Consumer Behaviour and Revealed Preference

How Revealing is Revealed Preference?

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- **I. Testing Rationality using Revealed Preference**
  - Afriat-Varian
  - Experiments, Observational Data and the SMP idea

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  - Using Nonparametric Expansion Paths
  - Unobserved Heterogeneity and Quantile Demands

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  - Intertemporal Preferences and Information

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- Background references in the intro lecture and on my website.

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- There are (at least) two key criticisms of the empirical application of revealed preference theory to consumer behaviour:
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- Modern RP analysis takes a nonparametric approach.
- To quote Dan McFadden: "parametric models interpose an untidy veil between econometric analysis and the propositions of economic theory"
- The aim of this lecture is to "lift 'McFadden's' untidy veil"!

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- ▶ Particular attention is given to application to observational data: nonseparable unobserved heterogeneity and endogeneity.
- ▶ New insights are provided about the price responsiveness and the degree of rationality, especially across different income and education groups.

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  - Collective choice
  - Habits
  - Intertemporal choice
  - Characteristics models

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- And **'Beyond'**...
  - Hyperbolic discounting
  - Choice under uncertainty
  - Consideration sets
  - Reference-dependent choice...

1. Nonparametric Revealed Preference

- Observe (a sample analog of) demands (continuous case), or choice probabilities (discrete case), and ask: Can the observable choices, or choice probabilities, be rationalized as an outcome of optimisation?
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- Focus mainly here on continuous choice models, following work with Browning and Crawford (BBC (2003,2008), and with Kristensen and Matzkin (BKM (2014, 2017), and also with Horowitz and Parey (BHP (2013, 2016).

- Should consider discrete choice models, following recent work of Kitamura and Stoye (2017) focussing on the Axiom of Stochastic Revealed Preference (ASRP) from McFadden and Richter (1991), McFadden (2005), and Manski (2012).

1. The consumer problem

- Assume every consumer is characterised by observed and unobserved heterogeneity $(\mathbf{h}, \varepsilon)$ and responds to a given budget $B(\mathbf{p}, x)$, with a unique, positive $J-$vector of demands

$$\mathbf{q} = \mathbf{d}(\mathbf{p}, x, \mathbf{h}, \varepsilon).$$

  Demand functions: $R^K_{++} \to R^J_{++}$,satisfy adding-up: $\mathbf{p}'\mathbf{q} = x$ for all prices and total outlays $x \in R$; $\varepsilon \in R^{J-1}$, $J-1$ vector of unobservable heterogeneity. Assume $\varepsilon \perp x \mid \mathbf{h}$, for now.
- The environment is described by a continuous distribution of $\mathbf{q}$, x and ε, for discrete types $\mathbf{h}$,
  - will often suppress observable heterogeneity $\mathbf{h}$.
- For *discrete prices* (finite set of markets), the demand curve for given prices defines the *expansion path (Engel curve)* for consumer $(\mathbf{h}, \varepsilon)$ as their total budget x (income) is varied:

$$\mathbf{q} = \mathbf{g}(x; \mathbf{h}, \varepsilon),$$

  this plays a central role in RP analysis of consumer demand.

1. The consumer problem (cont...)

- One key assumption in first generation studies was (additive) separability of ε. In the non-separable case we will assume conditions on preferences that ensure invertibility in ε, equivalent to monotonicity for the scalar heterogeneity case when $J = 2$.

  ▶ Application: illustrate importance of flexibility in price responses across the income distribution using gasoline demand, BHP (2013, 2016).

- Let's first abstract from heterogeneity and examine Afriat's Theorem for a single consumer, and the construction of support sets for counterfactual demands under new budgets.

Afriat's Theorem

- Assume consumer faces a finite number of budgets (markets) and index the budget sets as $B_t = B(\mathbf{p}(t), x)$, $t \in T$.
- The Afriat-Diewert-Varian Theorem allows us to characterise 'well behaved' preferences through a set of inequalities on observed price and quantity vectors $(\mathbf{p}^t, \mathbf{q}^t)$ across markets $t \in \{1, ..., T\}$.
- Provides the basis for a test of rationality which generalises to many alternative rationality concepts, for both observational and experimental data.

Afriat's Theorem (cont...)

Afriat's Theorem: The following statements are equivalent:

A. there exists a utility function $u(\mathbf{q})$ *which is continuous, non-satiated and concave which rationalises the data* $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,...,T}$

B1. there exist numbers $\{U_t, \lambda_t > 0\}_{t=1,...,T}$ *such that*

$$U_s \leq U_t + \lambda_t \mathbf{p}_t' (\mathbf{q}_s - \mathbf{q}_t) \;\; \forall \; s, t \in \{1, ..., T\}$$

B2. the data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,...,T}$ *satisfy the Generalised Axiom of Revealed Preference (GARP).*

GARP

Definition: *A dataset* $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,..,T}$ *satisfies GARP if and only if we can construct relations* R_0, R *such that*
(i) for all t, s *if* $\mathbf{p}_t\mathbf{q}_t \geq \mathbf{p}_t\mathbf{q}_s$ *then* $\mathbf{q}_t \ R_0 \ \mathbf{q}_s$;
(ii) for all $t, s, u, \ldots, r, v,$ *if* $\mathbf{q}_t \ R_0 \ \mathbf{q}_s$, $\mathbf{q}_s \ R_0 \ \mathbf{q}_u, \ldots, \mathbf{q}_r \ R_0 \ \mathbf{q}_v$ *then* $\mathbf{q}_t \ R \ \mathbf{q}_v$;
(iii) for all $t, s,$ *if* $\mathbf{q}_t \ R \ \mathbf{q}_s$, *then* $\mathbf{p}_s\mathbf{q}_s \leq \mathbf{p}_s\mathbf{q}_t$.

Condition (i) states that the quantities $\mathbf{q}_t$ are directly revealed preferred over $\mathbf{q}_s$ if $\mathbf{q}_t$ was chosen when $\mathbf{q}_s$ was equally attainable.

Condition (ii) imposes transitivity on the revealed preference relation R.

Condition (iii) states that if a consumption bundle $\mathbf{q}_t$ is revealed preferred to a consumption bundle $\mathbf{q}_s$, then $\mathbf{q}_s$ cannot be more expensive then q_t.

Figure 1a: illustrates a simple RP rejection.

Figure 1a

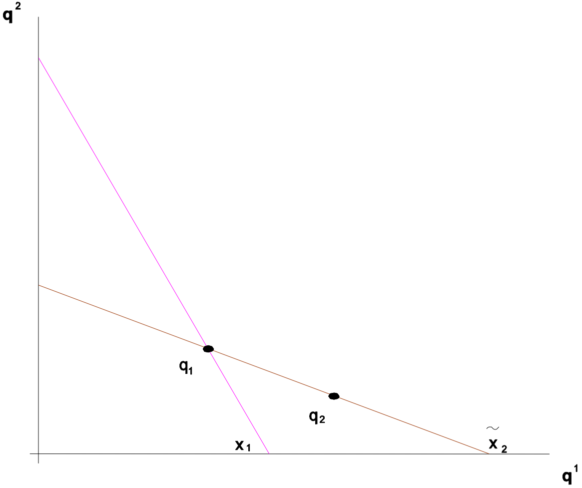
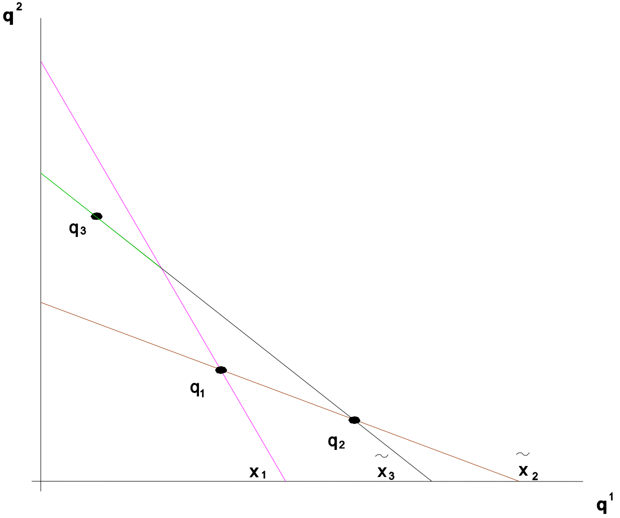


Figure 1a: Rejection



Support Sets and Bounds on Demand Responses

Suppose we observe a set of demand vectors $\{\mathbf{q}_1, \mathbf{q}_2, ...\mathbf{q}_T\}$ which record the choices made by a consumer when faced by the set of prices $\{\mathbf{p}_1, \mathbf{p}_2, ...\mathbf{p}_T\}$.

