Endogenous Knowledge Spillovers and Labor Mobility in Industrial Clusters

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Abstract

Knowledge spillovers and labor mobility in industrial clusters are interrelated phenomena. A firm's knowledge is embodied in the entrepreneur and in the specialized workers. Knowledge can spill over from one firm to another through two channels: direct revelation from one entrepreneur to another and labor mobility. We show that, in equilibrium, an entrepreneur can disclose information to another in order to avoid labor poaching. The incentive of firms to disclose information voluntarily is one of the novel contributions of our paper. In the absence of information disclosure, spillovers can still occur through labor poaching. Labor poaching and voluntary disclosure of information can also occur simultaneously in equilibrium. We also provide a rationale for the localized character of the spillovers and for the limited geographical extensions of industrial clusters.

1 Introduction

The last few years have seen a growing interest in industrial clusters. Much work has been devoted to the origins, the dynamics and the success of clusters in modern industrial economies (for surveys, see, e.g., Rosenthal and Strange, 2004, and Duranton and Puga, 2004). The analysis of clusters is of course not new: it dates back at least to Marshall (1890). Marshall related the competitive advantage of industrial clusters to three factors: the easy transmission and discussion of new ideas, the availability of skilled workers, and the availability of intermediate good suppliers.

Empirical analyses (see, e.g., Jaffe et al. 1993, Audretsch and Feldman, 1996, Baptista and Swan, 1996) have confirmed that knowledge spillovers are geographically localized and that there is a positive correlation between clustering and innovation activity. At the same time, according to anecdotal evidence and case studies, in some cases, technological spillovers take place through *voluntary* knowledge dissemination and are not the result of simple information leakage. Firms, even in the high-tech industries, adopt an open research environment, allowing other subjects to access their knowledge.[1]

These two findings seem clearly related. As scholars of technological progress have argued (see, e.g., Dosi, 1988), technological knowledge is com-

[1] For anecdotal evidence and references to case studies, see, e.g., Baumol (2002) and Lewis and Yao (2003).

plex, tacit and cumulative. Therefore, it is difficult to believe that *involuntary* leakage of information can be the main source of knowledge spillovers: having some spare information on a project is by no means close to knowing it. Technological knowledge is hard to transfer through formal means and often requires face-to-face communication. Firms that are located in the same area have big advantages in learning from one another. The ability of firms in the same cluster to exploit the same technology for market reasons suggests that they share information to a great extent, much more than what would result from sporadic *involuntary* information leakage.

As we said, in Marshall's view, another characteristic of industrial clusters is the large availability of skilled workers. A large pool of skilled worker in a small area makes easier for firms to find the labor skills that meet their needs. The presence of many similar firms in a small area can also lead to labor poaching and to a high rate of labor mobility. For instance, Saxenian (1994) reported that the labor turnover in the Silicon Valley was high. Clearly, skilled labor poaching is another source of knowledge spillovers, since workers can disseminate their knowledge to the rivals. Evidence on labor mobility is, however, mixed. Recent empirical work on Italian industrial districts (see, e.g., De Blasio and Di Addario, 2002), for instance, shows that in many districts the turnover rate is not higher than in the rest of the national economy.

This empirical and anecdotal evidence suggests many questions. Which market forces lead rival firms to voluntary dissemination of information? Why don't spillovers spread outside a certain area? What determines the extension of this area? Which incentives can be causal of labor mobility?

In this paper we will argue that knowledge spillovers and labor mobility are inter-related phenomena. Moreover, their inter-relation helps to explain why industrial clusters are limited to small geographical areas. In our economy, technological knowledge within a firm is embodied in the entrepreneur and in the specialized workers. Knowledge can spill over from one firm to another through two channels: 1) direct transmission of knowledge from one entrepreneur to another; and 2) knowledge flow resulting from skilled workers' mobility. The two channels are not independent. If the entrepreneur's and the workers' knowledge are substitutes, the entrepreneur can avoid the loss of his specialized workers by directly revealing knowledge to the rival firm. Knowledge transmission is a way to avoid labor poaching. This is perhaps our most novel and surprising result: in equilibrium, firms can have an incentive to disclose information voluntarily. We also show that when the entrepreneur decides not to disclose information, knowledge flows from one firm to another can still occur, since a competitor may find it optimal to poach the specialized workers. In fact, labor mobility and voluntary disclosure of information are not mutually exclusive. We will demonstrate that there are cases in which worker mobility may arise and, at the same time, the employer of the specialized worker discloses information.[2]

In many industrial districts, the labor market for specialized workers is localized in a specific geographical area. Clearly workers are much more inclined to change job in the same area than to move to another local labor market, in order to avoid moving costs.[3] Hence, the risk for a firm that its workers move outside that local area is low and the firm does not have any incentive to disclose information to competitors outside that area. This can explain why technological externalities are localized and the firms agglomeration does not expand beyond a geographical limit. The extension of the labor market determines the extension of the industrial district. While in some theoretical studies (see, e.g., Fujita and Ogawa, 1982) the informational externality is simply postulated to be decaying with distance, our analysis provides a justification for the spatial limits of spillovers.

Obviously we do not consider our theory as the only explanation of en-

[2] Baumol (2002) provides with some unsystematic evidence of our theory. He also argues that researchers and scientists are often unwilling to work for a firm that does not want them to share their knowledge with other researchers and this obliges the top management to accept the dissemination of many results of the firm. Similar considerations are also contained in Kornberg (1995). There is also evidence that worker mobility is an important channel of knowledge dissemination. For example, in a study conducted through a series of interviews, Von Hippel (1988) found that very frequently firms agree to train at no cost workers of competing enterprises and that a firm's personnel were sent to rival firms to help in the use of new processes or techniques. Other evidence of spillovers through specialized workers is provided by Saxenian (1994) and Almeida and Kogut (1999).

[3] The role of distance as a barrier to workers' mobility and to knowledge diffusion is also studied in Combes and Duranton (2001). They do not analyze voluntary spillovers.

dogenous spillovers in clusters. Many other factors can provide an incentive to entrepreneurs to share information. Local proximity can create good relations within the business community: a sense of reciprocity can lead a firm to reveal knowledge with the understanding that later the favor will be reciprocated. Although revealing information can result in a short term loss, it can be a profit maximizing choice in the long run as firms within the district obtain an advantage over external firms by sharing information. Firms can also choose information sharing as the effect of a technological cooperative agreement, even when they continue to compete in the market. Our analysis, however, shows that voluntary dissemination of knowledge does not necessarily presuppose reciprocity, since it can be simply explained by rivalry in the product and in the labor markets. The mechanism for the endogenous spillover that we identify also offers a rationale for the localized character of the spillovers and for the limited geographical dimension of the district.

A paper close in spirit to ours is that by Lewis and Yao (2003). They also are interested in explaining information dissemination and workers' turnover in clusters. They study the employment relation between a firm and a specialized worker. They show that contractual incompleteness makes it optimal for the firm to accept (to a certain degree) an open R&D environment (in which the employee can diffuse information outside the firm) and to accept a certain level of turnover. Their analysis is complementary to ours in that they focus on the contractual relation inside the firm and explain why firms do not necessarily adopt contracts that reduce labor mobility and information sharing.[4] Other papers close to ours are those by Combes and Duranton (2001), Fosfuri and Ronde (2004) and Vilalta i Bufi (2004). Theses papers analyze the role of labor poaching in the transfer of knowledge in industrial clusters. In particular, the first two papers discuss the trade-off between labor pooling and labor poaching. Combes and Duranton (2001) discuss this trade-off focusing on the role of product differentiation and of the market size. Fosfuri and Ronde (2004), instead, focus on cumulative innovations and trade secret laws. The paper by Vilalta i Bufi (2004), finally, discusses the efficiency of the worker mobility and of the resulting spillovers. None of these paper, however, considers the incentives of the firms (entrepreneurs) to voluntary information disclosure. The papers by Gersbach and Schmutzler (2003, 2003a) contemplate technological spillovers resulting from worker mobility too. They study the firms' incentives to invest in R&D in the presence of such spillovers. They do not discuss the localized character of spillovers and, as the previous papers, do not consider the possibility of voluntary information disclosure.

While our analysis helps to understand knowledge flows in industrial dis-

[4]Another paper that focuses on the relation between knowledge diffusion by the employees and the internal organization of the firm is Rodriguez-Palenzuela (1999).

tricts, it also contributes to the more general literature on technological spillovers. Almost all studies in this literature treat spillovers as involuntary information leakages. An exception is Katsoulakos and Ulph (1998). The mechanism identified by these authors is very different from ours. In Katsoulakos and Ulph (1998) firms voluntarily choose information sharing either because of technical complementarities or as the result of a technological cooperative agreement.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 illustrates the product and labor market competition. Section 4 discusses knowledge spillovers. Section 5 summarizes the equilibrium outcomes. Section 6 discusses the results.

2 The Model

2.1 Demand and Cost Functions

Two firms, denoted by 1 and 2, sell a homogeneous product. They face the following linear demand, in the quantity space where the price is positive:

$$p = 1 - q_1 - q_2,$$

where p denotes the price and q_i $(i = 1, 2)$ is the quantity produced by firm i.

Firm i's total cost is given by

$$C(q_i, K_i, w_i) = c(K_i)\, q_i + w_i,$$

where K_i $(0 \leq K_i \leq 1)$ is firm i' level of knowledge on the production process and w_i will be defined below. The higher K_i, the higher the productivity and the lower the cost of production. In particular, we assume that the marginal cost is linear in the level of knowledge, i.e., $c(K_i) = (1 - K_i)$. The firm's total knowledge level (K_i) depends on the knowledge of the entrepreneur, the knowledge of the employees and the knowledge spillover that the firm receives from the rival. Workers in the labor market are of two types: the skilled, or high-type, worker, with a specific knowledge (k_h) on the production process, and the standard, or low-type, worker, with no knowledge (i.e., $0 = k_l < k_h$). While there are many unskilled workers, available at a fixed wage in a competitive labor market, there is a limited supply of skilled workers. In particular, we assume that there is only one worker of such a type. The variable w_i $(w_i \geq 0)$ denotes the payment to the skilled worker for his knowledge. In other words, w_i is the quasi-rent that the skilled worker obtains for his knowledge contribution, independently of his work in the production process.[5]

As we said, firm i's knowledge level K_i is a function of the knowledge of the entrepreneur, $k_i > 0$, the knowledge of its workers, k_{W_i} (which can be either k_h or 0), and the amount of knowledge received from the rival, $\rho_j k_j$ ($j = 1, 2,\ j \neq i$). The latter amount is given by the technological knowledge of the other entrepreneur (k_j) times the proportion of knowledge that he decides to reveal, ρ_j ($0 \leq \rho_j \leq 1$).[6] In particular, we assume that $K_i = \min(k_i + k_{W_i} + \rho_j k_j, 1)$. The knowledge of an agent can be substituted by the knowledge of another. Moreover, we consider learning as a *multi-dimensional* heuristic process. Agents learn by trial and error, pursuing different paths. The probability that they know the "same things" is assumed to be zero and, therefore, the knowledge that they have can be added.[7] Notice, however, that when the maximum level of knowledge is reached, receiving more knowledge is useless. While decreasing returns to knowledge are crucial for our results, we use this particular formulation for simplicity's sake only.

