converting rounds into flows. If I_j is the average amount invested in round j, we convert this into a flow $I_t = \theta_t I_t$, where[11]

$$\theta_t \equiv \frac{1}{1.25} - \frac{1}{5}\left(\frac{1}{1.25} - \frac{1}{1.5}\right)t.$$

As an estimate for the typical yearly investment flow once the initial investment round is over, we obtain $\hat{k} = \frac{1}{T-1}\sum_{t=2}^{T}\theta_t I_t = \$3.68$m, again from Guler's data. Hence the target value becomes $\hat{k}/\hat{C} = 1.42$.

---

[11] Between the first and the sixth round, the termination hazard falls from 0.12 to 0.08. On the other hand, between year 1 and year 6, the termination hazard falls from 0.09 to 0.04. Thus the ratio of the two hazards rises from $\frac{12}{9} = 1.25$ to $\frac{8}{4} = \frac{1}{2}$. As a rough calculation, then, initially, rounds are once every 1.25 years, and by year 6, they are once every 1.5 years.

9.1.4 The data on $J(t)$

To estimate the cumulative cash-flow $J(t)$ of a typical project, we take the average cumulative cash-flow in Guler's data up to year t. Guler's data for π are the market valuations for the respective company at IPO or at acquisition. In line with the value $s^* = 0.40$ that we use (see below), we assume that 60% of these revenues accrue to VCs. Furthermore, for the positive cash-flows in the formula for $J'(t)$ in section 4.0.5 we calculate *conditional medians at* t instead of *conditional means* (as is done for the statistics inferred from the model, since it is easier to evaluate)— we do this because the mean is very sensitive to the extreme outliers that are present in Guler's data. We reproduce the series we obtain for $\hat{J}(t)$ in table A3 (plotted as the dashed line in Panel 4 of Figure 8.

Year	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{J}_t$	-9.0	-8.9	-7.3	-4.3	-0.7	0.7	2.1	5.1	5.7	6.0	6.2	7.1

Table A3

9.1.5 The data on s^*

The reference value $\hat{s}^* = 0.40$ is taken from Kaplan and Strömberg (2002), table 2.

9.1.6 The data on $S(t)$, $h(t)$ and $\psi(t)$

These data are from the *VentureExpert* database provided by *Venture Economics*, and are described in detail by Guler (2003). For clarity of the argument, the following table reproduces the Data Summary Table from Section 6 of the paper and will help us discuss in greater detail the data on successes and terminations and the construction of the variables.

'Age' is measured as the number of periods since the date of the first VC investment. Note that this measure of age is different from the measure in Figure 7 where 'age' is time elapsed since the firm was founded.

age	ipo	acq	term	#evnts	#left	$\hat{h}$	$\hat{\psi}$
0	12	8	0	20	1355	0.01	0.00
1	39	19	119	177	1335	0.04	0.09
2	54	49	103	206	1158	0.09	0.09
3	65	42	61	168	952	0.11	0.06
4	67	47	50	164	784	0.15	0.06
5	27	24	36	87	620	0.08	0.06
6	22	23	20	65	533	0.08	0.04
7	16	11	19	46	468	0.06	0.04
8	5	10	17	32	422	0.04	0.04
9	0	5	6	11	390	0.01	0.02
10	2	4	6	12	379	0.02	0.02
11	0	1	1	2	367	0.00	0.00
12	0	0	1	1	365	0.00	0.00

Table A4

The last three columns are plotted as the dashed lines in panels six (there normalized by dividing by 1355), seven and eight of Figure 8.

Here is how we calculated the hazards:

1. Column 6, "# left" is the empirical counterpart of $(1 - F[t])(1 - \Phi[t])$, i.e., of $S(t)$ in (33).

2. Column 7, "$\hat{h}$" is the ratio (ipo + acq)/(#left). That is, we treat IPOs and acquisitions as equivalent success realizations. So, e.g., the value of this ratio at age 1 is $\frac{39+19}{1335} = 0.043$, its value at age 2 is $\frac{54+49}{1158} = 0.089$, and so on. We now show that this is the empirical counterpart of $h(t)$. The sum of the columns (ipo + acq) we interpret as the number of successes at date t among firms for whom $T > t$. The probability of $\tau = t$ *and* its surviving beyond t is $f(t)(1 - \Phi[t])$. Therefore we equate these two concepts:

$$(\text{ipo} + \text{acq}) = f(t)(1 - \Phi[t]).$$

Therefore as calculated in Column 7, the "success hazard" is

$$\frac{\text{ipo} + \text{acq}}{\text{\# left}} = \frac{f(t)(1 - \Phi[t])}{(1 - F[t])(1 - \Phi[t])} = \frac{f(t)}{1 - F(t)} = h(t).$$

3. Column 8, "$\hat{\psi}$" is the ratio (term)/(#left). This is the empirical counterpart of $\psi(t)$; to see why, note that term is the number terminated at date t among firms for whom $T \geq t$ and $\tau \geq t$. The probability of $T = t$ *and* $\tau \geq t$ is $\Phi'(t)(1 - F[t])$. Therefore we equate the two concepts: (term) $= \Phi'(t)(1 - F[t])$ whereupon, as calculated in Column 8, the "termination hazard" is

$$\frac{\text{term}}{\text{\# left}} = \frac{\Phi'(t)(1 - F[t])}{(1 - F[t])(1 - \Phi[t])} = \frac{\Phi'(t)}{1 - \Phi[t]} = \psi(t).$$

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9.2 Numerical algorithm to solve the model

To solve the model, we make use of the monotonic relationship between W and C. As is apparent from equation (22), the value W of a generic project before the first investment increases monotonically in C. So when fixing all other model parameters and W in equation (22), there is only *one* level of C that is congruent with these parameter values.