- new price vector $\mathbf{p}_0$ with total outlay x_0, budget $B_0(\mathbf{p}_0, x_0)$.
- 'best' support set $S^V(\mathbf{p}_0, x_0)$ for $q(\mathbf{p}_0, x_0)$ is given by:

$$\left\{ \mathbf{q}_0 : \begin{array}{c} \mathbf{p}_0'\mathbf{q}_0 = x_0,\ \mathbf{q}_0 \geq \mathbf{0} \text{ and} \\ \{\mathbf{p}_t, \mathbf{q}_t\}_{t=0...T} \text{ satisfies GARP} \end{array} \right\}$$

$S(\mathbf{p}_0, x_0)$ is the identified set of demand responses for $\mathbf{p}_0, x_0$, with properties:

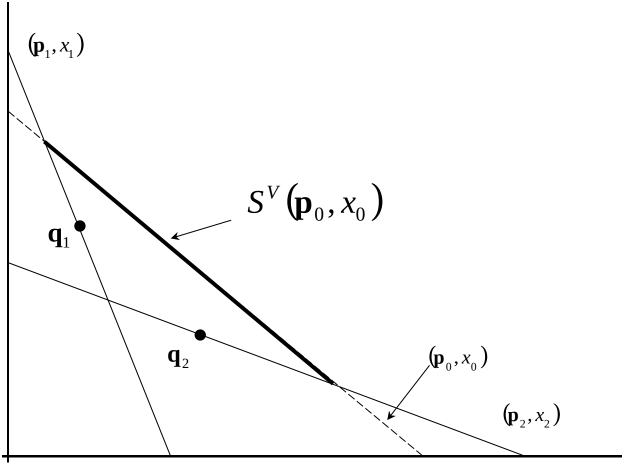
(1) $S(\mathbf{p}_0, x_0)$ is non-empty iff the data set $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,...T}$ satisfies GARP.

(2) If the data set $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1...T}$ satisfies GARP and $\mathbf{p}_0 = \mathbf{p}_t$ for some t then $S(\mathbf{p}_0, x_0)$ is the singleton $\{\mathbf{q}_t\}$.

(3) $S(\mathbf{p}_0, x_0)$ is convex.

Illustrated for the two dimensional case in Figure 1b:

Figure 1b: The 'Varian' Support Set with GARP



Extensions: (dynamic extensions later in slides)

1. Homothetic and weak separability (and conditional demands)
2. Characteristics models
3. Non-unitary models and altruism

- All the 'first generation' applications either:
  - follow individuals in short/small panels (and in experimental settings), or
  - treat unobserved heterogeneity as additive and work with conditional mean models.
- Illustrate the latter using a Kernel regression application to the improved bounds.
- Also generalise to nonseparable unobserved heterogeneity.

Weak Separability

- Partition our data into two sets of goods and prices

$$\left\{\left\{\mathbf{p}_t^1, \mathbf{q}_t^1\right\}, \left\{\mathbf{p}_t^2, \mathbf{q}_t^2\right\}\right\}_{t=1,\ldots,T}$$

- A utility function is separable in the group 1 goods, if

$$\{\mathbf{q}^1, \mathbf{q}^2\} \succeq \{\mathbf{q}_*^1, \mathbf{q}^2\} \iff \{\mathbf{q}^1, \mathbf{q}_\#^2\} \succeq \{\mathbf{q}_*^1, \mathbf{q}_\#^2\}$$

for all $\mathbf{q}^1, \mathbf{q}_*^1, \mathbf{q}^2$ and $\mathbf{q}_\#^2$.
- That is preferences within group 1 are independent of the composition of group 2.
- The functional representation is that a utility function u is (weakly) separable in the group 1 goods if we can find a "subutility function" $v\left(\mathbf{q}^1\right)$ and a "macro function" $w(v,\mathbf{q}^2)$ strictly increasing in v such that:

$$u\left(\mathbf{q}^1, \mathbf{q}^2\right) = w(v(\mathbf{q}^1), \mathbf{q}^2).$$

- Dimension reduction and two-stage budgeting.

Weak Separability

Theorem (weak separability). The following conditions are equivalent:
(1) there exists a weakly separable, concave, monotonic, continuous non-satiated utility function that rationalises the data;
(2) there exist numbers $\{V_t, W_t, \lambda_t > 0, \mu_t > 0\}_{t=1,...,T}$ *that satisfy:*

$$\begin{aligned} V_s &\leq V_t + \mu_t \mathbf{p}_t^{1\prime}\left(\mathbf{q}_s^1 - \mathbf{q}_t^1\right) \\ W_s &\leq W_t + \frac{\lambda_t}{\mu_t}\left(V_s - V_t\right) + \lambda_t \mathbf{p}_t^{2\prime}\left(\mathbf{q}_s^2 - \mathbf{q}_t^2\right) \end{aligned}$$

(3) the data $\left\{\mathbf{p}_t^1, \mathbf{q}_t^1\right\}_{t=1,...,T}$ *and* $\left\{1/\mu_t, \mathbf{p}_t^2, V_t, \mathbf{q}_t^2\right\}_{t=1,...,T}$ *satisfy GARP for some choice of* $\{1/\mu_t, V_t\}_{t=1,...,T}$ *that satisfies the Afriat inequalities.*

An easier strategy is to check for GARP for all goods and also for the weakly separable sub-set.

Characteristics and Revealed Preference

- Consumer choice model is extended to

$$\max_{\mathbf{q}} V(\mathbf{z}) \text{ subject to } \mathbf{z} = \mathbf{F}(\mathbf{q}) \text{ and } \mathbf{p}'\mathbf{q} \leq x, \mathbf{q} \geq 0.$$

- Blow, Browning and Crawford (REStud, 2006), extend the set of RP inequalities in BBC to the linear characteristics model, where $\mathbf{z} = \mathbf{A}'\mathbf{q}$.
- Underlying characteristics are not necessarily observed so a harder identification problem.
- They use scanner panel data. But otherwise identical approach.
- Note that GARP has to be satisfied on the original set of goods.

Collective Models

- Collective models - families versus individuals.
  - Data is typically for families, and may record some assignable and exclusive goods.
    - Individual labor supplies are often taken as examples.
  - Cherchye, De Rock and Vermeulen (2011, ...) extend the analysis presented in this lecture to the collective choice case. Very recently to non-corporative models as well.
  - A pair of utility functions $U^A(\mathbf{q}^A, \mathbf{Q})$ and $U^B(\mathbf{q}^B, \mathbf{Q})$ for two members A and B who consume private goods $(\mathbf{q}^A, \mathbf{q}^B)$ and public goods Q.
  - Observed demands satisfy collective rationalisation (CARP) if inequalities hold over "personalised" quantities.
  - Not all personalised quantities are observed so a harder identification problem. But otherwise identical approach.
    - Nice applications to family labour supply.

Altruism

- Models of Altruism - 'rational' altruistic preferences.
  - Andreoni and Miller (Ecta). Adapt measures in an experimental design to include payments to self and payments to others, $U(\pi_s, \pi_o; \gamma)$, where γ are the observable attributes of the game.
  - Extended set of RP inequalities.

Implementing Afriat's Theorem

Given some data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,\dots,T}$ to check for consistency with the theory we can either

1. Determine whether there exist numbers $\{U_t, \lambda_t > 0\}_{t=1,\dots,T}$ such that
$$U_s \leq U_t + \lambda_t \mathbf{p}_t' (\mathbf{q}_s - \mathbf{q}_t) \ \forall \ s, t \in \{1, \dots, T\}$$
or,
2. Determine whether the data satisfy GARP.