[5] This cost function can be thought of as deriving from a production function such as $q_i = \frac{1}{c(K_i)} n_i$, where n_i is the number of workers. The wage for the standard workers is exogenously determined in a competitive market and normalized to 1. If a firm employs only standard workers, it will have a total cost equal to $c(K_i)\, q_i$. If the firm employs the skilled worker, it will have a lower unit cost, but, in addition to the standard wage, it will have to pay w_i to the worker for his contribution in terms of knowledge.

[6] We are assuming that partial disclosure is possible and, moreover, useful to the other firm. For many technologies, this is indeed the case (see, e.g., Bhattacharya et al., 1990).

[7] For other papers that use this interpretation of knowledge, see, e.g., d'Aspremont and Jacquemin (1988), and Spence (1984).

Note that there are two possible sources of asymmetry between the two firms: one firm only can employ the skilled worker, and, furthermore, the entrepreneurs may have different levels of knowledge. Without loss of generality, we assume that $k_1 \geq k_2$. Furthermore, for expositional reasons, we assume that the skilled worker is initially employed by the entrepreneur with the higher level of knowledge.

2.2 Labor and Product Market Competition

We model the competition between the two firms as a three stage game. In the first stage, the entrepreneurs have the option to reveal, even partially, their knowledge to the rival.

In the second, they compete for the high type worker. Finally, they choose the quantity to sell. Formally, in the first stage, entrepreneur i chooses the level of technological spillover ρ_i. The technological spillover is exclusively endogenous, i.e., there is no involuntary information leakage. In the second stage, firms compete in the labor market for the high-type worker. They simultaneously offer a wage to the high-type worker. We assume that the reservation value for the high-type worker is equal to 1 (the standard wage), so that if a firm offers him $1 + w_i$, his participation constraint is always satisfied.[8] Moreover, we assume that the worker can move from firm 1 to

[8] See footnote 5 for a justification.

firm 2 at no cost and that only in the case of indifference (i.e., when the two firms offer him the same wage) he prefers to stay at firm 1.[9] The two firms compete for one worker only. Hence, the skilled worker will obtain a wage equal to the lower increase of profit that he can generate for either firm, as it will become clear in the following section.

In the third stage, firms choose simultaneously and independently their production quantities.

3 Production and Labor Market Decisions

In this section we start our equilibrium analysis. We solve the three-stage game backwards to find the subgame perfect equilibria.

3.1 The Production Stage

In the third stage, each firm chooses the level of production, q_i to maximize the profit function

$$(1 - q_i - q_j)q_i - (1 - K_i)q_i - w_i.$$

This function is strictly concave in q_i. Given our assumptions, both firms want to produce a strictly positive quantity as long as $K_i \leq 2K_j$. In this

[9]This last assumption is needed only to rule out some trivial multiple equilibria.

case, the equilibrium quantities are

$$q_i^* = q^*(K_i, K_j) = \frac{2}{3}K_i - \frac{1}{3}K_j,$$

and the equilibrium profits (gross of the skilled worker's wage) are

$$\Pi(K_i, K_j) = \frac{1}{9}(2K_i - K_j)^2 .$$

If $K_i > 2K_j$, in contrast, firm i will be a monopolist, producing the quantity $q^M(K_i) = \frac{1}{2}K_i$, and earning the profit $\Pi^M(K_i) = \frac{1}{4}K_i^2$.

3.2 The Hiring Decisions

The two firms choose simultaneously and independently the wage offer to the skilled worker. In equilibrium, this wage must be equal to the lower extra-profit that the worker is able to generate for either firm. Given the Bertrand competition in the labor market, the firm with the higher benefit from the worker will employ him at that wage.

Let us determine which firm employs the worker. First, let us consider the monopoly and let us assume that there are no knowledge spillovers, i.e., $\rho_1 = \rho_2 = 0$. In this case, there are two possibilities. Either potential entrant (firm 2) too can become a monopolist by hiring the worker (i.e., $k_2 + k_h > 2k_1$)

or not.[10] In the latter case, the worker is clearly more beneficial to firm 1. Indeed, by keeping the worker, this firm keeps its monopolistic position, while firm 2, by entering the market, can only earn the profit of a duopolist. In the former case, firm 2 can become a monopolist too, but its profit would still be lower than (or at most equal to) firm 1's, as it would have a lower (or equal) level of knowledge and, therefore, a lower productivity. Hence, firm 1 will keep the worker and pay him a salary equal to the profit of the potential entrant.

Lemma 1 *If the industry is a monopoly, i.e.,* $k_1 + k_h > 2k_2$*, then firm* 1 *keeps the worker and remains a monopolist.*

Proof. See Appendix ■

Now, let us move to the case in which the industry is a duopoly. By employing the worker, firm i earns a gross profit increase (with respect to the case in which the worker works for the rival) equal to

$$\Pi\left(\min\left(k_i + \rho_j k_j + k_h, 1\right), \min\left(k_j + \rho_i k_i, 1\right)\right) -$$
$$\Pi\left(\min\left(k_i + \rho_j k_j, 1\right), \min\left(k_j + \rho_i k_i + k_h, 1\right)\right).$$

When entrepreneur i employs the worker, he obtains two benefits: he in-

[10] A third case is that where firm 2 cannot enter the market even by hiring the worker. In this case, trivially, there is no scope for worker mobility.

creases his firm's knowledge level and avoids the worker working for the rival. Both effects are taken into account in the previous expression.

Firm 1 keeps the worker if and only if:

$$\Pi(\min(k_1+\rho_2 k_2+k_h,1),\min(k_2+\rho_1 k_1,1))-$$
$$\Pi(\min(k_1+\rho_2 k_2,1),\min(k_2+\rho_1 k_1+k_h,1))\geq$$
$$\Pi(\min(k_2+\rho_1 k_1+k_h,1),\min(k_1+\rho_2 k_2,1))-$$
$$\Pi(\min(k_2+\rho_1 k_1,1),\min(k_1+\rho_2 k_2+k_h,1)).$$

In the next lemma we find the conditions under which firm 1 (or firm 2) employs the worker in the absence of knowledge spillovers, i.e., for $\rho_1=\rho_2=0$.

Lemma 2 *If the market is a duopoly, i.e., $k_1+k_h\leq 2k_2$, then firm 1 keeps the worker under these conditions:*

1) $k_1+k_h\leq 1$, or

2) $k_1+k_h>1$, $k_2+k_h\leq 1$, $k_2\leq \frac{1}{2}\frac{8k_hk_1+5\left(1-k_1^2-k_h^2\right)}{4(1-k_1)+5k_h}$, and $k_h\leq\frac{1}{5}$, or

3) $k_1+k_h>1$, $k_2\leq\min\left(\frac{8}{5}-k_1,\frac{1}{2}\frac{8k_hk_1+5\left(1-k_1^2-k_h^2\right)}{4(1-k_1)+5k_h}\right)$ and $\frac{1}{5}<k_h<\frac{2}{5}$

4) $k_2+k_h>1$, $k_2\leq\frac{8}{5}-k_1$, and $k_h\geq\frac{2}{5}$.

Proof. See Appendix ■

Two forces determine which firm has a higher benefit from the skilled worker. The entrepreneur with a higher knowledge (lower unit cost) can spread the reduction of the unit cost over a larger production. The effect of the worker on the unit cost, however, may be lower for this firm because of the decreasing returns to knowledge. Remember that when a firm has reached a level of knowledge of 1, additional knowledge provided by the worker is useless. This explains the conditions in the lemma. When both entrepreneurs have "little" knowledge (i.e., when $k_1 + k_h \leq 1$), they can both completely benefit from the worker's knowledge, and, hence, firm 1 has a higher benefit from the worker. In fact, while both firms can completely benefit from the worker's knowledge, firm 1 can spread the reduction of the unit cost over a larger production. A necessary condition for firm 2 to have a higher benefit from the worker is that entrepreneur 1 has a "high" level of knowledge, i.e., $k_1 + k_h > 1$. Even under this condition, however, firm 1 has a higher benefit, provided that firm 2 has a level of knowledge lower than a threshold. In these circumstances (conditions 2 and 3 of the previous Lemma), although firm 1 cannot use the worker's knowledge completely, it has a higher benefit than the other firm, since the latter can exploit the increase in productivity for a low production only.

In summary, firm 2 is only able to hire the worker when entrepreneur 1 has so much knowledge that his return on the worker's knowledge is low. Moreover, entrepreneur 2's knowledge must be higher than a certain threshold. If not, firm 2's gains of productivity by hiring the worker would only be exploited for a small production level, while firm 1, by losing the worker, would suffer a loss of productivity for a large production level.

We can gain further intuition on the Lemma noting that when there is a big difference between the two entrepreneurs' knowledge, the market is close to a monopolistic structure. If the worker moved from firm 1 to firm 2, the market would evolve towards a more symmetric duopoly: the gain of the small duopolist by hiring the worker could not compensate for the loss suffered by the quasi monopolist.

We illustrate the previous Lemmas in Figures 1 and 2. In Figure 1 we show the line $k_1+k_2 = \frac{8}{5}$ (denoted by AB) and the function $k_2 = f(k_1, k_h) := \frac{1}{2}\frac{8k_h k_1+5\left(1-k_1^2-k_h^2\right)}{4(1-k_1)+5k_h}$ for three different values of k_h $(\frac{1}{5}, \frac{1}{3}, \frac{2}{5})$ (in dotted line in the region where it is not binding). Moreover, we also present the inverse function $k_1 = f^{-1}(k_2, k_h)$ above the 45 degree line (only in the region where it is binding).

Firm 1 has a higher benefit from the worker when k_2 is below either AB or $f(k_1, k_h)$, depending on the value of k_h. For a high k_h, i.e., $k_h \geq \frac{2}{5}$, the function $f(k_1, k_h)$ does not cross the line AB and it is not binding. For a small k_h, i.e., $k_h < \frac{1}{5}$, in contrast, the function crosses AB above the 45 degree line and AB becomes not binding. For intermediate values of k_h, both functions are binding.

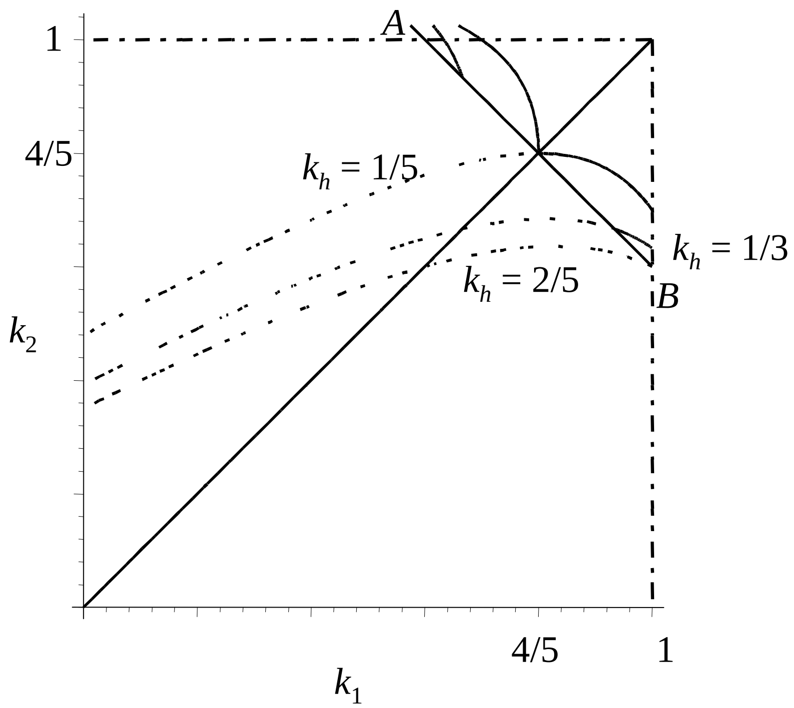

Figure 1: Areas in which firm 1 or firm 2 employ the worker (for different levels of k_h).