We make use of this fact by declaring W a parameter of the model and then backing out C as a result of it. This has two advantages: It is computationally easier, and it allows us to keep W in the positive range, which is a requirement to have a VC market at all. Note that, however, negative values for C may be obtained in certain parameter regions, i.e. a *payment* to the VC in the beginning is needed to reach a certain level W given the other parameters. However, negative levels of C are penalized in the estimation, so that this is not a problem in the estimation.

Throughout, we use quadrature methods to solve the various integrals in the model equations. The main advantage of these methods is that only a small number of function evaluations is needed to obtain a precise approximation. An obvious alternative is Monte-Carlo integration, i.e. the simulation of a large number of projects. This has the disadvantage that many more function evaluations are necessary to obtain a good approximation. We use Monte-Carlo simulation in the end to check the results we obtain by quadrature methods and find that quadrature performs very well.

It actually turns out that the Monte-Carlo integrals have some variance even when the number of simulated projects goes into the millions – this is because the distribution for π is Pareto and does not have finite variance under the estimated parameter $\lambda = 1.9$. Hence the speed of convergence of the Monte-Carlo integrals is lower than for finite-variance problems (but the Law of Large Numbers for the mean still holds, of course).

9.2.1 How to find $\pi_{\min}(x)$

There are two cases to consider when determining the lowest level of π tolerated by a VC when learning π at time x:

- $x \geq \tau_m$, where τ_m is the peak of the hazard rate: It is sufficient to look at the FOC:
$$\pi_{min}(x) = \frac{1+rW}{h(x)} = \frac{(1+rW)\tau_m}{\rho}$$

- $x < \tau_m$: In this case we cannot use the FOC. However, we know that the

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following function is increasing in π and that it will cut 0 at some point $\pi_{min}(x)$:

$$\tilde{V}(x, \pi) = \int_x^{\tilde{T}(\pi)} e^{-r(t-x)} \left(\pi + W - \frac{1}{h(t)} \right) \frac{dF(t)}{S(x)} \tag{50}$$

$$+ e^{r[\tilde{T}(\pi)-x]} \left[1 - \frac{F[\tilde{T}(\pi)] - F(x)}{S(x)} \right] W - W,$$

where $\tilde{T}(\pi) = h^{-1}[(1 - rW)/\pi]$ fulfills the FOC for optimal stopping. Note that this value is net of the opportunity cost W of scrapping the project at once (i.e. at x). The integral in the formula above can be approximated by Legendre Quadrature. As for all integrals calculated by quadrature methods in this algorithm, we use 30 quadrature nodes. The root of the equation is found by a *Matlab*-built-in numerical technique. We calculate $\pi_{min}(x)$ for all values on a grid between zero and τ_m; then we interpolate linearly between these points to obtain values for $\pi_{min}(x)$ between the grid points.

A plot of the $\pi_{min}(x)$ function under the estimated parameters can be found in the left panel of Figure 9.

9.2.2 $V_{rev}(x)$: the value of signal revelation at time x

Define $V_{rev}(x)$ as follows:

$$V_{rev}(x) = \int_{\pi_{min}(x)}^{\infty} \tilde{V}(x, \pi) dG(\pi),$$

where $\tilde{V}(x, \pi)$ is given in equation (50). The function $V_{rev}(x)$ value plays a crucial role when finding T_0, but also for the computation of C given W (or vice versa). Since the upper bound of integration is infinity and the integrand does not have closed form, computation of this integral is not straightforward.

To approximate the solution, note that the value of a project $\tilde{V}(x, \pi)$ becomes linear in π in the limit; to determine a (large) $\bar{\pi}$ where the function is very close to the asymptote, consider the following:

$$\tilde{V}(x, \pi) = \frac{\pi + W + 1/r}{S(x)} \int_x^{\tilde{\rho}\pi} e^{-r(t-x)} dF(t) +$$

$$+ e^{rx} \underbrace{e^{-r\tilde{\rho}\pi} \mu \left(\frac{\tilde{\rho}\pi}{\tau_m} \right)^{-\rho} \frac{W + 1/r}{S(x)}}_{=\varepsilon} - \frac{1}{r}$$

We want to find $\bar{\pi}$ sufficiently large so that the underbraced term equal to some very small ε. We solve numerically for the relevant $\bar{\pi}$ after taking logs (the terms e^{-rx} and

$S(x)$ are not taken into account here since they also appear in the other two terms in (**??**) – we want to make ε small *relative* to these terms):

$$\rho \ln \bar{\pi} + r\tilde{\rho}\bar{\pi} = -\ln \varepsilon + \ln \mu - \rho \ln \tilde{\rho} + \rho \ln(\tau_m)$$

We choose $\varepsilon = 10^{-5}$ in the implementation. The equation can be solved numerically for $\bar{\pi}$.