- Suppose we have just two market observations $\{\mathbf{p}_1, \mathbf{p}_2; \mathbf{q}_1, \mathbf{q}_2\}$.
- Then the Afriat Inequalities

$$U_s \leq U_t + \lambda_t \mathbf{p}_t' (\mathbf{q}_s - \mathbf{q}_t) \quad \text{and } \lambda_t > 0, \forall\ s, t \in \{1, 2\}$$

can be written as

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & -\mathbf{p}_2' (\mathbf{q}_1 - \mathbf{q}_2) \\ -1 & 1 & -\mathbf{p}_1' (\mathbf{q}_2 - \mathbf{q}_1) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\varepsilon \\ -\varepsilon \end{bmatrix}$$

where ε is an arbitrarily small constant, or

$$\mathbf{Ax} \leq \mathbf{b}$$

Applying Afriat's Theorem

- In essence we are asking whether there exist a solution to a set of linear inequalities. This is a linear programming problem and Dantzig's "simplex algorithm", can determine whether or not there is a feasible solution in a finite number of steps.
- In general checking for consistency requires a linear program with $2T$ variables and T^2 constraints.
- The fact that the number of constraints rises as the square of the number of observations can makes this condition computationally demanding in practice for very large datasets.
- Condition B2 (GARP) is sometimes more efficient. This requires us to compute the *transitive closure* of a finite relation. That is certainly a finite problem and Warshall (1962) gives a solution in T^3 steps. It is very easy to implement, see BBC (2003), for example.

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- Recall the simple RP rejection in: Figure 1a:

Figure 1a: a 'rejection' region

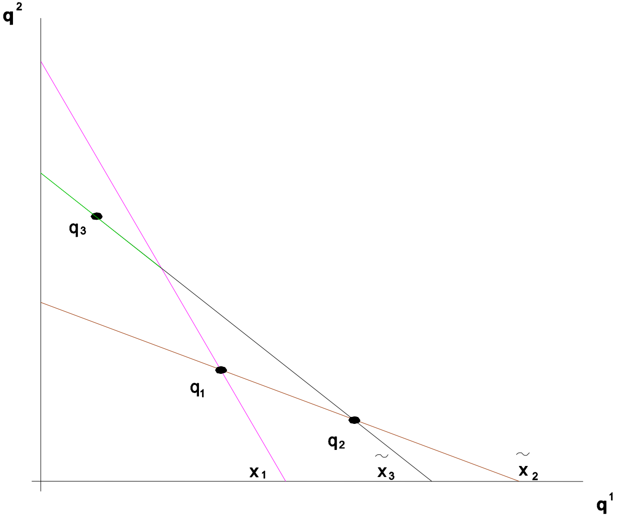
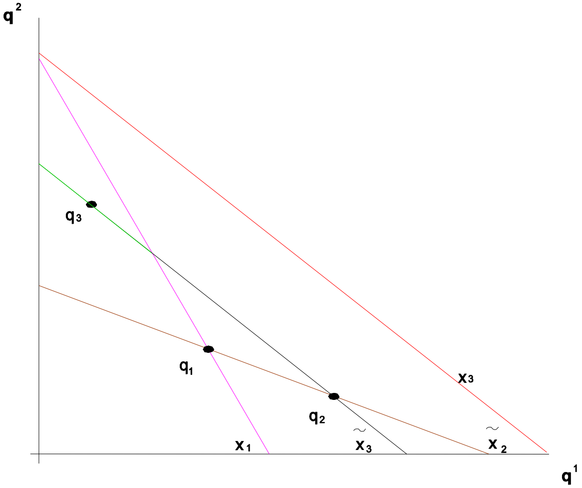


Figure 1a: An uninformative budget

$B(p_3, x_3)$



2. Intersection Demands and Improving Bounds

- Define sequential maximum power (SMP) path

$$\{\tilde{x}_s, \tilde{x}_t, \tilde{x}_u, ...\tilde{x}_v, x_w\} = \{\mathbf{p}_s'\mathbf{q}_t(\tilde{x}_t), \mathbf{p}_t'\mathbf{q}_u(\tilde{x}_u), \mathbf{p}_v'\mathbf{q}_w(\tilde{x}_w), x_w\}$$

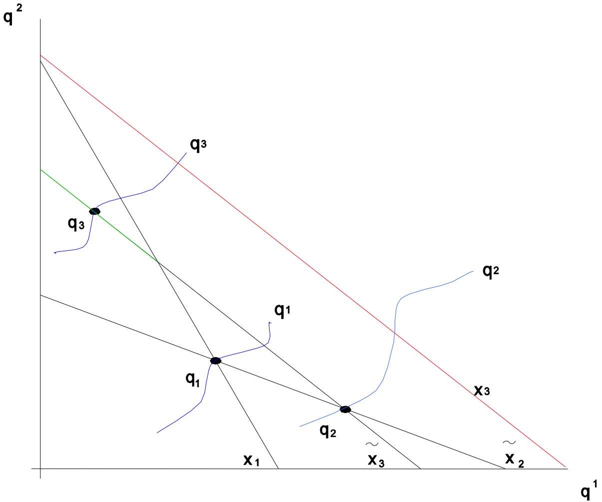
- Proposition 2.1 (BBC, 2003): Suppose that the sequence

$$\{\mathbf{q}_s(x_s), \mathbf{q}_t(x_t), \mathbf{q}_u(x_u)..., \mathbf{q}_v(x_v), \mathbf{q}_w(x_w)\}$$

  rejects RP. Then SMP path also rejects RP. ■

- This result has been used in the design of RP experiments and also extended this result to models of collective choice, habits, in the referenced papers...
- Key idea for observational data: - use expansion paths (Engel curves) to mimic the experimental design.
- Observe consumers across a finite set of markets (in each market they face the same relative prices). Using expansion paths $q_t(x)$ (Engel curves) for each market t, we are able to generate the SMP path. See Fig 2a.

Figure 2a: Using Expansion Paths



Bounds on Demand Responses Using Engel Curves

- The expansion paths (Engel Curves) $\{\mathbf{q}_t(x)\}_{t=1,..T}$, define intersection demands $q_t(\tilde{x}_t)$ by $p_0' q_t(\tilde{x}_t) = x_0$.
- The set of points that are consistent with observed expansion paths and utility maximisation is given by the support set:

$$S(\mathbf{p}_0, x_0) = \left\{ \mathbf{q}_0 : \begin{array}{c} \mathbf{q}_0 \geq \mathbf{0},\ \mathbf{p}_0'\mathbf{q}_0 = \mathbf{x}_0 \\ \{\mathbf{p}_0, \mathbf{p}_t; \mathbf{q}_0, \mathbf{q}_t(\tilde{x}_t)\}_{t=1,\dots,T} \text{ satisfy GARP} \end{array} \right\}$$

  The support set $S(\mathbf{p}_0, x_0)$ that uses expansion paths and intersection demands defines e-bounds on demand responses

- $S(\mathbf{p}_0, x_0)$ is the identified set for the parameter $\mathbf{q}(\mathbf{p}_0, x_0)$.
- Proposition 2.2 (BBC2): the set is sharp and is convex, refined in BBCDV (AEJ-Micro, 2015).
- See Figures 2 b,c,d