Finally, notice that when k_h increases (up to $\frac{2}{5}$), the area for which firm 2 has a higher benefit from the worker increases as well. This is quite intuitive: when k_h is higher, the worker's knowledge starts becoming irrelevant for the big firm for smaller values of k_1.

In Figure 2 we present the same analysis for $k_h = \frac{1}{3}$ only. The curve with negative slope results from merging the relevant (i.e., binding) parts of AB, f and f^{-1}. It represents values of k_1 and k_2 for which firm 1 and firm 2 have the same benefit from employing the worker. Of course the two firms have the same benefit also when they are identical, i.e., along the 45° line. Firm 1 has a higher benefit in the area denoted by I (and I'). In a specular way, firm 2 has a higher benefit from hiring the worker in the area denoted by II (and II'). Given our assumption that $k_1 \geq k_2$, only the triangle below the 45° line is relevant for the present analysis. Nevertheless, we also consider the area above it, given that – as we shall see – after the spillover, firm 2 may happen to have a bigger knowledge level than firm 1.

4 Technological Spillovers

In the previous section we identified the levels of knowledge for which the specialized worker will keep working for firm 1 or will be hired by firm 2. Now, we discuss the possibility that either entrepreneur wants to give knowledge

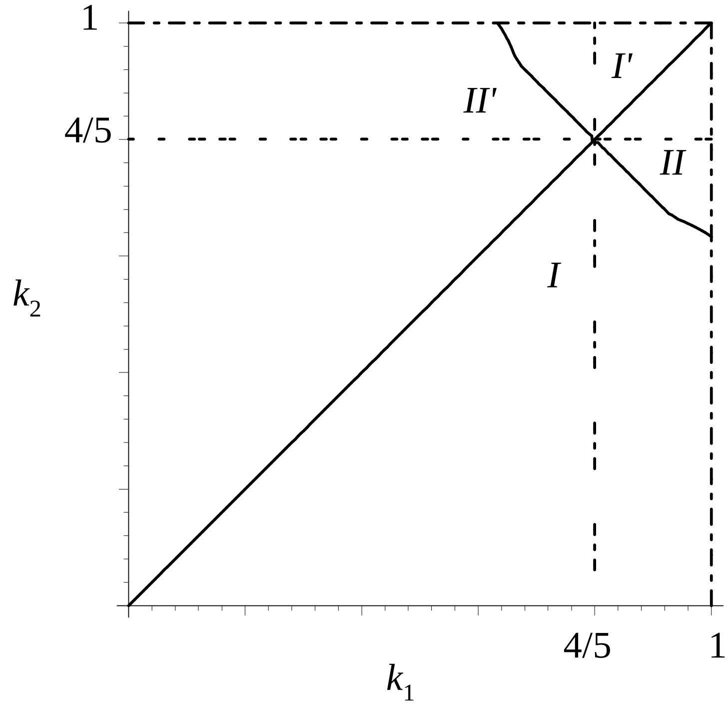

Figure 2: Areas in which firm 1 or firm 2 employ the worker (for $k_h = \frac{1}{3}$).

to the other in the first stage of the game. There are two reasons for which an entrepreneur may be willing to do so.

First, the entrepreneur who – in the status quo – employs the worker, may give knowledge to the rival for the purpose of lowering the wage. Recall that the wage paid by an entrepreneur is just equivalent to the extra-profit that the worker is able to generate for the competitor. Moreover, the knowledge of the worker and that of the entrepreneurs are substitute. Therefore, giving knowledge to the rival may lower the surplus that the rival can obtain from the worker and, thus, may reduce the wage. Clearly, giving knowledge to the rival has a negative effect too, since the rival becomes stronger. The voluntary spillover is profitable if the positive effect on the labor cost overwhelms this negative effect.

A second reason for endogenous spillovers is that an entrepreneur may try, through the spillover, to change the status quo, i.e., the conditions of the second stage of the game and, therefore, to be able to employ the worker. Consider the case in which–for the original levels of knowledge–firm 1 loses the worker. If entrepreneur 1 reveals knowledge to firm 2, he may reduce the worker's importance for the rival, and, therefore, be able to keep him. Similarly, consider the case in which, in the status quo, firm 1 is able to keep the worker. Entrepreneur 2 may be willing to reveal knowledge to firm 1 in order to make the worker less important for firm 1 and, therefore, be able to poach him.

Before we start our analysis, it is just worth mentioning that neither reason is relevant in the case of a monopolist. Clearly, the monopolist cannot lower the worker's wage by giving knowledge to the potential entrant. Indeed, the worker's wage is equal to the potential entrant's profit: this profit is not decreasing in the level of spillover. It is also obvious that the monopolist does not need to poach the worker either, and, therefore, also the second reason is irrelevant. Given these considerations, we can focus on the case of a duopoly. We start by considering the possibility of spillovers in area I of Figure 2.[11]

4.1 Technological Spillovers in Area I

In area I firm 1 is able to keep the worker. Therefore, in the absence of spillovers, its net profit is

$$\Pi(\min(k_1 + k_h, 1), k_2) - \tag{1}$$
$$[\Pi(\min(k_2 + k_h, 1), k_1) - \Pi(k_2, \min(k_1 + k_h, 1))].$$

[11]While Figure 2 is drawn for a specific value of k_h, a similar figure obtains for other values.

By revealing knowledge to the rival, firm 1 reduces its gross profit (first term in the expression above), as the total knowledge of firm 2, K_2, goes up from k_2 to $\min(k_2 + \rho_1 k_1, 1)$. At the same time, the increase in K_2 may lower the labor cost represented by the difference in bracket in the expression above.

Now we prove that, as long as firm 2 is "big enough," firm 1 finds the spillover profitable.

Proposition 3 *In Area I, firm* 1 *finds it profitable to spill over knowledge to firm* 2 *if and only if* $k_2 \geq \max\left\{\frac{3}{5}, 1 - k_h\right\}$*. The optimal spillover level is* $\rho_1 \geq \frac{1-k_2}{k_1}$*.*

Proof. See Appendix. ■

To understand this result, it is useful to refer to Figure 3. The Figure illustrates the values (shaded region) for which the spillover is profitable for the cases in which $k_h \geq \frac{2}{5}$. Consider a point in Area I close to the 45 degree line, where the two entrepreneurs have almost the same level of knowledge. In this case, the worker is almost as useful to one firm as he is to the other. Thus, he can appropriate most of the surplus that he is able to generate for either firm.[12] Therefore, firm 1 may be willing to transfer knowledge to firm 2 in order to reduce the worker's wage and to appropriate more of the surplus that he generates. The worker's wage is decreasing in the spillover if and

[12] In the extreme case in which the two firms are identical, the worker can appropriate the entire surplus he generates and nothing would be left to the entrepreneur.

only if $k_2 + k_h \geq 1$: this explains one of the conditions in the Proposition. Now let us explain the other. When both entrepreneurs have a high level of knowledge, as required by the first condition, the net profit of firm 1 is $\Pi(1, k_2) - \Pi(1, k_1) + \Pi(k_2, 1)$. After maximum spillover (which turns out to be the optimal level of spillover), this profit is higher than in the status quo for $k_2 > \frac{3}{5}$, which is indeed the other condition in the Proposition.

The level of optimal spillover is the maximum one. In order to reduce the specialized worker's wage, firm 1 prefers to reveal as much knowledge to firm 2 as possible up to reaching $K_2 = 1$. In Figure 3, this means moving from Area I to Area I'. Clearly, given that the maximum level of knowledge is 1, any spillover level $\rho_1 \geq \frac{1-k_2}{k_1}$ is equally profitable:[13] through the spillover, entrepreneur 1 wants to reach the maximum attainable level of asymmetry with the competitor.

Now, let us discuss the possibility of a spillover from firm 2 to firm 1. In Area I, in the absence of spillovers, firm 2 will be unable to hire the specialized worker. By revealing knowledge, however, it may lower the importance of the worker for the other firm and be able to poach him. After poaching the worker from the rival, its profit may be higher. In the next Proposition, we find the conditions under which this is true.

[13] Given that $K_2 = \min\{1, k_2 + \rho_1 k_1\}$, clearly $K_2 = 1$ for any $\rho_1 \geq \frac{1-k_2}{k_1}$. Moreover, given that $k_2 \geq \frac{3}{5}$, $\frac{1-k_2}{k_1} < 1$.

Proposition 4 *Firm 2 finds it profitable to spill over knowledge to firm* 1 *in Area I if and only if*

1) $k_2 \geq \max\left\{\frac{3}{5}, 1 - k_h\right\}$ *or,*

2) $k_2 + k_h < 1$ *and* $k_2 \geq \frac{4}{5} - \frac{1}{2}k_h$.

Moreover, the optimal level of spillover is $\rho_2 \geq \frac{1-k_1}{k_2}$.

Proof. See Appendix ■

It is easy to see that condition 2 in the Proposition implies that k_2 be greater than or equal to $\frac{3}{5}$. Therefore, the Proposition shows that, to find the spillover profitable, entrepreneur 2 must have a level of knowledge higher than a certain threshold.[14] By looking at Figure 3, this is easily understood. The purpose of entrepreneur 2 is "to move" from Area I to Area II. The spillover from firm 2 increases firm 1's knowledge. Only for high levels of k_2, however, this has the effect of moving from Area I to Area II. When entrepreneur 2 has a low level of knowledge, the minimum level of K_1 needed to pass from one area to the other is never reached, even if he discloses all his knowledge. When his level of knowledge respects the constraints indicated above, not only is entrepreneur 2 able to hire the worker, he also finds it profitable. The optimal level of spillover turns out to be the maximum one. When both entrepreneurs have a high level of knowledge, the specialized

[14] Note that both conditions also imply $k_1 + k_2 > 1$. In condition 1 we have $k_1 \geq k_2 \geq \frac{3}{5}$ and hence $k_1 + k_2 \geq \frac{6}{5}$. Similarly, for condition 2. Moreover, note that in the Proposition there is an implicit upper bound for k_h, i.e., $k_h < \frac{2}{5}$.