In the calculations for $V_{rev}(x)$, we evaluate the integral from π_{min} to $\bar{\pi}$ with Legendre quadrature; for the region above we use the following approximation:

$$V(\pi) = A\pi + A(W + 1/r) - \frac{1}{r}$$

where

$$A = \int_0^\infty e^{-rt} dF(t)$$

for $\pi \geq \bar{\pi}$. A is obtained with Legendre Quadrature for the part up to τ_m and by Laguerre Quadrature for the part above.

Then we can get an approximation for the value from πs above $\bar{\pi}$ in closed form, since our approximation $V(\pi)$ for high π is linear in π

$$\int_{\bar{\pi}}^\infty V(\pi) dG(\pi) = \left[-1/r + A(W + 1/r) \right] \int_{\bar{\pi}}^\infty \lambda \pi_{min}^\lambda \pi^{-\lambda-1} d\pi +$$

$$+ A\pi \int_{\bar{\pi}}^\infty \lambda \pi_{min}^\lambda \pi^{-\lambda-1} d\pi =$$

$$= \left[AW - \frac{1-A}{r} \right] \pi_{min}^\lambda \bar{\pi}^{-\lambda} + A\pi_{min}^\lambda \frac{\lambda}{\lambda-1} \bar{\pi}^{1-\lambda}$$

The part of the integral below $\bar{\pi}$ is again obtained by Legendre Quadrature; it can be verified that the function $V(\pi)$ is smooth (i.e. infinitely often differentiable), so quadrature yields very good results.

9.2.3   Find T_0

Note that the FOC for T_0 as given in equation (23) is decreasing in T_0 for values greater than τ_m: $\pi_{min}(x)$ is increasing in x since the hazard is declining, and the function below the integral is declining in T_0. The first-order condition at t is:

$$FOC(t) = \delta V_{rev}(t) + h(t)E[\pi] - (a + k) - rW$$

The root of this function above τ_m is found numerically. Should the FOC be already negative at τ_m—in this case, the VC would (in almost all cases) *never* want to keep a project alive—we penalize heavily in the solution algorithm.

9.2.4 Find C given W

As described before, we have made C an outcome in terms of the parameter W, and we have to solve the following equation to obtain C:

$$C = -W - 1/r + \int_0^{T_0} V(x)dB(x) + e^{-\delta T_0}V(T_0).$$

We use integration by parts here to account for the variable costs: First, pay the lifetime value $1/r$ for all projects, then re-gain e^{-rt}/r when the project comes to an end – note that this yields the correct value $(1 - e^{rt})/r$ for each project. $V(x)$ is the time-0 value of all projects with revelation time x; this value does not change anymore for $x > T_0$ since the policy is the same for all these projects; they yield $E[\pi]$ in all periods before T_0, and then they are shut down.

This is the formula for $V(x)$:

$$V(x) = \int_0^{\pi_{min}(x)} H(\pi, x)dG(\pi) + \int_{\pi_{min}(x)}^{\infty} H(\pi, T^*(\pi))dG(\pi),$$

where

$$H(x, \pi) = \int_0^x e^{-rt}(\pi + W + 1/r)dF(t) + e^{-rx}[1 - F(x)](W + 1/r).$$

$H(x, \pi)$ can be calculated in a similar way as described for $\tilde{V}(x, \pi)$ before when fixing x and π. The first term of $V(x)$ is easier to calculate since the policy does not vary – either the project succeeds before x, or it is shut down at signal revelation:

$$\int_0^{\pi_{min}(x)} H(\pi, x)dG(\pi) = G[\pi_{min}(x)](W + 1/r)\left[\int_0^x e^{-rt}dF(t) + e^{-rx}[1 - F(x)]\right] +$$

$$+ \underbrace{\int_{\pi_l}^{\pi_{min}(x)} \pi dG(\pi)}_{=\frac{\lambda}{\lambda-1}\pi_l^{\lambda}\left[\pi_l^{-\lambda+1}-\pi_{min}(x)^{-\lambda+1}\right]} \int_0^x e^{-rt}dF(t)$$

We have to split up the second term in the formula for $V(x)$ at $\bar{\pi}$ as described before in the calculations for $V_{rev}(x)$ to get an expression that is computable:

$$\int_{\pi_{min}(x)}^{\infty} H(\pi, T^*(\pi))dG(\pi) = \int_{\pi_{min}(x)}^{\bar{\pi}} H(\pi, \tilde{\rho}\pi)dG(\pi)$$

$$+ [1 - G(\bar{\pi})](W + 1/r)\int_0^{\infty} e^{-rt}dF(t) + \int_0^{\infty} e^{-rt}dF(t)\frac{\lambda}{\lambda - 1}\pi_l^{\lambda}\bar{\pi}^{-\lambda+1},$$

The first integral is calculated by Legendre Quadrature; again, it can be checked that we are integrating over a smooth function. Note that in the above expression we have

used

$$\int_a^b \pi dG(\pi) = \frac{\lambda}{\lambda - 1} \pi_l^\lambda \left[-\pi^{-\lambda+1} \right]_a^b .$$

Now we can integrate over the smooth function $V(x)$ from 0 to T_0 in the above formula for C. We do this by Legendre Quadrature.