Figure 2b: The 'Varian' Support Set with RP

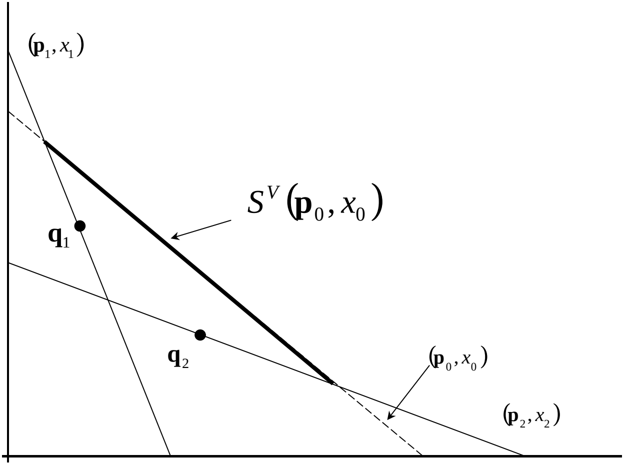


Figure 2c. Support set with Expansion Paths

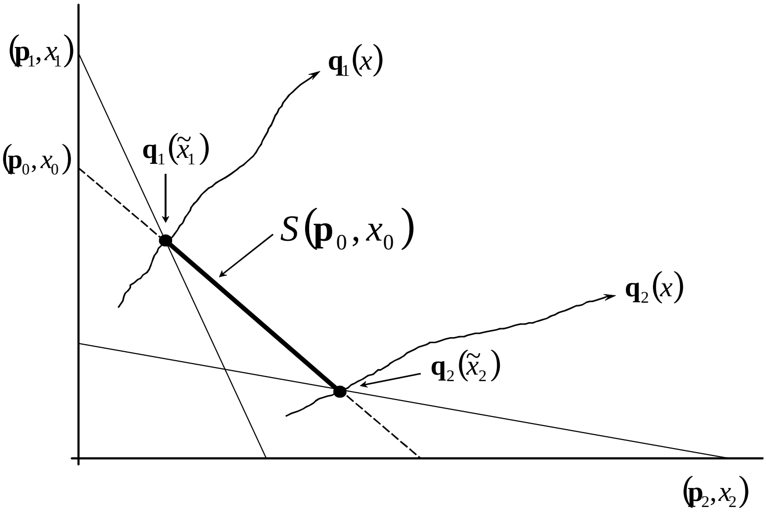
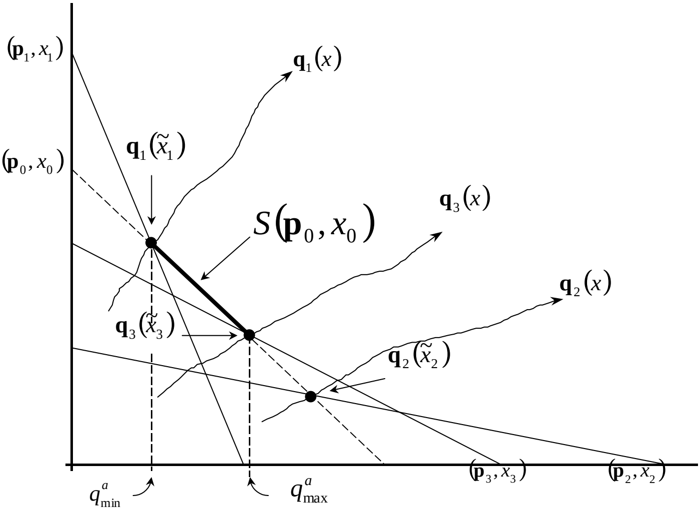


Figure 2d: Support Set with Many Markets

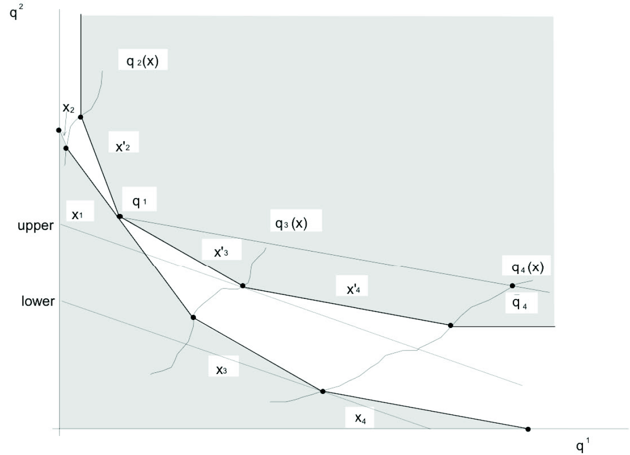


Transitivity

- Transitivity, like symmetry, adds nothing in the two good case.
- Many good application to UK FES diary records on foods, services and some categories of other goods, BBC (Ecta, 2008).
- Implement the SMP idea using average local Engel curves for each market estimated by nonparametric regression.
  - ▶ *Engel curves can be quite nonlinear (see QUAIDS and other references).*
  - ▶ *Test rationality through RP inequality restrictions based on intersection demands.*
  - ▶ *Findings: periods of time for certain demographic groups for which the RP restrictions cannot be rejected.*
  - ▶ *Use restricted Engel curves to estimate bounds on counterfactual demand responses.*
- Assume additive unobserved heterogeneity on Engel curves, BCK (2007) account for endogeneity.
- Also construct (and estimate) bounds on welfare costs of prices (tax) changes....

Bounds on indifference surface and cost of living

- provide bounds on compensating variations.



Note that as the data becomes dense

- - the RP test for consistency becomes more demanding
- - the bounds on indifference curves become tighter
- - the bounds on demand responses become tighter.

- If the data become perfectly dense (effectively an infinite dataset) we have the indifference curve map and demand curves themselves.

- In this case the RP conditions become equivalent to the usual integrability conditions - the Slutsky condition and homogeneity.

Will give some examples of the nonparametric implementation of Slutsky condition for this case.

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Summary so far....

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  - focus on unobserved heterogeneity with some examples
  - formalise the notion of taste change within the RP approach, again with an application.
- If time will also show how the approach can be extended to a life-cycle model with habit formation, and look at discrete choice models. All this is in the lecture notes anyway!

RP for Heterogeneous Consumers

- ▶ Assume every consumer is characterised by unobserved heterogeneity (ε) and responds to a given budget $(\mathbf{p}, \mathbf{x})$, with a unique, positive $J-$vector of demands

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- ▶ for $J = 2$, invertibility is equivalent to monotonicity in unobserved heterogeneity ε
- for example: ▶

Invertible Preferences, J=2

- For example, if preferences take the form:

$$U_i^t\left(q_{1i}, q_{0i}\right) = v\left(q_{1i}, q_{0i}\right) + w(q_{1_i}, \varepsilon_i)$$

  preference heterogeneity ε_i
  w strictly increasing and concave with positive cross derivative guarantees q_1 is invertible in ε.
- Note that RP consistent responses to price and income changes will be represented by a shift in the distribution of demands.
- We will assume baseline demands are monotonic in scalar unobserved heterogeneity so that quantile demands, conditional on x income and price regime, identify individual demands.

Figure 2a: The distribution of heterogeneous consumers

- Distribution of consumer tastes in a market:

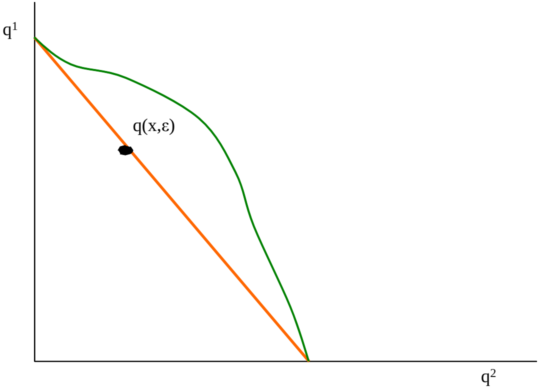


Figure 2b: Monotonicity and rank preserving changes

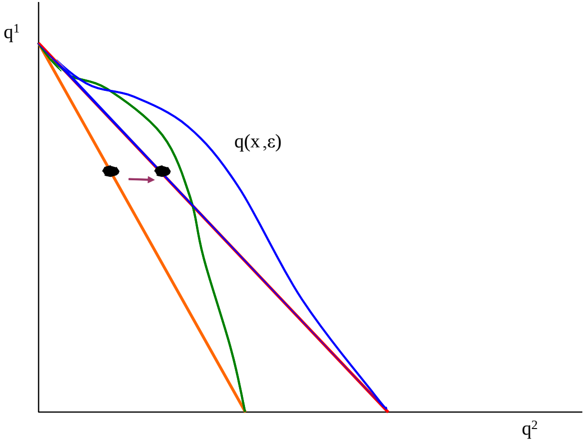
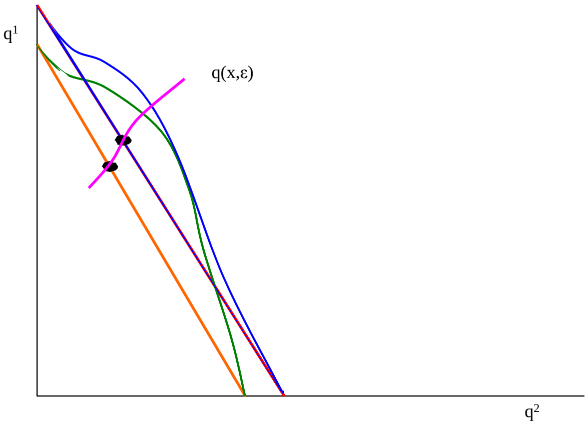


Figure 2c: The quantile expansion path



- $>$ Quantile structural function - quantile Engel curve.

- Consider the identification and estimation of demands

$$\mathbf{q}\,(t) = \mathbf{d}(x(t), t, \boldsymbol{\varepsilon}), \qquad t = 1, ..., T,$$

where the demand function in any price regime $p\,(t)$ is exactly the stochastic *expansion path (Engel curve)* for prices for market t.