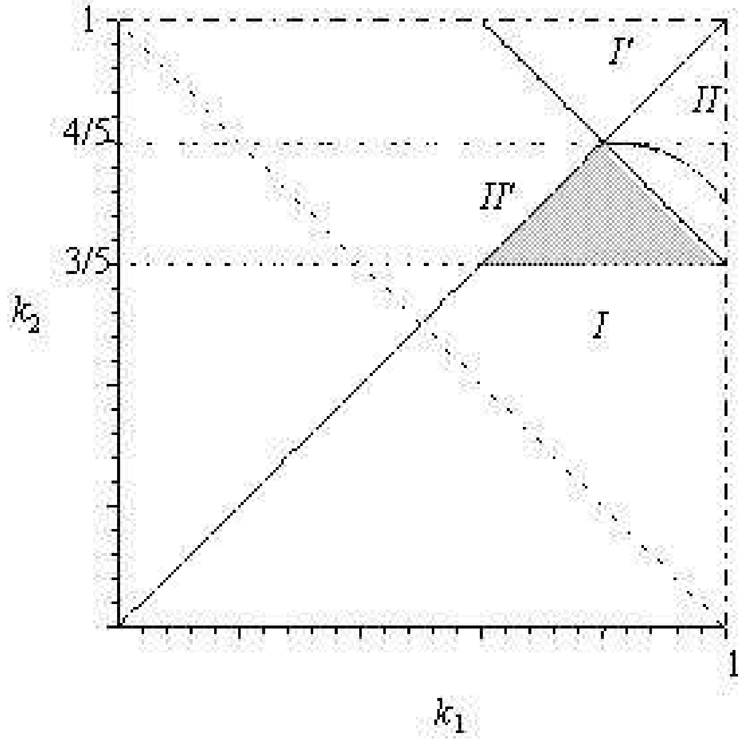

Figure 3: Region where there is spillover from firm 1 to firm 2 and from firm 2 to firm 1 in area I.

worker extracts most of the surplus that he generates for his employer, i.e., he earns a high wage, as we have already noticed above. The spillover increases the difference in knowledge between the two entrepreneurs and reduces this wage. This labor cost reduction overwhelms the cost of competing with a stronger rival and the entrepreneur prefers to reveal as much as he can.By inspection of conditions 1 and 2 in the Proposition, it is easy to note that an increase in the worker's knowledge makes the spillover profitable to firm 2 for a larger set of values of k_1 and k_2, as intuition would suggest. When the specialized worker can give a higher contribution in terms of productivity, firm 2 is more easily willing to disclose information in order to hire him.

Finally, notice that the conditions indicated in Proposition 4 are less restrictive than those in Proposition 3, as can also be checked in Figure 4. In fact, Proposition 4 contemplates the same condition present in Proposition 4 and, in addition, condition 2, i.e., $k_2 + k_h < 1$ and $k_2 \geq \frac{4}{5} - \frac{1}{2}k_h$. The light grey area in Figure 4 represents the values of k_h and k_2 for which both firms find it profitable to spill over knowledge in Area I. The dark grey area represents the values for which the spillover is profitable for firm 2 only. The result is not surprising: firm 2 has high incentives to spill over knowledge in order to poach the skilled worker, while firm 1, through the spillover, can only lower the worker's salary.To conclude the analysis of Area I,we need to study the conditions under which either firm finds it profitable to *accept* knowledge from the rival. When the knowledge disclosure comes from firm 1, the spillover is, obviously, profitable to firm 2. Since the latter would not be able to hire the specialized worker without receiving knowledge, the only effect of the spillover is to increase its profit. In contrast, the effect of firm 2's spillover on firm 1's net profit is less obvious. On the one hand, firm 1 is strengthened by more knowledge. On the other, by accepting more knowledge, the firm loses the specialized worker. In the next Proposition we show that whenever it is profitable for firm 2 to reveal knowledge, it is also

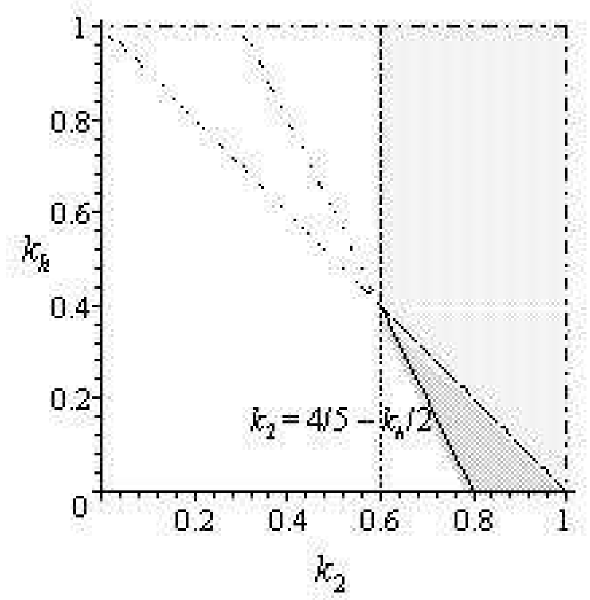

Figure 4: Regions in the $k_h - k_2$ space in which firm 2 only finds the spillover profitable (dark grey region) or both firms find the spillover optimal (ligh grey region).

profitable for firm 1 to accept it.

Proposition 5 *Consider the levels of knowledge for which firm* 2 *chooses* $\rho_2 \geq \min\left[1, \frac{1-k_1}{k_2}\right]$ *in Area* I. *For these levels, the net profit of firm* 1, *after receiving the maximum spillover from firm* 2, *is higher than without spillover.*

Proof. See Appendix ■

After the spillover, firm 2 reaches a level of knowledge $K_2 = 1$ but has to pay part of the gross profits to the worker in terms of wage. Firm 1, in contrast, keeps its original level of knowledge ($K_1 = 1$) (i.e., its level of knowledge when employing the worker), but does not have to pay the worker anymore. Clearly, it has to face a stronger rival, but this market externality is low, as noted above. On the other hand, the wage that firm 2 has to pay is even higher than what the worker received from firm 1, as it is easy to check. Therefore, whenever firm 2 finds it profitable to disclose knowledge, firm 1 finds it profitable to accept it.

4.2 Technological Spillovers in Area II

In Section 3.2 we proved that firm 2 will hire the specialized worker if the two entrepreneurs have levels of knowledge defining Area II. In Area II also there is scope for knowledge spillovers. Entrepreneur 2 may be willing to give knowledge to entrepreneur 1 in order to reduce the worker's wage. And entrepreneur 1 may be willing to give knowledge to entrepreneur 2 in order to avoid losing the specialized worker.

We start our analysis by looking at the first case. By revealing information, firm 2 certainly reduces its gross profit, as it strengthens the rival, but it may be able to reduce the cost of human capital. We can show that, indeed, the latter effect is always dominant in this area and, therefore, the spillover is always profitable.

Proposition 6 *In Area II, the net profit of firm 2 is increasing in the spillover level ρ_2. The optimal level of the spillover is $\rho_2 \geq \min\left[1, \frac{1-k_1}{k_2}\right]$.*

Proof. See Appendix. ■

The intuition for this result is similar to that for Proposition 1. When the two entrepreneurs have a similar level of knowledge, the worker is almost as useful to one firm as he is to the other. Therefore, he can appropriate most of the surplus that he generates for either firm. Hence, in a situation of almost symmetry, entrepreneur 2 has an incentive to reveal knowledge to entrepreneur 1 in order to appropriate more of the surplus created by the worker. With the spillover, the two entrepreneurs become indeed more asymmetric in knowledge and this reduces the salary to be paid to the worker. Despite the fact that the spillover increases the rival's knowledge, the firm finds it profitable. In our model this is true for all the values of area II: for all these values, the two entrepreneurs's level of knowledge is so similar that the spillover is always profitable. Notice, in particular, that, after maximum spillover, the worker's knowledge is useless for firm 1's production process. Therefore, firm 2 just pays him the competitive wage, since his contribution to the other firm's knowledge is nought.

Let us analyze, now, the possibility that entrepreneur 1 reveals knowledge. He may do so to avoid that the skilled worker leaves for firm 2. Indeed, after the spillover, the worker may be less profitable for firm 2 and, hence, firm 2 may have a lower incentive to poach him. In Figure 2 this is equivalent to moving from a point in Area II to a point in Area I'.

Proposition 7 *Firm* 1 *finds it profitable to spill over its knowledge to firm* 2 *in Area* II *if and only if* $k_2 + k_h \geq 2k_1 - \sqrt{4k_1 - 2 - k_1^2}$. *Moreover, the optimal level of spillover is* $\rho_1 \geq \min\left[1, \frac{1-k_2}{k_1}\right]$.

Proof. See Appendix. ■

If entrepreneur 1 reveals knowledge, he finds it profitable to reveal all his knowledge. Consider Figure 5 and take a point in Area II. The purpose of firm 1's knowledge revelation is to avoid the loss of the worker. Therefore, after the spillover the levels of the entrepreneurs' knowledge should belong to Area I'. In this area, when the two entrepreneurs have a similar knowledge level (i.e., for points close to the 45 degree line), the worker is able to appro-

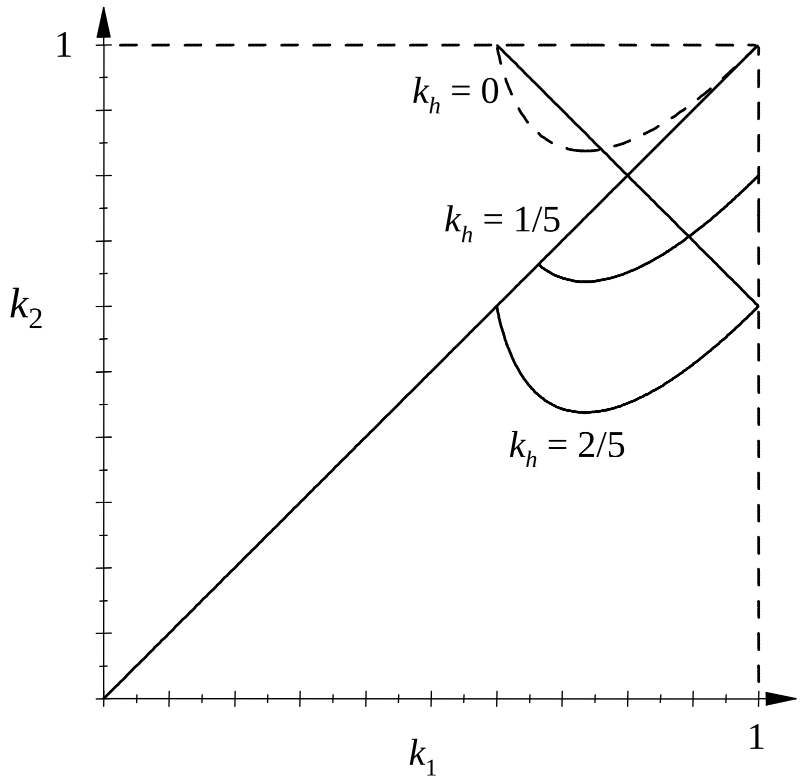

Figure 5: Spillover from firm 1 to Firm 2 in area II.

priate most of the surplus that he creates. Therefore, entrepreneur 1 finds it profitable to reveal all his knowledge to create the maximal asymmetry with the other entrepreneur and lower the worker's wage. The optimal spillover is again a corner solution.

For the spillover to be profitable to firm 1, the knowledge level of firm 2 has to be higher than a critical threshold. This threshold is illustrated in Figure 5 for different values of k_h. Note that – as the figure shows – the threshold becomes irrelevant for $k_h > \frac{2}{5}$. Indeed, for $k_h \geq \frac{2}{5}$, the function

$k_2 + k_h = 2k_1 - \sqrt{4k_1 - 2 - k_1^2}$ does not belong to Area II, and firm 1 finds the spillover profitable for all values belonging to this area. While there is an upper bound for the threshold to be biding $\left(k_h \leq \frac{2}{5}\right)$, Figure 5 shows that there is no lower bound, since for $k_h = 0$ the threshold crosses area II in the point $k_1 = k_2 = 1$.