9.2.5 Weights for the project pool

The mass of screened projects that is still alive at time t is the following:

$$H_{scr}(t) = \int_0^t \left[1 - G\left(\max\{\pi_{min}(x), \pi_{min}(t)\} \right) \right] dB(x)$$

For details of the calculation strategy in this case, refer to the description of the numerical implementation of the solution for $R(t)$. In terms of terminations, we can write for $t < T_0$:

$$H_{scr}(t) = 1 - \underbrace{e^{-\delta t}}_{\text{unscreened}} - \underbrace{H_{term}(t)}_{\text{screened and terminated}}$$

So the weight of screened projects in successes, i.e. the probability that a project had been screened conditional on success, is

$$h_{scr}(t) = \frac{H_{scr}(t)}{H_{scr}(t) + e^{-\delta t}}$$

A plot of this function under the estimated parameter values can be found on the right panel of Figure 9.

9.2.6 Calculate α

Compute the expected present value $C_{PV} = C_{PV}^M + C_{PV}^C$ of all costs for the VC: Clearly, the part coming from marginal cost must be $C_{PV}^M = k/r$, since the VC will never be idle and hence always pay a flow cost k. The part C_{PV}^C stemming from the repeated payment of the fixed cost C can be obtained in a recursive fashion:

$$C_{PV}^C = C + E[e^{-rT} C_{PV}^C],$$

where $T = \min\{\tau, T^*(\pi)\}$ is the time of success or termination of the project, whichever comes first. From this, we obtain

$$C_{PV}^C = \frac{C}{1 - E[e^{-rT}]}, \qquad E[e^{-rT}] = \int_0^\pi e^{-r \min\{\tau, T^*(\pi)\}} g(\pi) d\pi.$$

$E[e^{-rT}]$ can be computed in a manner quite similar to C, so the computation procedure is not explained again here. Now, we can obtain the α implied by the model: $\alpha = W/C_{PV}$, as given in equation (55).

9.2.7 Calculate $R(t)$

We now calculate the implied returns as determined by (27). We first need the density of projects in the (x, π)-space at a given time t, i.e., the density on the pool of projects that is still not terminated at time t. To obtain this density, first define

$$\hat{q}(x, \pi|t) = \begin{cases} b(x)g(x)I(\pi \geq \pi_{min}(x, t)) & \text{if } x \leq t \\ b(x)g(x)I(t \leq T_0) & \text{if } x > t \end{cases}$$

where $\pi_{min}(x, t) = \max\{\pi_{min}(x), \pi_{min}(t)\}$ is the lowest project quality left from the pool of projects screened at time x, and $I(\cdot)$ is the indicator function for an event.

Then the density of a pair (π, x) at time t is

$$q(x, \pi|t) = \frac{\hat{q}(x, \pi|t)}{C_t},$$

where

$$C_t = \int_0^\infty \int_{\pi_l}^\infty \hat{q}(x, \pi|t)d\pi dx.$$

C_t may be calculated as

$$C_t = \int_0^t \left[1 - G(\pi_{min}(x, t))\right] dx + I(t \leq T_0)[1 - B(t)]$$

The left-hand side here is approximated by Legendre Quadrature over the smooth parts of the integrand; notice that for $t > \tau_m$, $\pi_{min}(x, t)$ will be flat over some interval on x and vary on the rest of the values for x (at least under reasonable parameters). In this case, we employ quadrature on each of these parts in order to be operating over a smooth integrand.

In a similar fashion we can then calculate the expected π for a succeeding project at t:

$$E[\pi|t] = \underbrace{\int_0^t \pi q(\pi, x|t)dx}_{\text{screened pool}} + \underbrace{I(t \leq T_0)[1 - B(t)]E[\pi]}_{\text{unscreened pool}}$$

Note that the integrand in the term on the left can be evaluated in closed form since π is distributed Pareto. Again, we use Legendre Quadrature over the smooth parts of the integrand to approximate the solution. A plot of the function $E[\pi|t]$ under the estimated parameters can be found in the left panel of Figure 9. Now we have everything in place to compute the conditional return at time t

$$R(t) = \frac{1}{t} \ln \left(\frac{e^{-rt}s^* E[\pi|t]}{C + (1 - e^{-rt})k/r} \right),$$
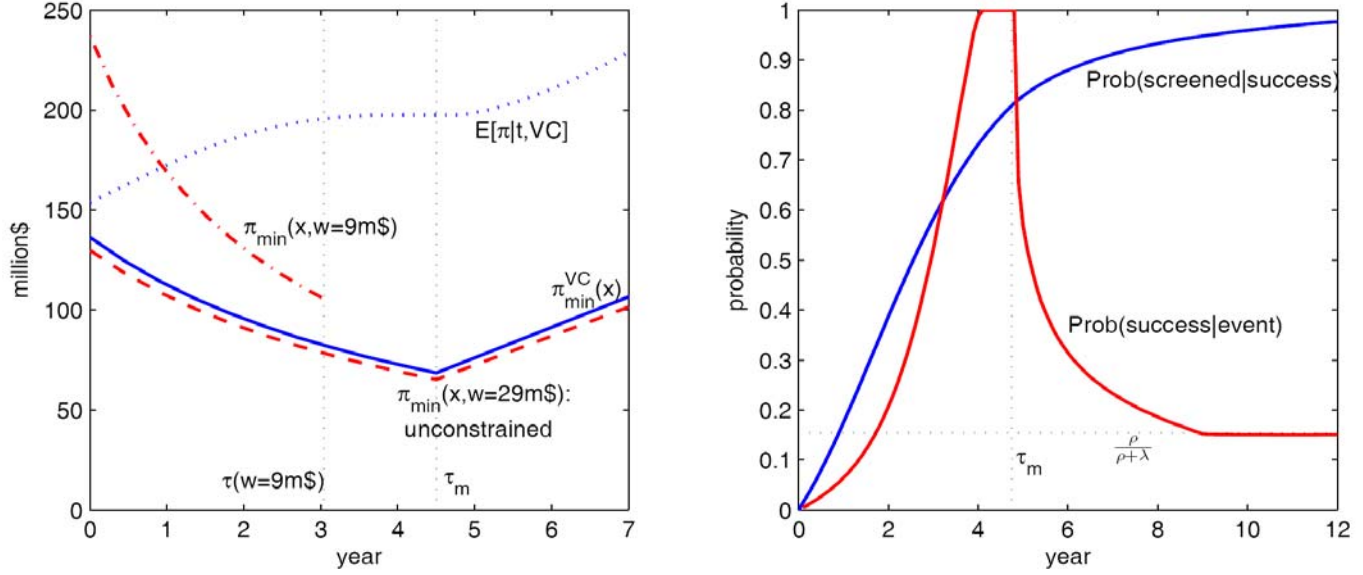
where s^* is given in paragraph (9.2.10).