- For the case with scalar heterogeneity, $J = 2$, the conditions for in $\boldsymbol{\varepsilon}$ invertibility correspond to monotonicity. For this case:

$$\varepsilon \in \mathbb{R}, \mathbf{d}(x(t), t, \varepsilon) = (d_1(x(t), t, \varepsilon), d_2(x(t), t, \varepsilon))$$

we make the following assumptions:

- A 3.1: The variable $x\,(t)$ has bounded support, $x\,(t) \in X = [a, b]$ for $-\infty < a < b < +\infty$, and is independent of $\varepsilon \sim U\,[0, 1]$, *for now*.
- A 3.2: The demand function $d_1\,(x, t, \varepsilon)$ is invertible in ε and is continuously differentiable in (x, ε).

Nonseparable Demand

- Identification Result: $d_1(x, t, \tau)$ is identified as the τth quantile of $q_1|x(t)$:

$$d_1(x, t, \tau) = F^{-1}_{q_1(t)|x(t)}(\tau|x).$$

Thus, we can employ standard quantile regression techniques to estimate d_1.

- Let

$$\rho_\tau(y) = (\mathbb{I}\{y < 0\} - \tau)\, y, \quad \tau \in [0, 1],$$

be the check function used in quantile estimation. The budget constraint defines the path for d_2. We let D be the set of feasible demand functions,

$$\mathcal{D} = \left\{ \mathbf{d} \geq 0 : d_1 \in \mathcal{D}_1,\ d_2(x, t, \tau) = \frac{x - p_1(t)\, d_1(x, t, \varepsilon(t))}{p_2(t)} \right\}.$$

Estimation

- Let $(\mathbf{q}_i(t), x_i(t))$, $i = 1, ..., n$, $t = 1, ..., T$, be i.i.d. observations from a demand system, $\mathbf{q}_i(t) \in R^2$.
- Then estimate $d(t, \cdot, \tau)$ by

$$\hat{\mathbf{d}}(\cdot, t, \tau) = \arg\min_{d_n \in \mathcal{D}_n} \frac{1}{n} \sum_{i=1}^{n} \rho_\tau \left(q_{1i}(t) - d_{1n}(x_i(t)) \right), \quad t = 1, ..., T,$$

where D_n is a sieve space ($D_n \to D$ as $n \to \infty$).

- Let $B_i(t) = (B_k(x_i(t)) : k \in \mathcal{K}_n) \in R^{|\mathcal{K}_n|}$ denote basis functions spanning the sieve D_n.
- Then $\hat{d}_1(x, t, \tau) = \sum_{k \in \mathcal{K}_n} \hat{\pi}_k(t, \tau) B_k(x)$, where $\hat{\pi}_k(t, \tau)$ is a standard linear quantile regression estimator:

$$\hat{\pi}(t, \tau) = \arg\min_{\pi \in \mathbb{R}^{|\mathcal{K}_n|}} \frac{1}{n} \sum_{i=1}^{n} \rho_\tau \left(q_{1i}(t) - \pi' \mathbf{B}_i(t) \right), \quad t = 1, ..., T.$$

- BKM (2014) derive rates and asymptotic distribution of the sieve estimator.

RP-Restricted Estimation

- There is no particular reason why estimated expansion paths for a sequence of prices (markets) $t = 1, ..., T$ satisfies RP for any type ε. To impose the RP restrictions, we simply define the constrained function set as: $D_C^T = D^T \cap \{d(\cdot,\cdot,\tau) \text{ satisfies RP}\}$.
- Define the constrained estimator by:

$$\begin{aligned} & \{\hat{\mathbf{d}}_C(\cdot, t, \tau)\}_{t=1}^T \\ = \quad & \arg \min_{\{\mathbf{d}_n(\cdot,t,\tau)\}_{t=1}^T \in \mathcal{D}_C^T} \frac{1}{n} \sum_{t=1}^T \sum_{i=1}^n \rho_\tau (q_{1,i}(t) - d_{1,n}(t, x_i(t))), \tau \in [0, 1 \end{aligned}$$

- Since RP imposes restrictions across t, the above estimation problem can no longer be split up into T individual sub problems as in the unconstrained case. Adapting results on nonparametric estimation under shape constraints, show the constrained sieve estimator $\hat{d}_C$ converges with the same rate as $\hat{d}$.
- BKM (2014) demonstrate that as $n \longrightarrow \infty$, the unrestricted estimator, $\hat{d}$, satisfies RP almost surely. Conclude that $\hat{d}_C$ is asymptotically equivalent $\hat{d}$, and all the asymptotic properties of $\hat{d}$ are inherited by $\hat{d}_C$.

Estimating Counterfactual Demand Bounds

- For new budget $(\mathbf{p}_0, x_0)$ define the estimated income levels $\hat{x} = (\hat{x}(1), ..., \hat{x}(T))$ as the solutions to

$$\mathbf{p}_0' \hat{\mathbf{d}}_C(\hat{x}(t), t, \tau) = x_0, \quad t = 1, ..., T,$$

  the support set estimator is $\hat{S}_{\mathbf{p}_0, x_0} = \{\mathbf{q} \in \mathcal{B}_{\mathbf{p}_0, x_0} | \hat{\mathbf{x}} - \mathbf{P}\mathbf{q} \leq 0\}$.
- A valid confidence set can be constructed for the demand bounds, and is akin to the result found in, for example, CHT's Theorem 5.2. Use modified bootstrap from Bugni (2009, 2010), Andrews and Soares (2010).
- We would also like to test for whether the consumers in the sample are rational (i.e. obey the RP restrictions).

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- We would also like to test for whether the consumers in the sample are rational (i.e. obey the RP restrictions).
- Idea: Compute unrestricted demand estimates, and see how far they are from satisfying RP restrictions.
- Measure discrepancies between a given alternative set of demands, $\underline{\mathbf{q}} = (\mathbf{q}(1), ..., \mathbf{q}(T)) \in R^{2T}$, and $\underline{\hat{\mathbf{q}}}_0$ by:

$$MD_n(\{\mathbf{q}(t)\} | \mathbf{p}_0, x_0)$$

$$= \sum^{T} (\mathbf{q}(t) - \hat{\mathbf{q}}_0(t, \mathbf{p}_0, x_0))' W_t (\mathbf{q}(t) - \hat{\mathbf{q}}_0(t, \mathbf{p}_0, x_0)),$$

Data

- A sub-population from the UK FES diary records

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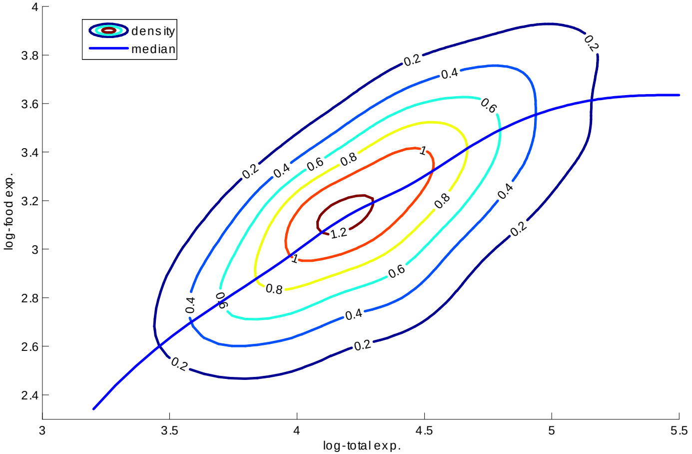
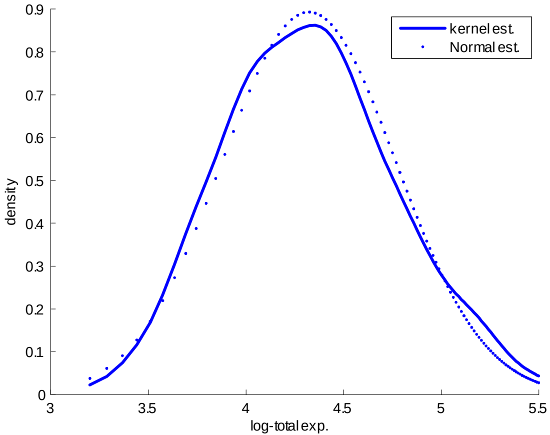
- Analyse spending on food and other non-durables.