When entrepreneur 2's knowledge is below the threshold, firm 2's total knowledge is $K_2 = k_2 + k_h < 1$ and firm 1's profit in the absence of spillover is $\Pi(k_1, k_2 + k_h)$. By revealing information, firm 1 would strengthen firm 2 (whose total knowledge would reach 1) and have a net profit of $\Pi(1,1) - (\Pi(1,k_1) - \Pi(1,1))$. Avoiding labor poaching by firm 2 in such a case has a lower benefit than the cost of the information disclosure.

In other words, when the specialized worker has a level of knowledge lower than $\frac{2}{5}$, entrepreneur 1 finds it profitable to give knowledge to the rival only if the rival is already strong enough. If, instead, the other entrepreneur has a low level of knowledge, then the cost of strengthening him is higher than the benefit of keeping the worker. When the specialized worker has a level of knowledge higher than $\frac{2}{5}$, he is so beneficial to the firm's productivity that entrepreneur 1 prefers to spill over knowledge to the rival for all values belonging to Area II in order to avoid labor poaching.

To conclude the analysis of spillovers in area II we need to study the conditions under which either firm finds it profitable to accept knowledge from the rival. The analysis is similar to that of Area I. When the knowledge revelation comes from firm 2, the spillover is, obviously, always profitable to the other firm. Without receiving knowledge, this firm would not be able to employ the specialized worker. Hence, the only effect of the spillover is to increase its profit. The effects of the spillover from firm 1 on firm 2's net profit is not so obvious. This firm is strengthened by more knowledge. By accepting more knowledge, however, the firm loses the opportunity to poach the specialized worker. Now we show that whenever revealing knowledge is profitable for entrepreneur 1, accepting it is profitable for firm 2.

Proposition 8 *The net profit of firm* 2 *after maximal spillover from firm* 1 *in Area* II *is always higher than the net profit in the absence of spillover.*

Proof. See Appendix. ■

After maximal spillover, firm 2 has profit $\Pi(1,1)$. In the absence of spillovers, its profit would be

$$\Pi\left(\min\left(k_2+k_h,1\right),k_1\right)-\left[\Pi\left(1,k_2\right)-\Pi\left(k_1,\min\left(k_2+k_h,1\right)\right)\right].$$

Intuitively, in Area II this second expression is always lower than the first, since the salary to pay to the worker is high, relative to the surplus that he produces. In Area II the two entrepreneurs have similar levels of knowledge.

Therefore, even if firm 2 hires the worker, it cannot completely exploit the quasi-rent that he generates. For this reason, firm 2 finds it profitable to accept the spillover and not to poach the worker.

5 Knowledge Spillovers and Labor Mobility in Equilibrium

We are now ready to discuss the equilibrium outcomes in terms of knowledge spillovers and labor mobility. To start our analysis we prove a simple but useful result. In equilibrium, it is impossible to have bilateral spillovers: the profit of the firm who does not employ the worker is decreasing in the other firm's level of knowledge, therefore the former has no incentive to give knowledge to the latter.

Lemma 9 *If, in equilibrium, firm j employs the workers, then firm i $(i \neq j)$ has no incentive to spill over knowledge to firm j.*

Proof. See Appendix. ■

Given this result, we have only to study the cases in which there is spillover from firm 1 to firm 2 or vice versa. The analysis of the previous Section delivers the equilibrium outcomes as immediate corollaries. We summarize the outcomes in the two areas identified above in the following propositions. The first refers to Area I.

Proposition 10 *If there is a duopoly (i.e., $k_1 + k_h \leq 2k_2$), Area I can be divided into three regions.*

Region Ia, characterized by the following conditions:

1) $k_1 + k_2 \leq 1$, or

2) $k_1 + k_h < 1$, or

3) $k_1 + k_2 \geq 1$, $k_2 + k_h \geq 1$ and $k_2 < \frac{3}{5}$, or

4) $k_1 + k_2 \geq 1$, $k_1 + k_h \geq 1$, $k_2 + k_h < 1$ and $k_2 < \frac{4}{5} - \frac{1}{2}k_h$.

In this region, there exists a unique equilibrium: no spillover occurs, and firm 1 keeps the worker.

Region Ib, characterized by the following conditions:

$k_1 + k_h \geq 1$, and $\frac{4}{5} - \frac{1}{2}k_h \leq k_2 < 1 - k_h$.

In this region, there exists a unique equilibrium: there is spillover from firm 2 to firm 1, $\rho_2 \geq \frac{1-k_1}{k_2}$, and the worker moves to firm 2.

Region Ic, characterized by the following condition:

$k_2 \geq \max\left\{1 - k_h, \frac{3}{5}\right\}$.

In this region there exist two equilibria: the first one is identical to that of Region Ib; in the second, there is spillover from firm 1 to firm 2, $\rho_1 \geq \frac{1-k_2}{k_1}$, and the worker remains at firm 1. Firms are indifferent between the two equilibria, since $K_1 = K_2 = 1$ in both of them.

In the next proposition we illustrate the equilibrium outcomes in Area II.

Proposition 11 *If there is duopoly (i.e., $k_1 + k_h \leq 2k_2$), Area II can be divided into two regions.*

Region IIa, characterized by the following condition:

$k_2 + k_h < 2k_1 - \sqrt{4k_1 - 2 - k_1^2}$.

In this region there exists only one equilibrium: there is spillover from firm 2 to firm 1, $\rho_2 \geq \min\left[1, \frac{1-k_1}{k_2}\right]$, and the former firm hires the worker.

Region IIb, characterized by the following condition:

$k_2 + k_h \geq 2k_1 - \sqrt{4k_1 - 2 - k_1^2}$.

In this region there exist two equilibria: the first is identical to that of region IIa; in the second, there is spillover from firm 1 to firm 2, $\rho_1 \geq \min\left[1, \frac{1-k_2}{k_1}\right]$, and no labor mobility. Firms obtain the same profit in the two equilibria since $K_1 = K_2 = 1$ in both of them.

Given that both propositions are an immediate corollary of the previous ones, we do not provide a proof. We illustrate the propositions in Figure 6. The figure is drawn for a level of $k_h = 0.3$.

For low levels of k_2, (i.e., $k_1 + k_h > 2k_2$) the industry is a monopoly (area M). In this case there is no spillover and no labor mobility.

For higher levels of k_2 (firm 2 is more efficient), the industry is a duopoly. Nevertheless, for k_2 belonging to area Ia, there is no spillover and no labor mobility. In fact, for k_2 just sufficient to have a duopoly, firm 2 is unable to poach the worker from firm 1. At the same time, firm 1 has no incentive to share knowledge with firm 2, since the worker's salary decreases significantly only if the two firms have a similar level of knowledge.

When k_2 is even higher (areas Ib and IIa), we have spillover from firm 2 to firm 1 and labor poaching. Hence, in equilibrium the worker will be hired by firm 2. Finally, for a large k_2 (areas Ic and IIb), also firm 1 has an incentive to spill over knowledge to firm 2, in order to reduce the wage (area Ia) or to avoid labor poaching and lowering the wage (in area IIb). Hence, in these areas there are multiple equilibria: one equilibrium with spillover from firm 1 to firm 2 and no labor mobility and one with spillover form firm 2 to firm 1 and labor poaching. In both equilibria the firms obtain the highest level of efficiency, i.e., $K_1 = K_2 = 1$.

For different levels of k_h the figure is only slightly different. For $k_h \leq \frac{1}{5}$, region Ic disappears, i.e., in Area I there are only equilibria with no spillover or with spillovers from firm 2 to firm 1. In fact, when the worker has a low level of knowledge, the benefit for firm 1 of lowering his salary through the spillover is always lower than the cost of strengthening the other firm.

Finally, for $k_h \geq \frac{2}{5}$, regions Ib and IIa disappear and only three possibilities survive: monopoly, no spillover and multiple equilibria. Indeed, in this case, the worker is so important that, unless there is no possibility for labor poaching (in areas M and Ia), both firms have an incentive to spill over knowledge to the rival in order to employ him and lower his salary.

6 Discussion

We have studied how technological knowledge can spill over from one firm to another through two channels: entrepreneurs can decide to reveal knowledge to their competitors; and specialized workers can transfer information by moving from one employer to another. Knowledge revelation is a way through which firms can lower labor cost. It is also an instrument to compete on the skilled labor market. In the absence of direct knowledge revelation, still spillovers can occur, since specialized workers trained at one firm can be hired by another. Labor mobility and knowledge revelation are not mutually exclusive. There are indeed cases in which both phenomena occur in equilibrium.

Direct knowledge revelation occurs in equilibrium only when both entrepreneurs have a relatively high level of knowledge. Spillovers and labor mobility are more likely when entrepreneurs have similar levels of knowl-

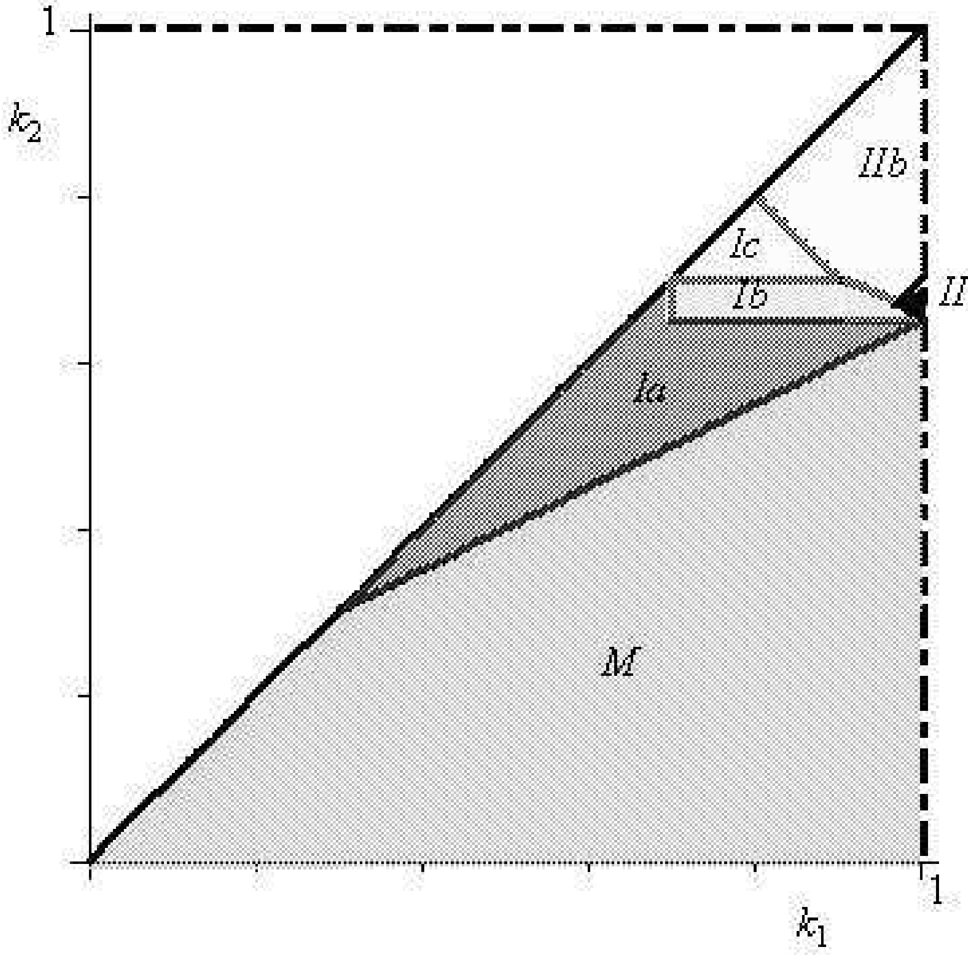

Figure 6: Characterization of the equilibria.
M: Monopoly
Ia: No spillover. Firm 1 hires the worker
Ib and IIa: Spillover from 2 to 1 and 2 hires the worker
Ic and IIb: Multiple equilibria

edge and, therefore, a similar market share. Under these circumstances, the firms are more willing to reveal information in order to reduce the labor cost. Not surprisingly, knowledge spillovers and labor mobility are also more likely when the specialized worker has a higher level of knowledge and, therefore, a higher impact on productivity.