Figure 9: SELECTION OF π

9.2.8 Calculate k/C

The proportion of fixed cost to flow investment is k/C.

9.2.9 Calculate $J(t)$

We have

$$\hat{p} = C - p^*,$$

where we use

$$p^* = \frac{C(1-k)}{1+rW}.$$

Compute $J(t)$ for $t = 1, 2, \ldots, 12$ by numerically integrating as follows:

$$J(t) = -\hat{p} + \int_0^t J'(s)ds \simeq -\hat{p} + \sum_{i=1}^{t/\Delta t} J'[(i-1)\Delta t]\Delta t,$$

where we choose $\Delta t = 0.1$ and where $J'(t)$ is obtained from the following equation:

$$J'(t) = \left(-k + h(t)(1-s^*) \int_{\frac{a+k+rW}{h(t)}}^{\infty} \pi d\Gamma_t(\pi) \right) S(t)$$

50

9.2.10 Calculate s^*

The predicted share of the entrepreneur in the profit π is

$$s^* = \frac{1-k}{1+rW}.$$

Although we set $s^* = 0.4$, A more recent sample that Kaplan, Sensoy and Strömberg (2006) analyze shows that the founders now tend to retain a smaller share, 10% to 19%.

9.2.11 Calculate $S(t), h(t)$ and $\psi(t)$

For the estimation, we now compute the C.D.F.s of terminations. Let $\Phi^{VC}(t)$ be the VC's terminations C.D.F..By (4), $\pi = (a + k + rW)/h(T^{VC})$, and so the fraction of projects terminated by date t *conditional on no success* until date t, i.e., $\Pr(T \leq t \mid \tau \geq t)$, is

$$\Phi^{VC}(t) = \begin{cases} 0 & \text{for } t = 0 \\ G\left(\pi_{\min}^{VC}\right) & \text{for } t \in \left(0, h^{-1}\left[\frac{a+k+rW}{\pi_{\min}^{VC}}\right]\right) \\ G\left(\frac{a+k+rW}{h(t)}\right) & \text{for } t \geq h^{-1}\left(\frac{a+k+rW}{\pi_{\min}^{VC}}\right). \end{cases} \tag{51}$$

whence the *termination hazard* for the VC-backed firms, $\psi^{VC}(t) \equiv \frac{\left(\Phi^{VC}\right)'(t)}{1-\Phi^{VC}(t)}$, is

$$\psi^{VC}(t) = \begin{cases} \infty & \text{for } t = 0 \\ 0 & \text{for } t \in \left(0, h^{-1}\left[\frac{a+k+rW}{\pi_{\min}^{VC}}\right]\right) \\ (a + k + rW)\,\theta\left(\frac{a+k+rW}{h(t)}\right)\left(\frac{-h'(t)}{[h(t)]^2}\right) & \text{for } t \geq h^{-1}\left(\frac{a+k+rW}{\pi_{\min}^{VC}}\right) \end{cases} \tag{52}$$

where $\theta(\pi) \equiv g(\pi)/(1 - G(\pi))$ is the hazard rate of G. The terminations and hazards of rich Es with wealth w, written as $\Phi^{E,w}(t)$ and $\psi^{E,w}(t)$, also satisfy (51) and (52), but with W set equal to zero, and with $\pi_{\min}^{VC}$ replaced by $\pi_{\min}^{E}$. The same can also be said for Es with wealth $w < C + \frac{k}{r}$, except that $\pi_{\min}^{VC}$ replaced by $\pi_{\min}^{E}(w)$, and that $\psi^{E,w}(t)$ becomes infinite at $t = \tau(w)$ defined in (5). We compute the survival function $S(t)$, $t = 0, 1, .., 12$ as follows:

$$S(t) = [1 - F(t)]\left[1 - \Phi^{VC}(t)\right],$$

where the terminations C.D.F. $\Phi^{VC}(t)$ is just the converse of the variable C_t calculated before to obtain $R(t)$ in paragraph 9.2.7. We calculate $S(t)$ at the exact points $t = 0$, $t = 1$ etc. since the survival is a stock variable which is measured exactly at these points in the data, too.