Engel curve distribution for food in one market



Total Expenditure (Budget) Distribution



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- Use these results to develop a test of the RP inequalities.
  - Use 3rd order pol. spline with 5 knots
  - RP restrictions imposed at 100 x-points over the empirical support x across markets.

Figure 4a. Unrestricted Quantile Expansion Paths: Food

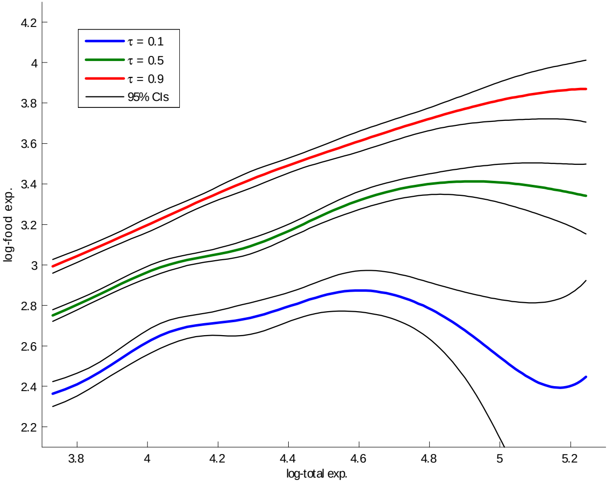


Figure 4b. RP Restricted Quantile Expansion Paths: Food

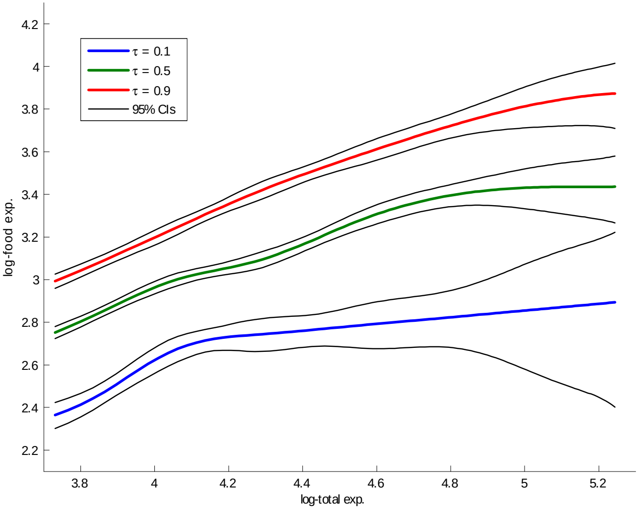


Figure 5a: Quantile Counterfactual Demand Bounds at Median Income and Median Heterogeneity

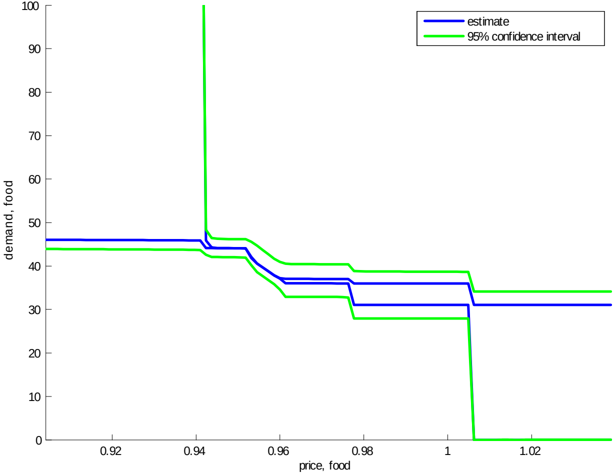
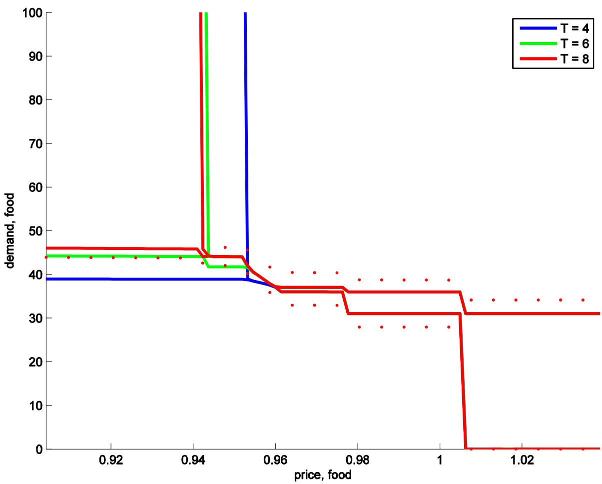


Figure 5b: Estimated Counterfactual Demand Bounds as More Markets are Added



Notes on the Estimates

- Note the 'local' nature of the analysis - the bounds vary with income, heterogeneity and the number of markets
- Demand (e-)bounds (support sets) are defined at the quantiles of x and ε
  - tightest bounds given information and RP.
  - show how vary with income and heterogeneity
- To account for the endogeneity of x we can utilize IV quantile estimators developed in Chen and Pouzo (2009) and Chernozhukov, Imbens and Newey (2007). The basic results remain valid in the quantile demand case except that the convergence rate stated there has to be replaced by that obtained in Chen and Pouzo (2009) or Chernozhukov, Imbens and Newey (2007).
- Alternatively, use the control function approach taken in Imbens and Newey (2009) to recover the QSF. They use this data and the exact same instrument. Specify

$$\ln x = \pi(\mathbf{z}, v)$$

where π is monotonic in v, z are a set of instrumental variables.

The Slutsky Inequality

- When prices and demand are continuous, the RP conditions for a single good become equivalent to the Slutsky inequality shape restriction on the single good demand (normalised to the outside good)

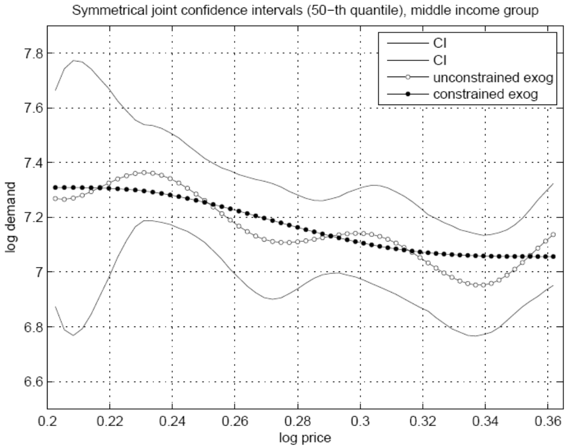
$$q_1 = d_1(p, x, \varepsilon).$$

- Blundell, Horowitz and Parey (2016) make the monotonicity assumption on ε, and impose the Slutsky condition on the nonparametric estimate of the conditional quantile function.
- The constrained estimator is obtained by solving a nonparametric quantile estimation problem subject to the Slutsky condition for all (p, x).
- This problem has unaccountably many constraints. They replace the continuum of constraints by a discrete set, imposing the restriction on a grid.
- Also implement an exogeneity test for prices and develop an IV estimator.

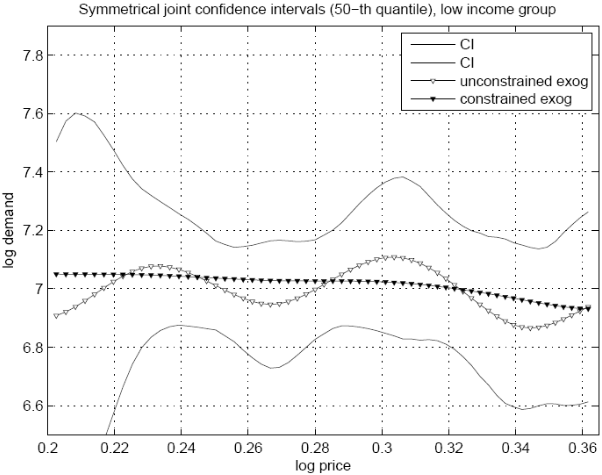
The National Household Travel Survey

- Apply to gasoline demand in the US National Household Travel Survey (2001) - a household-level survey that was conducted by telephone and complemented by travel diaries and odometer readings.
- To minimize heterogeneity, restrict the sample to a specific set of demographics.
- Take vehicle ownership as given and do not investigate how changes in gasoline prices affect vehicle purchases or ownership.
- The resulting sample contains 5,254 observations.
- As an instrument for gasoline price we use distance from Gulf supply point.