Almost all the studies in the literature on technological innovation consider spillovers as an involuntary leakage of information (due to reverse engineering, industrial espionage, ineffectiveness of the patent law). Our work, in contrast, shows that entrepreneurs have incentives to disseminate information *voluntarily*. This is a particularly novel contribution of our paper. Furthermore, the few theoretical papers that admit endogenous spillovers (see, e.g., Combes and Duranton, 2001, Gersbach, and Schmutzler, 2003, and Fosfuri and Ronde, 2004) show that spillovers occur through workers turnover. We have shown that knowledge can (voluntarily) flow from one firm to another even *in the absence* of labor mobility. Our analysis can account for knowledge spillovers even in industrial clusters where there is no evidence of a high rate of turnover.[15]

There is anecdotal evidence of voluntary information sharing in industrial clusters. The literature on industrial clusters, however, typically explains it

[15] A low rate of turnover is, for instance, documented for the Italian districts by De Blasio and Di Addario (2002).

by invoking reciprocity or the cooperative environment that can be sustained when the same firms operate for years in the same area. While we do not deny the importance of such factors in explaining localized technological spillovers, we suggest that voluntary dissemination of information could simply stem out of firms' competition in the labor and in the good markets.

In our analysis we have assumed, for simplicity, that workers can move from one firm to another at no cost. If, in contrast, moving is costly, the analysis would change in a straightforward way. Clearly, the higher this cost, the lower the outside option for the worker and the lower the incentive of the entrepreneurs to share knowledge with the rivals. Indeed, if this cost were high enough, no spillover and no labor mobility should be expected. This consideration leads to an immediate explanation of the dimension of industrial districts. Industrial districts typically do not expand beyond a particular geographical area. It is also well known that the labor market for skilled workers is itself localized in that area. Our model indicates that, indeed, the dimension of the labor market determines the dimension of the district. Firms belonging to the district do not have any incentive to spill over knowledge to firms far away, as these are not competitors on the labor market, provided that the moving cost for the workers is sufficiently high. At the same time, the transportation costs impede the transfer of knowledge through labor mobility. Therefore, the technological externalities that fuel industrial districts do not exceed some geographical borders.

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7 Appendix

Proof of Lemma 1. We must consider three cases:
Case 1: $k_2 + k_h \leq \frac{1}{2}k_1$. This case is trivial, as the potential entrant would not be able to enter the market even by hiring the worker. Therefore, there is no scope for worker mobility.
Case 2: $\frac{1}{2}k_1 < k_2 + k_h \leq 2k_1$. In this case if the worker moves to firm 2, in equilibrium there will be a duopoly. By avoiding that the worker leaves for the entrant firm, firm 1 avoids the following loss: $\Pi^M(\min(k_1 + k_h, 1)) - \Pi(k_1, \min(k_2 + k_h, 1))$. On the other hand, by hiring the worker, the potential entrant would earn $\Pi(\min(k_2 + k_h, 1), k_1)$. Hence, we have to prove that

$$\Pi^M(\min(k_1 + k_h, 1)) - \Pi(k_1, \min(k_2 + k_h, 1)) \geq \\ \Pi(\min(k_2 + k_h, 1), k_1).$$

This follows immediately from the following inequalities:

$$\Pi^M(\min(k_1 + k_h, 1)) \geq \Pi^M(\min(k_2 + k_h, 1)) > \\ \Pi(k_1, \min(k_2 + k_h, 1)) + \Pi(\min(k_2 + k_h, 1), k_1),$$

where the second inequality is easily verified, as the profit of a monopolist is higher than the sum of the profits of two duopolists.
Case 3: $k_2 + k_h > 2k_1$. For these parameter values, if the worker moves to firm 2, firm 2 will become monopolist. Therefore we have

$$\Pi^M(\min(k_1 + k_h, 1)) \geq \Pi^M(\min(k_2 + k_h, 1)).$$

Given that we have assumed that the worker prefers to stay at firm 1 even in the case he is offered the same wage from the two firms, the proposition follows immediately.■

Proof of Lemma 2. By hiring the worker, each firm obtains the following increase of gross profits:

$$\Pi(\min(k_i + k_h, 1), k_j) - \Pi(k_i, \min(k_j + k_h, 1)).$$

Therefore, firm 1 has a higher (not smaller) incentive to hire the worker if and only if

$$\Pi(\min(k_1 + k_h, 1), k_2) - \Pi(k_1, \min(k_2 + k_h, 1)) \geq \\ \Pi(\min(k_2 + k_h, 1), k_1) + \Pi(k_2, \min(k_1 + k_h, 1))$$

We have to distinguish three cases.
Case 1: $k_1 + k_h < 1$. The above inequality is equivalent to

$$\Pi(k_1 + k_h, k_2) - \Pi(k_1, k_2 + k_h) -$$
$$\Pi(k_2 + k_h, k_1) + \Pi(k_2, k_1 + k_h) = 2\,(k_1 - k_2)\,k_h \geq 0,$$

which is always satisfied.
Case 2: $k_1 + k_h \geq 1$ and $k_2 + k_h < 1$. The inequality is equivalent to

$$\Pi(1, k_2) - \Pi(k_1, k_2 + k_h) - \Pi(k_2 + k_h, k_1) + \Pi(k_2, 1) =$$
$$-\frac{2}{9}\,(4\,(1 - k_1) + 5k_h)\,k_2 + \frac{1}{9}\left(8k_h k_1 + 5\left(1 - k_1^2 - k_h^2\right)\right) \geq 0,$$

which is satisfied for

$$k_2 \leq \frac{1}{2}\frac{8k_h k_1 + 5\,(1 - k_1^2 - k_h^2)}{4\,(1 - k_1) + 5k_h}.$$

Case 3: $k_2 + k_h \geq 1$. The inequality is equivalent to

$$\Pi(1, k_2) - \Pi(k_1, 1) - \Pi(1, k_1) + \Pi(k_2, 1) =$$
$$\frac{1}{9}\,(5k_2 - 8 + 5k_1)\,(k_2 - k_1) \geq 0$$

which is satisfied for $k_1 + k_2 \leq \frac{8}{5}$, or

$$k_2 \leq \frac{8}{5} - k_1.$$

Finally, notice the following two observations.

First, for $k_h < \frac{1}{5}$ Case 3 is irrelevant. Indeed, if $k_h < \frac{1}{5}$, the inequality $k_2 + k_h \geq 1$ implies that $k_2 > \frac{4}{5}$. However, given that $k_1 > k_2$, it is never possible that the constraint $k_2 \leq \frac{8}{5} - k_1$ be satisfied.

Second, the function $f\,(k_1, k_h) = \frac{1}{2}\frac{8k_h k_1 + 5\left(1 - k_1^2 - k_h^2\right)}{4(1 - k_1) + 5k_h}$ crosses $\frac{8}{5} - k_1$ from the left in the point where $k_1 = k_h + \frac{3}{5}$, which implies $k_2 = 1 - k_h$. For $k_h = \frac{1}{5}$ the two curves cross at $k_1 = k_2 = \frac{4}{5}$, and for $k_h = \frac{2}{5}$ they cross at $k_1 = 1$. Hence, for $\frac{1}{5} \leq k_h \leq \frac{2}{5}$, we are in Case 2 to the right of the intersection and in Case 3 to the left of it. For $k_h > \frac{2}{5}$ we are always in Case 3. For an illustration, see Figure 1 in the text.■

Proof of Lemma 9. The spillover from firm i to firm j can be represented by an increase in K_j, provided that $K_j < 1$. The profit of the firm that does not employ the worker is:

$$\Pi(K_i, K_j) = \frac{1}{9}\left(2\min(1, K_i) - \min(1, K_j)\right)^2$$

The effect of the spillover is given by the following derivative:

$$\frac{\partial}{\partial K_j}\Pi(K_i, K_j) = \begin{cases} \frac{2}{9}\left(2\min(1, K_i) - \min(1, K_j)\right) < 0 \text{ if } K_j < 1 \\ 0 \text{ if } K_j \geq 1. \end{cases}$$

■

Proof of Proposition 3. We, first, prove that the optimal spillover must be a corner solution. We have to distinguish three cases:
Case 1: $k_1 + k_h < 1$.
Firm 1's profit is

$$\Pi(k_1 + k_h, k_2) - \Pi(k_2 + k_h, k_1) + \Pi(k_2, k_1 + k_h),$$

whose derivative is

$$\frac{\partial}{\partial k_2}\left(\Pi(k_1 + k_h, k_2) - \Pi(k_2 + k_h, k_1) + \Pi(k_2, k_1 + k_h)\right) = \frac{2}{9}k_2 - \frac{4}{9}k_1 - \frac{16}{9}k_h.$$

This derivative is non negative if and only if

$$k_2 \geq 2k_1 + 8k_h.$$

Even if firm 1 transfer all of its knowledge to firm 2, the final level of knowledge for firm 2 is $k_2 + k_1$ and it is not possible that

$$K_2 = k_2 + k_1 \geq 2k_1 + 8k_h.$$

Therefore, the derivative with respect to k_2 is always negative and the optimal level of spillover is $\rho_1 = 0$.
Case 2: $k_1 + k_h \geq 1$, $k_2 + k_h < 1$.
Firm 1's profit is

$$\Pi(1, k_2) - \Pi(k_2 + k_h, k_1) + \Pi(k_2, 1),$$

whose derivative is

$$\frac{\partial}{\partial k_2}\left(\Pi(1, k_2) - \Pi(k_2 + k_h, k_1) + \Pi(k_2, 1)\right) = \frac{4}{9}k_1 + \frac{2}{9}k_2 - \frac{8}{9}k_h - \frac{8}{9},$$

which is positive if and only if

$$2k_1 + k_2 - 4k_h - 4 \geq 0,$$

which is clearly impossible. Hence $\rho_1 = 0$.
Case 3: $k_2 + k_h \geq 1$.
Firm 1's profit is

$$\Pi(1, k_2) - \Pi(1, k_1) + \Pi(k_2, 1),$$

whose derivative is

$$\frac{\partial}{\partial k_2}\left(\Pi(1, k_2) - \Pi(1, k_1) + \Pi(k_2, 1)\right) = \frac{10}{9}k_2 - \frac{8}{9},$$

which is either always positive or first negative and then positive. Hence, the optimal spillover is a corner solution. To find the conditions under which the optimal solution is $\rho_1 = 1$ we compare the net profit of firm 1 for $\rho_1 = 0$ and $\rho_1 = 1$, i.e., we check when the following inequality is satisfied:

$$\Pi(1, k_2) - \left(\Pi(1, k_1) - \Pi(k_2, 1)\right) \leq \Pi(1, 1) - \left(\Pi(1, k_1) - \Pi(1, 1)\right).$$