The success hazard $h(t), t = 1, .., 12$ is computed according to the formula given in section 9.5. Note that since success is purely exogenous, this hazard is solely

governed by τ_m and ρ. We evaluate the hazard in the model in the middle of the period corresponding to a year in the data, e.g. at $t = 0.5$ for the first year, since the data to calculate the empirical hazard stem from successes that are spread out over the entire first year; in other words, the hazard is a flow concept.

Also the terminations hazard $\psi(t)$ is calculated for $t = 1, .., 12$ since it is a flow variable in the data, too. In this case we compute the exact sample analog: For the hazard over year 2, for example, we take the difference between the terminations C.D.F. $\Phi^{VC}(t)$ at $t = 1$ and $t = 2$ and then divide by the survival $S(1)$.

9.3 Loss Function

The set of unknown parameters is $\{\rho, \pi_0, \lambda, C, k\}$. To estimate them, we fix $\tau_m = 4.5$ since this is the peak of the empirical hazard rate. Furthermore, we set $r = 0.127$ as described before in paragraph 9.1.1.

We specify the loss function RSS for a set of parameters $(\rho, \pi_0, \lambda, k, W)$ and the moments from the data as follows:

$$\text{RSS} = w_1[\ln \hat{\alpha} - \ln \alpha]^2 + w_2\left[\ln(\hat{k}/\hat{C}) - \ln(k/C)\right]^2 +$$
$$+ w_3 \sum_{i=2}^{8}[\hat{R}(t) - R(t)]^2 + w_4 \sum_{t=0}^{12}[\hat{J}_t - J_t]^2 + w_5[\ln \hat{s}^* - \ln s^*]^2 +$$
$$+ w_6 \sum_{t=1}^{12}[\hat{h}_t - h_t]^2 + w_7 \sum_{t=0}^{12}[\hat{S}_t - S_t]^2 \tag{53}$$

We only penalize two out of the three functions $h(t)$, $S(t)$ and $\psi(t)$ since knowledge of two pins down the third. We chose penalties on logarithms for some moments since a simple quadratic scheme did not penalize absurd behavior sufficiently; for example, when the ratio k/C goes from $1/10$ to $1/100$, this is a big qualitative change in our eyes but would not be penalized much by a purely quadratic scheme.

For the weighting scheme, we chose $w = (5, 1, 1, 0.1, 0.1, 0.5, 0.01)$. The high penalty on α reflects both the importance we attach to this parameter and its low scale. The other penalties are chosen to reflect the scale of the respective parameters.

We use a *Matlab*-built-in line-search method to minimize RSS with respect to the parameters of the model. The minimization process proved to be very robust; the algorithm converged to the same solution for almost any of about 100 randomly chosen starting points. Using numerical gradients and standard errors of the moments, it is possible to obtain (approximate) standard errors for our estimators. This is an important next step in this project.

The described weighting scheme yielded the estimates reported in Table 2A. Estimates of R, S, and h are plotted in Figure 8.

9.4 Statistics from the solo entrepreneur's problem

9.4.1 How to find $\pi_{\min}(w, x)$

For the solo entrepreneur with wealth w and an associated time $\tau(w)$ where she runs out of wealth, we have the following value of a project with known π at x, for simplicity here in terms of time 0:

$$V_s(w, x, \pi) = \int_x^{\min\{\rho\pi, \tau(w)\}} e^{-rt} \left(\pi - \frac{1}{h(t)} \right) dF(t)$$

The root of this equation is called $\pi_{min}(w, x)$; this is the payoff where the solo entrepreneur of wealth π is just indifferent between terminating and continuing a project. The numeric solution to this problem is implemented just as the one in the analogous equation for the VC described in paragraph 9.2.1.

We first take a grid over the wealth levels of interest, $[C, C + k/r]$. Then, for each wealth level w on the grid, we calculate $\pi_{\min}(w, x)$ for a grid point $x_i \in [0, \tau(w)]$. By linearly interpolating between the grid points in the (w, x)-plane it is then possible to obtain an approximate $\pi_{\min}(w, x)$ for any point (w, x).

A plot of the function $E[\pi|t]$ for the unconstrained entrepreneur and for a constrained entrepreneur with w=9m\$ can be found in the left panel of Figure 9.

9.4.2 The solo entrepreneur's FOC for $T_0(w)$

For entrepreneurs with $\tau(w) < \tau_m$, we set $T_0 = \tau(w)$, i.e. the entrepreneur does not terminate projects of unknown value before she runs out of money. In the reasonable parameter region, the first-order condition for T_0 is increasing below τ_m, so that the entrepreneur would terminate everything at $t = 0$ – we want to avoid this to see where the entrepreneurs profits cut zero. The entrepreneur's first-order condition for T_0 is indeed

$$FOC(w, x) = \delta \underbrace{\frac{e^{rx}}{S(x)} \int_{\pi_{min}(w,x)}^{\infty} V_s(w, \pi, x) dG(\pi)}_{=V_{s,rev}(w,x)} + h(x) E[\pi] - 1$$

For $\tau(w) > \tau_m$, we check this condition at τ_m and $\tau(w)$: If it is not positive at τ_m, we set $T_0 = \tau_m$—the reasoning for this is the same as before for the case $\tau(w) < \tau_m$. If it is still positive at $\tau(w)$, then the solo entrepreneur will let all unscreened projects survive until the end, and we set $T_0 = \tau(w)$. Otherwise, we solve the above equation for a root – this root must be unique since the FOC is downward-sloping for $t > \tau_m$.