The Shape Restricted Demand Curve (Median Demand at Median Income)



The Shape Restricted Demand Curve (Median Demand at Low Income)



Findings

- Imposing the Slutsky restriction on an otherwise fully nonparametric estimate of the demand function produces well-behaved estimates of the demand function, avoiding arbitrary and possibly incorrect parametric or semiparametric restrictions.
- The Slutsky constrained nonparametric estimates revealed features of the demand function that are not present in simple parametric models, especially on price responses across the income distribution.

Findings

- Imposing the Slutsky restriction on an otherwise fully nonparametric estimate of the demand function produces well-behaved estimates of the demand function, avoiding arbitrary and possibly incorrect parametric or semiparametric restrictions.
- The Slutsky constrained nonparametric estimates revealed features of the demand function that are not present in simple parametric models, especially on price responses across the income distribution.
- In recent work we note we do not observe the true transactions price. Instead, we observe a local (county) average price p that is related to p^* by

$$p = p^* + \zeta$$

where ζ is an unobserved random variable.
- The resulting errors in variables are called "Berkson errors" and are common in economics data - the opposite of classical errors in variables.
- Show this can produce important biases in the quantile estimation of demands but these are limited once the Slutsky inequality condition is imposed.

Multiple Goods and Many Errors

- Multiple goods bring the full power of transitivity (symmetry) but, together with non-separable heterogeneity also raise additional invertibility, identification and estimation issues, and nonseparable heterogeneity is 'essential' in multiple good demand models.
- Generalise the previous many good example of BBC (2008) to avoid the use of average local demands. This requires invertibility of demand, see Matzkin (2007, 2010), Beckert and Blundell (2008), Berry, Gandhi, and Haile (2013).
- The idea is to introduce variables Z that are correlated with unobserved heterogeneity ε. BKM (2017) limit the dimensionality of the unobserved heterogeneity, and focus on individual demands. Averages over a subpopulation are investigated in Hausman and Newey (2013) and Blomquist and Newey (2013). Related to the random coefficients models of Lewbel and Pendakur (2013).
- Dette, H., S. Hoderlein, and N. Nuemeyer (2011) consider integrability conditions without invertibility but not necessarily for a particular individual.

- Consider a demand system where $G = J - 1$ unobservable variables can enter in nonlinear, nonadditive ways.

$$\begin{aligned}
q_1 &= d^1\left(\mathbf{p}, x, \varepsilon_1, ..., \varepsilon_G\right) \\
q_2 &= d^2\left(\mathbf{p}, x, \varepsilon_1, ..., \varepsilon_G\right) \\
&\quad \cdots \\
q_G &= d^G\left(\mathbf{p}, x, \varepsilon_1, ..., \varepsilon_G\right)
\end{aligned}$$

  where the vector of unobserved heterogeneity (tastes) $\left(\varepsilon_1, ..., \varepsilon_G\right)$ is independent of $\left(\mathbf{p}, x\right)$ conditional on Z.
- We make an invertibility assumption

$$\begin{aligned}
\varepsilon_1 &= r^1\left(q_1, ..., q_G, \mathbf{p}, x\right) \\
\varepsilon_2 &= r^2\left(q_1, ..., q_G, \mathbf{p}, x\right) \\
&\quad \cdots \\
\varepsilon_G &= r^G\left(q_1, ..., q_G, \mathbf{p}, x\right)
\end{aligned}$$

- In the application, $\boldsymbol{\varepsilon} = \mathbf{r}\left(\mathbf{q}, \mathbf{p}, x\right)$ satisfies the Revealed Preference restrictions.

- Berry and Benkard (2006) and Matzkin (2007, 2008) note that without further restrictions the system is not identified. Our solution is based on variables that are excluded from the functions of interest. Show identification when:

  A unimodal restriction with respect to Z on the conditional density of the vector of unobserved heterogeneity.

- Develop methods to estimate the value of the vector of unobserved tastes of each consumer and the demand function of each consumer.
- All methods are constructive and the estimators are shown to be consistent and asymptotically normal.
- *Assumption M* : For some invertible H and given ε, there exists unique z such that

$$\frac{\partial f_{\varepsilon|Z=z}(\varepsilon)}{\partial z} = 0 \quad <=> \quad \varepsilon = H(z)$$

which requires: $\dim(z) = \dim(\varepsilon)$ and $f_{\varepsilon,Z}$ differentiable at z

- An example:

$$\varepsilon = H\left(g(z) + \eta\right); \quad \eta \text{ independent of } Z; \text{ mode of } \eta \text{ known}$$

A special case of which is:

$$\varepsilon = \eta - z; \quad \eta \text{ independent of } Z; \text{ mode of } \eta \text{ known}$$

as used in Matzkin (2007).

- BKM are able to show identification of finite changes in budgets for any individual defined by a particular ε.
- BBC (2008) application from the Family Expenditure Survey in the UK
    - food share and services share as functions of log(expenditure) and two unobserved tastes
    - $z1 =$ family size calculated using equivalence scales
    - $z2 =$ cohort, adjusted by education, of head of household

III. Rationality and Taste Change

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  - If rejection, allow for taste change
- Investigate the degree of 'taste change' for tobacco and other 'bads'
- Address a specific question: How much of the fall in tobacco consumption in the UK was due to a rise in the relative price of tobacco and how much can be attributed to taste change?

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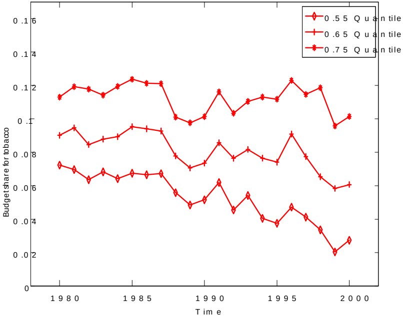
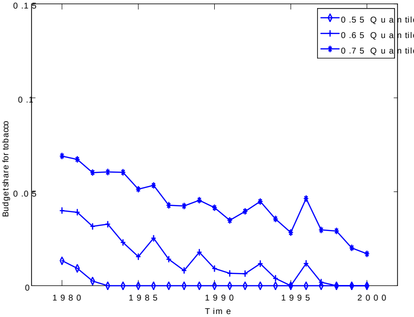
- Aim to inform policy on the balance between information/health campaigns and tax reform.
- ABBC (2017) also consider how tastes evolve across different education strata. Do tastes change differentially across education groups?

Taste changes and prices

UK Budget shares for Tobacco: Quantiles



(a) Low Education



(b) High Education

Taste Change

- Consumer i's maximisation problem can be expressed as:

$$\max_{\mathbf{q}} u^i(\mathbf{q}, \boldsymbol{\alpha}_t^i) \text{ subject to } \mathbf{p}'\mathbf{q} = x$$

where $\mathbf{q} \in \mathbb{R}_+^K$ denotes the demanded quantity bundle, $\mathbf{p} \in \mathbb{R}_{++}^K$ denotes the (exogenous) price vector faced by consumer i and x gives total expenditure.

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- We also allow for unobserved permanent heterogeneity *across* consumers.

- Using this framework we derive RP inequality conditions that incorporate minimal perturbations to individual preferences to account for taste change.

Marginal utility (MU) perturbations

- MU perturbations represent a simple way to incorporate taste variation: McFadden & Fosgerau, 2012; Brown & Matzkin, 1998, represent taste heterogeneity as a linear perturbation to a base utility function.

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- Characterising taste change in this way yields the temporal series of utility functions:

$$u^i(\mathbf{q}, \boldsymbol{\alpha}_t^i) = v^i(\mathbf{q}) + \boldsymbol{\alpha}_t^{i\prime}\mathbf{q}, \text{ where } \boldsymbol{\alpha}_t^i \in \mathbb{R}^K.$$

- Under this specification, $\alpha_t^{i,k}$ can be interpreted as the taste shift in the marginal utility of good k at time t for individual i.
- The theorems below imply this specification is not at all restrictive.

Afriat conditions

- For individual *i* we seek the Afriat inequalities that would allow us to rationalise observed prices $\{\mathbf{p}^1, ...\mathbf{p}^T\}$ and quantities $\{\mathbf{q}^1, ...\mathbf{q}^T\}$.
- We can '*good* 1 *taste rationalise*' the observed prices and quantities if there is a function $v(\mathbf{q})$ and scalars $\{\alpha_1, \alpha_2, ...\alpha_T\}$ such that:

$$v\left(\mathbf{q}^t\right) + \alpha_t q_1^t \geq \psi\left(\mathbf{q}\right) + \alpha_t q_1$$

for all $\mathbf{q}$ such that $\mathbf{p}^t\mathbf{q} \leq \mathbf{p}^t\mathbf{q}^t$.