This is equivalent to

$$\Pi(1, k_2) + \Pi(k_2, 1) - 2\Pi(1, 1) \leq 0,$$

which is true for $k_2 \geq \frac{3}{5}$.
Finally, we have to check that, after the spillover, firm 1 will indeed be able to keep the worker. After the spillover, $K_2 = 1$. Moreover, by Lemma 2, firm 1 will be able to keep the worker after the spillover if $K_2 \geq \frac{8}{5} - k_1$ (notice that for $K_2 > K_1$ the signs of the inequalities in the Lemma are reversed), which means $1 \geq \frac{8}{5} - k_1$. This is clearly satisfied, given that $k_1 > k_2 \geq \frac{3}{5}$.■

Proof of Proposition 4. In the absence of spillovers, firm 2's profit is

$$\Pi\left(k_2, \min\left(k_1 + k_h, 1\right)\right).$$

If it spills knowledge over to firm 1 and hires the worker, its profit will be

$$\Pi\left(\min\left(k_2 + k_h, 1\right), \min\left(k_1 + \rho_2 k_2, 1\right)\right) -$$
$$\left(\Pi(\min\left(k_1 + \rho_2 k_2 + k_h, 1\right), k_2) - \Pi\left(\min\left(k_1 + \rho_2 k_2, 1\right), \min\left(k_2 + k_h, 1\right)\right)\right).$$

We first prove that the solution for the optimal spillover must be a corner one. We have to distinguish three cases.
Case 1: $k_2 + k_h > 1$.

In this case firm 2's profit is

$$\Pi(1, k_1) - \Pi(1, k_2) + \Pi(k_1, 1),$$

whose derivative is

$$\frac{\partial}{\partial k_1}\left(\Pi(1, k_1) - \Pi(1, k_2) + \Pi(k_1, 1)\right) = \frac{2}{9}(5k_1 - 4).$$

Therefore, the profit has a minimum for $k_1 = \frac{4}{5}$ and the optimal spillover is a corner solution.
Case 2: $k_1 + k_h > 1$, $k_2 + k_h \leq 1$.
In this case, firm 2's profit is

$$\Pi(k_2 + k_h, k_1) - \Pi(1, k_2) + \Pi(k_1, k_2 + k_h),$$

whose derivative with respect to k_1 is

$$\frac{\partial}{\partial k_1}\left(\Pi(k_2 + k_h, k_1) - \Pi(1, k_2) + \Pi(k_1, k_2 + k_h)\right) = \frac{2}{9}(5k_1 - 4k_2 - 4k_h).$$

Therefore, the profit has a minimum in $k_1 = \frac{4}{5}(k_2 + k_h)$ and the optimal spillover is a corner solution.
Case 3: $k_1 + k_h \leq 1$.
In this region the profit for firm 2 is

$$\Pi(k_2 + k_h, k_1) - \Pi(k_1 + k_h, k_2) + \Pi(k_1, k_2 + k_h),$$

whose derivative is:

$$\frac{\partial}{\partial k_1}\left(\Pi(k_2 + k_h, k_1) - \Pi(k_1 + k_h, k_2) + \Pi(k_1, k_2 + k_h)\right) = \frac{2}{9}(k_1 - 2k_2 - 8k_h),$$

which is positive if

$$k_1 \geq 2k_2 + 8k_h.$$

Hence the solution is a corner one.
Therefore, in all three cases, the optimal spillover must be $\rho_2 \in \{0, 1\}$. Let us find the solution in the various cases.
Case 1a: $k_2 + k_h > 1$ and $k_1 + k_2 > 1$.

In this case the spillover is profitable if

$$\Pi(1,1) - \Pi(1,k_2) + \Pi(1,1) - \Pi(k_2,1) =$$
$$-\frac{1}{3} - \frac{5}{9}k_2^2 + \frac{8}{9}k_2 \geq 0,$$

which is satisfied for $k_2 > \frac{3}{5}$. After (maximal) spillover, firm 1 will have a level of knowledge $K_1 = 1$. Notice that the levels of knowledge $K_1 = 1$ and $k_2 > \frac{3}{5}$ belong to area II (i.e., respect the conditions of Lemma 2 for firm 2 to hire the worker) and, hence, firm 2 will, indeed, be able to hire the worker.
Case 1b: $k_2 + k_h > 1$ and $k_1 + k_2 \leq 1$.
In this case, the spillover is profitable if

$$\Pi(1,k_1+k_2) - \Pi(1,k_2) + \Pi(k_1+k_2,1) - \Pi(k_2,1) =$$
$$\frac{1}{9}\left(5k_1^2 + 10k_1k_2 - 8k_1\right) \geq 0,$$

which is satisfied for $k_2 \geq \frac{4}{5} - \frac{1}{2}k_1$. Notice, however, that this inequality, along with $k_1 + k_2 \leq 1$, implies $k_1 \leq \frac{2}{5}$ and $k_2 \geq \frac{3}{5}$, which is, clearly a contradiction. Thus there cannot be spillover.
Case 2a: $k_1 + k_h > 1$, $k_2 + k_h \leq 1$ and $k_1 + k_2 > 1$.
In this case the spillover is profitable if

$$\Pi(k_2+k_h,1) - \Pi(1,k_2) + \Pi(1,k_2+k_h) - \Pi(k_2,1) =$$
$$\frac{1}{9}k_h(-8 + 10k_2 + 5k_h) \geq 0$$

which is satisfied for $k_2 > \frac{4}{5} - \frac{1}{2}k_h$. Notice that $k_2 + k_h \leq 1$ and $k_2 > \frac{4}{5} - \frac{1}{2}k_h$ imply $k_2 > \frac{3}{5}$ and $k_h < \frac{2}{5}$. Therefore, after the spillover, firm 1 will have a level of knowledge $K_1 = 1$. By Lemma 2, the level of k_2 compatible with area II must satisfy

$$k_2 > \frac{1}{2}\frac{8k_hk_1 + 5(1 - k_1^2 - k_h^2)}{4(1-k_1) + 5k_h}.$$

It can be proved that the rhs is decreasing in $k_1$in the relevant range. Therefore:

$$\frac{4}{5} - \frac{1}{2}k_h \geq \frac{1}{2}\frac{8k_hk_1 + 5(1 - k_1^2 - k_h^2)}{4(1-k_1) + 5k_h}.$$

Hence, after the spillover, firm 2 will indeed be able to hire the worker.
Case 2b $k_1 + k_h > 1$, $k_2 + k_h \leq 1$, $k_1 + k_2 \leq 1$.

In this case the spillover is profitable if

$$\Pi\left(k_2+k_h,k_1+k_2\right)+\Pi\left(k_1+k_2,k_2+k_h\right)-\Pi(1,k_2)-\Pi\left(k_2,1\right)=$$
$$\frac{5}{9}k_1^2+\frac{2}{9}k_1k_2-\frac{8}{9}k_1k_h-\frac{1}{3}k_2^2+\frac{2}{9}k_2k_h+\frac{5}{9}k_h^2+\frac{8}{9}k_2-\frac{5}{9}\geq 0,$$

whose solution is

$$k_2>\frac{1}{3}\left(k_h+k_1+4-\sqrt{(16k_h^2-22k_1k_h+8k_h+16k_1^2+8k_1+1)}\right).$$

This inequality, along with $k_1+k_2\leq 1$, implies (after some manipulation) $15k_h-30k_1k_h+6k_h>0$, whose solution is $k_h>2k_1-\frac{2}{5}$. Remember, however, that in this case $k_h<\frac{2}{5}$. These two last equalities imply that $k_1<\frac{2}{5}$. Therefore, $k_1+k_h<1$, which is a contradiction. Hence, in this case there is no spillover.
Case 3a: $k_1+k_h\leq 1$, $k_1+k_2>1$.
In this case the spillover is profitable if and only if

$$\Pi\left(k_2+k_h,1\right)-\Pi(1,k_2)+\Pi\left(1,k_2+k_h\right)-\Pi\left(k_2,k_1+k_h\right)=$$
$$\frac{1}{9}\left(2\left(2k_1-2+7k_h\right)k_2+1-8k_h+4k_h^2-k_1^2-2k_1k_h\right)\geq 0.$$

This inequality can never be satisfied. In fact a necessary condition for $\rho_2=1$ is that, starting from a certain level, the profit increases in the knowledge of firm 1. But, as proven above, this is true only for $k_1\geq 2k_2+8k_h$. This last condition implies that $k_2\leq\frac{1}{2}$ and $k_h\leq\frac{1}{8}$. Using these conditions into the previous inequality, we have

$$2\left(2k_1-2+7k_h\right)k_2+1-8k_h+4k_h^2-k_1^2-2k_1k_h\leq$$
$$2k_1-2+7k_h+1-8k_h+4k_h^2-k_1^2-2k_1k_h\leq$$
$$2k_1-2+\frac{7}{8}+1-1+4\left(\frac{1}{8}\right)^2-k_1^2-2k_1\left(\frac{1}{8}\right)=$$
$$\frac{7}{4}k_1-\frac{17}{16}-k_1^2<0,$$

where the last inequality comes from the fact that the last polynomial does not have real roots.
Case 3b: $k_1+k_h\leq 1$, $k_1+k_2\leq 1$.
The proof for this case uses an argument similar to that of case 2b.■

Proof of Proposition 5. Firm 2 finds it profitable to spill over knowledge when

$$\Pi\left(\min\left(k_2+k_h,1\right),\min\left(k_1+\rho_2k_2,1\right)\right)-$$
$$\left(\Pi(\min\left(k_1+\rho_2k_2+k_h,1\right),k_2\right)-\Pi\left(\min\left(k_1+\rho_2k_2,1\right),\min\left(k_2+k_h,1\right)\right))\geq$$
$$\Pi\left(k_2,\min\left(k_1+k_h,1\right)\right).$$

Firm 1 finds it profitable to accept the spillover if and only if

$$\Pi\left(\min\left(k_1+k_h,1\right),k_2\right)-\left(\Pi(\min\left(k_2+k_h,1\right),k_1\right)-$$
$$+\Pi\left(\min\left(k_1+k_2,1\right),\min\left(k_2+k_h,1\right)\right))\geq\Pi\left(k_2,\min\left(k_1+k_h,1\right)\right).$$

Remember that firm 2 spills over knowledge to firm 1 in two cases
1) $k_2+k_h>1$, $k_1+k_2>1$, and $k_2\geq\frac{3}{5}$,
2) $k_1+k_h>1$, $k_1+k_2>1$, $k_2+k_h<1$, and $k_2\geq\frac{4}{5}-\frac{1}{2}k_h$.
In case 1) the first inequality becomes

$$\Pi\left(1,1\right)-\Pi(1,k_2)+\Pi\left(1,1\right)\geq\Pi\left(k_2,1\right),$$

while the second becomes

$$\Pi\left(1,1\right)-\Pi\left(1,k_2\right)+\Pi(1,k_1)\geq\Pi\left(k_2,1\right).$$