Again, we encounter the problem that the integral's upper limit in the FOC is infinity, as was the case for the analogous problem of the VC in paragraph 9.2.3. However, in the entrepreneurs case we have a natural upper bound for the linear

asymptote: The function will already be linear for π such that $\rho\pi \geq \tau(w)$. Hence we re-state the revelation value as follows:

$$e^{-rx}S(x)V_{s,rev}(w,x) = \int_{\pi_{min}(x)}^{\tau(w)/\rho} V_s(\pi, x, w)dG(\pi) +$$

$$+ [1 - G(\tau(w)/\rho)]\frac{-1 + \int_x^{\tau(w)} e^{-rt}dF(t) + [1 - F(\tau(w))]}{r} +$$

$$+ \left[\int_x^{\tau(w)} e^{-rt}dF(t)\right]\left[\underbrace{\int_{\tau/\rho}^{\infty} \pi dG(\pi)}_{=\frac{\lambda}{\lambda-1}\left(\frac{\tau/\rho}{\pi_l}\right)^{-\lambda}\frac{\tau}{\rho}}\right]$$

Again, we use Legendre Quadrature to approximate the integrals involved here, just as in the VC's case described in paragraph 9.2.3.

9.4.3 The VC's and entrepreneur's net worth and excess return

Derivation of (55).—To arrive at W and C_{PV} we discount by r, the rate of return required given the risk characteristics of the income stream that the VC faces. That rate depends partly on the covariance of the VC's income stream with the market index, i.e., β. Let r_f denote the risk-free interest rate, and let $r_{S\&P}$ denote the expected return on the market index for which we shall use the S&P 500 as a proxy. Then the CAPM prediction for the expected return on venture capital is

$$r = r_f + \beta\left(r_{S\&P} - r_f\right).$$

To verify (55) in another way, let time be discrete and assume that each company matures and yields π for sure at the end of one period. Let $p = 1$ and $k = 0$. Then one dollar today yields π dollars a period from now, and so the excess return on the investment would be

$$\alpha = \pi - (1 + r). \tag{54}$$

Now let us instead calculate the excess return using (55): The VC then invests in a new company every period, and his discount factor is $\frac{1}{1+r}$. Therefore

$$W = \frac{\pi - (1 + r)}{1 - \frac{1}{1+r}} \text{ and } C_{PV} = \frac{1}{1 - \frac{1}{1+r}}.$$

Substituting these values into (55) gives us the same value of α as (54) does.

The lifetime value of venture capital is W, of which roughly a fraction $1 - \int e^{-rt}|dS(t)|$ stems from the current project and the remainder from future project. The expected present value of lifetime investment of a VC in current units is $C_{PV} \equiv$

$C + \frac{(a+k)}{r}\left(1 - \int e^{-rt}\left|dS\left(t\right)\right|\right)$. Therefore the lifetime value per lifetime dollar invested is

$$\alpha \equiv \frac{W\left(1 - \int e^{-rt}\left|dS\left(t\right)\right|\right)}{C_{PV}}. \tag{55}$$

This is also the VC's flow excess return per period, as explained further in the Appendix A. Let $C_{PV}^w = C + (a+k)\frac{1 - \int e^{-r\min(t,\tau[w])}\left|dS^{E,w}(t)\right|}{r}$ be the PV of costs on an E project, where $\tau\left(w\right)$ is defined in (5), where $S^{E,w}\left(t\right) = \left(1 - F\left[t\right]\right)\left(1 - \Phi^{E,w}\left[t\right]\right)$, and where $\Phi^{E,w}$ is defined in the remark following (52). The rate of return of E in excess of r is

$$\varepsilon\left(w\right) = \frac{Q^E\left(w\right) - C}{C_{PV}^w}. \tag{56}$$

At w^* E is indifferent between going solo and abandoning the project, hence $\varepsilon\left(w^*\right) = 0$. Since $\partial Q^E\left(w\right)/\partial w > 0$, the numerator rises with w, but so does the denominator. It rises with E's level of wealth. The excess return becomes flat at the point $C + k/r$, i.e., the point where the solo E ceases to be liquidity constrained in any state of the world, i.e., for any realization of π.

To calculate the present value of a project at $t = 0$ and the present value of the cost, we first define the optimal scrapping time as $T^*(w,\pi) = \min\{\rho\pi, \tau(w)\}$. Then we have

$$V(w) = -C - 1/r + \int_0^{T_0(w)} \delta e^{-\delta x} V_x(w,x)dx + e^{-\delta T_0(w)} V_x(w, T_0(w))$$

$$V_x(w,x) = \int_{\pi_l}^{\pi_{min}(w,x)} A(x)\pi + \frac{B(x)}{r}dG(\pi) + \int_{\pi_{min}(w,x)}^\infty V_s^0(w,x,\pi)dG(\pi)$$

$$A(x) = \int_0^x e^{-rt}dF(t)$$

$$B(x) = \int_0^x e^{-rt}dF(t) + [1 - F(x)]e^{-rx}$$

$$V_s^0 = \int_0^{T^*(w,\pi)} e^{-rt}(\pi + 1/r)dF(t) + \frac{1 - F(T^*(w,\pi))}{r}$$

Unlike the other integrals, we compute the value of the project for the solo entrepreneur by Monte-Carlo methods. We simulate three million projects, apply the optimal policies for a wealth level w which are given by the function $\pi_{min}(w,x)$ and the number $T_0(w)$, and then calculate the averages corresponding to the integral for $V(w)$. We do this for the entire grid of wealth levels defined before.

The solo return for the entrepreneur is then simply

$$Q_S(w) = w + V(w)$$

where $V(w)$ is given above. The excess return $\varepsilon(w)$ is given by the ratio of expected discounted revenue to expected discounted cost (both discounted at the same r used in the VC model), which are again computed by Monte-Carlo integration.

We use Monte-Carlo integration here instead of quadrature methods because they are easier to implement; notice that we only calculate the statistics for the solo entrepreneur after having found the optimal parameter vector, so computational parsimony is not a concern here.

9.5 Derivation of h

We use n as the exponent in order to clarify the algebra, and then we shall evaluate the result at $n = 2$. We write the mixing parameter as μ. Then

$$F(t) = (1 - \mu) \left(\frac{\min(t, t_{\min})}{\tau_m} \right)^n + I_{[\tau_m, \infty)} \mu F^P(t),$$

Then For $t < \tau_m$, $f(t) = \frac{1}{\tau_m} (1 - \mu) n \left(\frac{t}{\tau_m} \right)^{n-1}$, and therefore

$$
\begin{aligned}
\frac{f(t)}{1 - F(t)} &= \frac{1}{\tau_m} \frac{(1 - \mu) n \left(\frac{t}{\tau_m} \right)^{n-1}}{1 - (1 - \mu) \left(\frac{t}{\tau_m} \right)^n} = \frac{1}{\tau_m} \frac{(1 - \mu) n t_{\min}^{1-n} t^{n-1}}{1 - (1 - \mu) \tau_m^{-n} t^n} \\
&= \frac{(1 - \mu) n t_{\min}^{1-n} t^{n-1}}{\tau_m - (1 - \mu) \tau_m^{1-n} t^n}
\end{aligned}
$$

and

$$\lim_{t \nearrow 1} \frac{f(t)}{1 - F(t)} = \frac{1}{\tau_m} \frac{1 - \mu}{\mu} n$$

For $t \geq \tau_m$

$$
\begin{aligned}
\frac{f(t)}{1 - F(t)} &= \frac{\mu \rho t_{\min}^\rho t^{-\rho-1}}{1 - \left[(1 - \mu) + \mu \left(1 - \left(\frac{t}{\tau_m} \right)^{-\rho} \right) \right]} \\
&= \frac{\mu \rho \tau_m^\rho t^{-\rho-1}}{1 - 1 + \mu - \mu + \mu \left(\frac{t}{\tau_m} \right)^{-\rho}} \\
&= \frac{\rho \tau_m^\rho t^{-\rho-1}}{\mu - \mu \left(1 - \left(\frac{t}{\tau_m} \right)^{-\rho} \right)} = \frac{\rho}{t}
\end{aligned}
$$

Therefore the hazards are equal at τ_m if $\frac{1}{\tau_m} \frac{1-\mu}{\mu} n = \frac{\rho}{\tau_m}$, i.e., if $\mu = \frac{1}{1 + \frac{\rho}{n}}$. After setting $n = 2$, this leads to $\mu = \frac{2}{2+\rho}$, $1 - \mu = \frac{\rho}{2+\rho}$, and, hence for $t < \tau_m$,

$$\frac{f}{1 - F} = \frac{1}{\tau_m} \frac{(1 - \mu) n t_{\min}^{1-n} t^{n-1}}{1 - (1 - \mu) \tau_m^{-n} t^n} = \frac{1}{t_{\min}} \frac{\frac{\rho}{2+\rho} n \tau_m^{-1} t}{1 - \frac{\rho}{2+\rho} \tau_m^{-2} t^2} = \frac{1}{\tau_m} \frac{2 \rho \tau_m^{-1} t}{2 + \rho - \rho \tau_m^{-2} t^2}.$$

9.6   Procedure behind Figure 7

This how the empirical distribution of company's age at IPO was obtained *excluding rollups and spinoffs*:

- First, we matched the data on all firms on Ritter's website ($N = 8,309$) with the list he sent to us, which are the ones that are backed by VCs ($N = 2,899$).

- We only consider the year of the offering date, getting rid of the month and the day. The difference between this year and the founding year is the age at IPO. `IssueAge`.

- All records for which one of the two years is missing (27 companies) and for which the age at IPO turns out to be negative (3 companies, these are presumably errors in the data) are excluded.

- This leaves $5,522$ companies that were solo at IPO and $2,784$ companies that were VC-backed.

- Now, these data are matched to the data from Ritter about spinoffs. Companies that are spinoffs are excluded, which eliminates 810 records.

- Also, the company data are manually matched to the data from Brown, Dittmar and Servaes (2005) on rollups: This creates five matches, but none of these matches results in new exclusions since the records had been excluded in one of the earlier stages.

- In the end, we have $N_S = 4,850$ solo companies and $N_{VC} = 2,675$ VC-backed companies left in the sample.

The two graphs show the following:

1. Empirical pdf: This is the proportion of companies that had IPO exactly at age t.

2. Empirical cdf: This is the proportion of IPOs up to (and including) age t. The dotted lines are confidence bands at 95%-confidence level (two-sided); they come from the *Matlab*-built-in Kaplan-Meier method.