Clearly the first implies the second. Similarly, in case 2) the first inequality is

$$\Pi\left(k_2+k_h,1\right)-\Pi(1,k_2)+\Pi\left(1,k_2+k_h\right)\geq\Pi\left(k_2,1\right),$$

and the second is

$$\Pi\left(1,k_2\right)-\left(\Pi(k_2+k_h,k_1)-\Pi\left(1,k_2+k_h\right)\right)\geq\Pi\left(k_2,1\right).$$

Again, the first implies the second.■

Proof of Proposition 6. We first prove that firm 2's profit is increasing in the spillover. We have to consider two cases:
Case 1: $k_2+k_h\geq1$.
In this case, firm 2's net profit is

$$\Pi\left(1,k_1+\rho_2k_2\right)-\left(\Pi\left(1,k_2\right)-\Pi\left(k_1+\rho_2k_2,1\right)\right),$$

whose derivative w.r.t. ρ_2 is

$$\frac{\partial}{\partial\rho_2}\left(\Pi\left(1,k_1+\rho_2k_2\right)+\Pi\left(k_1+\rho_2k_2,1\right)\right)=k_2\left(\frac{10}{9}k_1+\frac{10}{9}\rho_2k_2-\frac{8}{9}\right).$$

This derivative is always positive, as $k_1 > \frac{4}{5}$ in area II (since $\frac{4}{5}$ is the solution of $k_1 = \frac{8}{5} - k_1$).
Case 2: $k_2 + k_h < 1$.
In this case, firm 2's net profit is

$$\Pi\left(k_2 + k_h, k_1 + \rho_2 k_2\right) - \left(\Pi\left(1, k_2\right) - \Pi\left(k_1 + \rho_2 k_2, k_2 + k_h\right)\right),$$

whose derivative w.r.t. ρ_2 is

$$\frac{\partial}{\partial \rho_2}\left(\Pi\left(k_2 + k_h, k_1 + \rho_2 k_2\right) + \Pi\left(k_1 + \rho_2 k_2, k_2 + k_h\right)\right) = \frac{2}{9} k_2 \left(5\rho_2 k_2 + 5k_1 - 4\left(k_2 + k_h\right)\right),$$

which is always positive, as $k_1 > \frac{4}{5}$ in area II.
Now we show that, after maximal spillover, firm 2 is still able to hire the worker, i.e., we are still in Area II. Given that the spillover is equivalent to an increase in firm 1's knowledge, by Lemma 2, we have to show that $\frac{1}{2}\frac{8k_h k_1 + 5\left(1 - k_1^2 - k_h^2\right)}{4(1-k_1)+5k_h}$ and $\frac{8}{5} - k_1$ are decreasing in k_1 in the Area under analysis. The latter expression is, indeed, decreasing for any value of k_1, as it is immediate to check. Let us consider the former. Its derivative w.r.t. k_1 is

$$\frac{16k_h - 25k_1 k_h + 10k_h^2 - 20k_1 + 10k_1^2 + 10}{\left(4\left(1 - k_1\right) + 5k_h\right)^2},$$

which is positive (negative) for $k_1 < (>)\ 1 + \frac{5}{4}k_h - \frac{3}{20}\sqrt{\left(40k_h + 25k_h^2\right)}$. Therefore the expression represents a maximum. Consider, first, the case $k_h > \frac{1}{5}$. In this case, the function $\frac{1}{2}\frac{8k_h k_1 + 5\left(1 - k_1^2 - k_h^2\right)}{4(1-k_1)+5k_h}$ crosses $\frac{8}{5} - k_1$ at $k_1 = k_h + \frac{3}{5}$. Therefore, if we prove that in the maximum k_1 is lower than $k_h + \frac{3}{5}$, we are sure that in our area the function is always decreasing. This is indeed the case, as the inequality

$$1 + \frac{5}{4}k_h - \frac{3}{20}\sqrt{\left(40k_h + 25k_h^2\right)} \leq k_h + \frac{3}{5}$$

is equivalent to $280k_h + 200k_h^2 - 64 = 8\left(5k_h + 8\right)\left(5k_h - 1\right) \geq 0$, whose solution is $k_h > \frac{1}{5}$.
Consider, now, the case $k_h < \frac{1}{5}$. In this case, the expression $\frac{1}{2}\frac{8k_h k_1 + 5\left(1 - k_1^2 - k_h^2\right)}{4(1-k_1)+5k_h}$ crosses the 45 degree line for $k_1 = 1 - k_h$. Hence, we must prove that in the

maximum k_1 is lower than $1-k_h$. This is true, as the inequality

$$1+\frac{5}{4}k_h-\frac{3}{20}\sqrt{(40k_h+25k_h^2)}<1-k_h$$

is equivalent to $360k_h-1800k_h^2=-360k_h(5k_h-1)<0$, whose solution is $k_h<\frac{1}{5}$.■

Proof of Proposition 7. We know from Lemma 2 that in this Area $k_1+k_h>1$. Moreover, from the same Lemma, it is straightforward to check that $k_1+k_2>1$. We first prove that the solution for the optimal spillover is a corner one. Firm 1's profit after the spillover is

$$\Pi(1,k_2)-(\Pi(1,k_1)-\Pi(k_2,1)).$$

Its derivative w.r.t. the knowledge of firm 2 is

$$\frac{\partial}{\partial k_2}(\Pi(1,k_2)-(\Pi(1,k_1)-\Pi(k_2,1)))=\frac{10}{9}k_2-\frac{8}{9}.$$

Given that this derivative is positive for $k_2>\frac{4}{5}$, the spillover must be a corner solution.
Given this result, we must compare firm 1's profit for $\rho_1=0$ and $\rho_1=1$.
For $\rho_1=0$, firm 1 earns

$$\Pi(k_1,\min(k_2+k_h,1)),$$

while for $\rho_1=1$ it earns

$$\begin{aligned}\Pi(1,1)-(\Pi(1,k_1)-\Pi(1,1)) &=\\ 2\Pi(1,1)-\Pi(1,k_1)&.\end{aligned}$$

There are two cases:
<u>Case 1</u>: $k_2+k_h\geq 1$.
Firm 1 chooses to give spillover iff

$$\Pi(k_1,1)-2\Pi(1,1)+\Pi(1,k_1)\leq 0,$$

true for $k_1\geq\frac{3}{5}$, which in Area II is certainly satisfied for Lemma 2. Moreover, after the spillover the level of knowledge of firm 2 is $K_2=\min(k_1+k_2,1)$ $=1$, since $k_1+k_2\geq\frac{7}{5}$. Therefore, the levels of knowledge after the spillover belong to Area I' and firm 1 will hire the worker.
<u>Case 2</u>: $k_2+k_h<1$.

Firm 1 chooses to give spillover iff

$$\Pi(k_1, k_2 + k_h) - \Pi(1,1) + (\Pi(1, k_1) - \Pi(1,1)) \leq 0,$$

true for $k_2 \geq 2k_1 - k_h - \sqrt{4k_1 - 2 - k_1^2}$ (notice that $4k_1 - 2 - k_1^2$ is positive as $k_1 > 2 - \sqrt{2}$ is always satisfied in Area II, since $\frac{4}{5} > 2 - \sqrt{2}$).
Note that we can rewrite the inequality $k_2 \geq 2k_1 - k_h - \sqrt{4k_1 - 2 - k_1^2}$ as $k_2 + k_h \geq 2k_1 - \sqrt{4k_1 - 2 - k_1^2}$. Furthermore, $2k_1 - \sqrt{4k_1 - 2 - k_1^2} < 1$ for $k_1 \geq \frac{3}{5}$, because we are always in Area II.
By combining Case 1 and Case 2 we find the condition expressed in the Proposition.
To conclude the proof, we have to show that, after the spillover, firm 1 will be able to keep the worker. Since after the spillover $k_1 + k_2 \geq \frac{7}{5}$, as in the previous point, after the spillover the levels of knowledge are in region I'.■

Proof of Proposition 8. In Area II, without spillovers firm 2 earns

$$\Pi(\min(k_2 + k_h, 1), k_1) - (\Pi(1, k_2) - \Pi(k_1, \min(k_2 + k_h, 1))),$$

while with spillover from firm 1 it obtains

$$\Pi(1,1).$$

Hence it will accept the spillover iff:

$$\Pi(\min(k_2 + k_h, 1), k_1) - \Pi(1, k_2) + \Pi(k_1, \min(k_2 + k_h, 1)) - \Pi(1,1) \leq 0 \quad (2)$$

There are two cases:
1) $k_2 + k_h \geq 1$ and
2) $k_2 + k_h < 1$.
Case 1). Recall that from Lemma 2 in this Area we have

$$k_2 \geq \frac{8}{5} - k_1,$$

which implies that $k_1 \geq \frac{4}{5}$. Therefore, condition (2) becomes:

$$\Pi(1, k_1) - \Pi(1, k_2) + \Pi(k_1, 1) - \Pi(1,1) = \frac{1}{9}\left(5k_1^2 - 8k_1 - k_2^2 + 4k_2\right) \leq 0,$$

which is satisfied for

$$k_2 \leq 2 - \sqrt{4 + 5k_1^2 - 8k_1}.$$

Indeed, this inequality is satisfied since the right hand side is no smaller than k_1. In fact,

$$2-\sqrt{4-8k_1+5k_1^2}\geq k_1$$

implies

$$\left(2-k_1\right)^2-\left(4+5k_1^2-8k_1\right)=4k_1\left(1-k_1\right)\geq 0,$$

which is always satisfied.

Case 2). From Lemma 2 in this Area we have

$$k_2\geq\frac{1}{2}\frac{8k_hk_1+5\left(1-k_1^2-k_h^2\right)}{4\left(1-k_1\right)+5k_h}. \tag{3}$$

Moreover, in this Area the acceptance rule is

$$\begin{aligned}\Pi\left(k_2+k_h,k_1\right)-\Pi\left(1,k_2\right)+\Pi\left(k_1,k_2+k_h\right)-\Pi\left(1,1\right) &= \\ \frac{1}{9}\left(5k_1^2-8k_1k_2-8k_1k_h+4k_2^2+10k_2k_h+5k_h^2+4k_2-5\right) &\leq 0.\end{aligned} \tag{4}$$

The condition (2) is satisfied for what previously proved. If we compare (4) with (2) and we can prove that

$$\begin{gathered}\left(5k_1^2-8k_1k_2-8k_1k_h+4k_2^2+10k_2k_h+5k_h^2+4k_2-5\right)\leq \\ \left(5k_1^2-8k_1-k_2^2+4k_2\right),\end{gathered}$$

then we have proved the Proposition. The above expression is equivalent to

$$-8k_1-5k_2^2+8k_1k_2+8k_1k_h-10k_2k_h-5k_h^2+5\geq 0$$

which is satisfied for

$$\frac{8}{5}k_1-1-k_h\leq k_2\leq 1-k_h$$

The second inequality is certainly satisfied, and so is the first since (3) holds and

$$\frac{1}{2}\frac{8k_hk_1+5\left(1-k_1^2-k_h^2\right)}{4\left(1-k_1\right)+5k_h}\geq\frac{8}{5}k_1-1-k_h.$$

In fact this expression is equivalent to

$$\frac{1}{10}\frac{-80k_hk_1+65+39k_1^2+25k_h^2-104k_1+90k_h}{4\left(1-k_1\right)+5k_h}\geq 0.$$

The denominator is positive, and so is the numerator for

$$k_1 \leq \frac{5}{13} k_h + 1,$$

which is clearly satisfied.■