Afriat conditions

Theorem: The following statements are equivalent:

1. Individual observed choice behaviour, $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,...,T}$, can be good-1 rationalised by the set of taste shifters $\{\alpha_t\}_{t=1,...,T}$.

2. One can find sets $\{v_t\}_{t=1,...,T}$, $\{\alpha_t\}_{t=1,...,T}$ and $\{\lambda_t\}_{t=1,...,T}$ with $\lambda_t > 0$ for all $t = 1, ..., T$, such that there exists a non-empty solution set to the following inequalities:

$$\begin{aligned} \left(v\left(\mathbf{q}^t\right) - v\left(\mathbf{q}^s\right)\right) + \alpha_t\left(q_1^t - q_1^s\right) &\leq \lambda_t\left(\mathbf{p}^t\right)'\left(\mathbf{q}^t - \mathbf{q}^s\right) \\ \alpha_t &\leq \lambda_t p_t \end{aligned}$$

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- These inequalities are a simple extension of Afriat (1967).
- When they hold there exists a well-behaved base utility function and a series of taste shifters on good-1 that perfectly rationalise observed behaviour.

A surprising result

- We can then show, under mild assumptions on the characteristics of available choice data, that we can always find a pattern of taste shifters on a single good that are sufficient to rationalise any finite time series of prices and quantities:

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- We can then show, under mild assumptions on the characteristics of available choice data, that we can always find a pattern of taste shifters on a single good that are sufficient to rationalise any finite time series of prices and quantities:

  Definition: There is 'perfect intertemporal variation' (PIV) in good 1 if $q_1^t \neq q_1^s$ for all $t \neq s = 1, ..., T$.

  Theorem: Given observed choice behaviour, $\{\mathbf{p}^t, \mathbf{q}^t\}$ for $t = 1, \ldots T$ where good-1 exhibits PIV, one can *always* find a set $\{v_t, \alpha_t, \lambda_t\}$ with $\lambda_t > 0$ for all $t = 1, ..., T$, that satisfy the Afriat inequalities.

- PIV is sufficient for rationalisation but not necessary.

Taste changes as price adjustments

- We can reinterpret the rationalisability question as a 'missing price problem'.
- We can find scalars $\{v_1, ...v_T\}$, positive scalars $\{\lambda_1, ...\lambda_T\}$, and a weakly positive taste-adjusted price vector, $\{\widetilde{\mathbf{p}}^t\}_{t=1,..,T}$, such that

$$v\left(\mathbf{q}^t\right) - v\left(\mathbf{q}^s\right) \geq \lambda_t \left(\widetilde{\mathbf{p}}^t\right)' \left(\mathbf{q}^t - \mathbf{q}^s\right)$$

  where

$$\widetilde{\mathbf{p}}^t = \left[p_1^t - \alpha_t/\lambda_t, \mathbf{p}_{\neg 1}^t\right].$$

- We refer to α_t/λ_t as the *taste wedge*.
- The change in demand due to a positive taste change for good 1 $(\alpha_t > 0)$ can be viewed as a price reduction in the price of good 1.
- This provides a link between two of the levers (*taxes and information*) available to governments.

Recovering taste change perturbations

- Given the no rejection result, we can always find a non-empty *set* of scalars that satisfy the Afriat conditions.
- Pick out values $\{v_t, \alpha_t, \lambda_t\}_{t=1,\ldots T}$ that solve:

$$\min \sum_{t=2}^{T} \alpha_t^2 \text{ subject to the Afriat inequalities}$$

- This a quadratic-linear program.
- Minimizing the sum of squared α's subject to the set of RP inequalities ensures that the recovered pattern of taste perturbations are sufficient to rationalise observed choice behaviour.
- With $\alpha_1 = 0$, we interpret $\{\alpha_t\}_{t=2,\ldots,T}$ as the minimal rationalising marginal utility perturbations to good-1 relative to preferences at $t = 1$.
- Can also impose more structure on the evolution of taste change over time. For example, monotonicity: $\alpha_{t+1} \leq \alpha_t$.

Empirical Strategy

- Our empirical analysis uses data drawn from the U.K. Family Expenditure Survey (FES) between 1980 and 2000.
- The FES records detailed expenditure and demographic information for 7,000 households each year.
- It is not panel data so we follow birth-cohorts of individuals stratified by education level.

Estimation

- To operationalise we estimate censored quantile expansion paths at each price regime (see Chernozhukov, Fernandez-Val and Kowalski (2010)) subject the RP inequalities.

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- Separately by birth cohort and by education group $E^i \in \{L, H\}$.
- We use a quantile control function approach to correct for the endogeneity of total expenditure.
- We recover shifts in the distribution of demands and ask what are the minimal perturbations to tastes that maintain the RP inequalities at each particular quantile.

Results

- Minimal virtual prices along each birth cohort's SMP path τth quantile and education group E are recovered as:

$$\widehat{\pi}_t^{E,\tau} = p_t^1 - \frac{\widehat{\alpha}_t^{E,\tau}}{\widehat{\lambda}_t^{E,\tau}}$$

  The "taste wedge", $\widehat{\alpha}_t^{E,\tau}/\widehat{\lambda}_t^{E,\tau}$ represents the change in the marginal willingness to pay for tobacco relative to base tastes.

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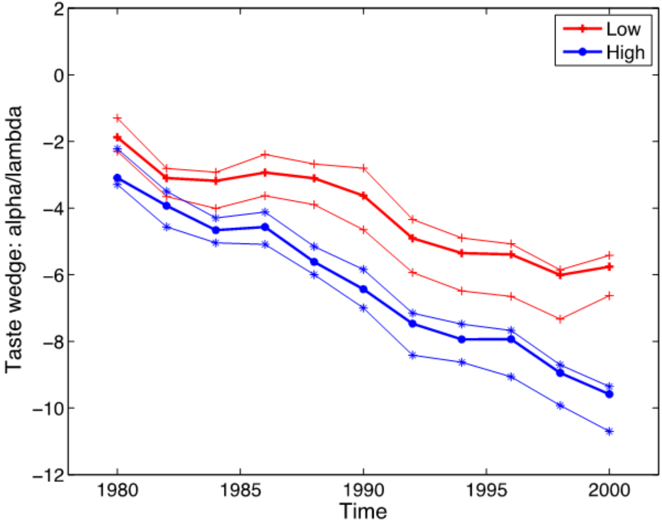
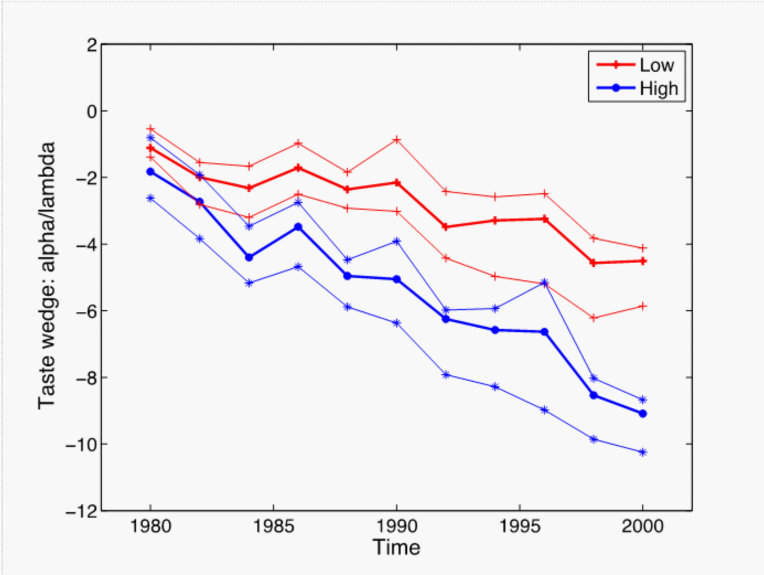
2. There are significant differences in the path of *systematic taste change* between education cohorts for light and moderate smokers.
3. The taste change trajectories for light and moderate smokers in the high education cohort are similar.
4. Education is irrelevant for explaining the evolution of virtual prices amongst heavy smokers.

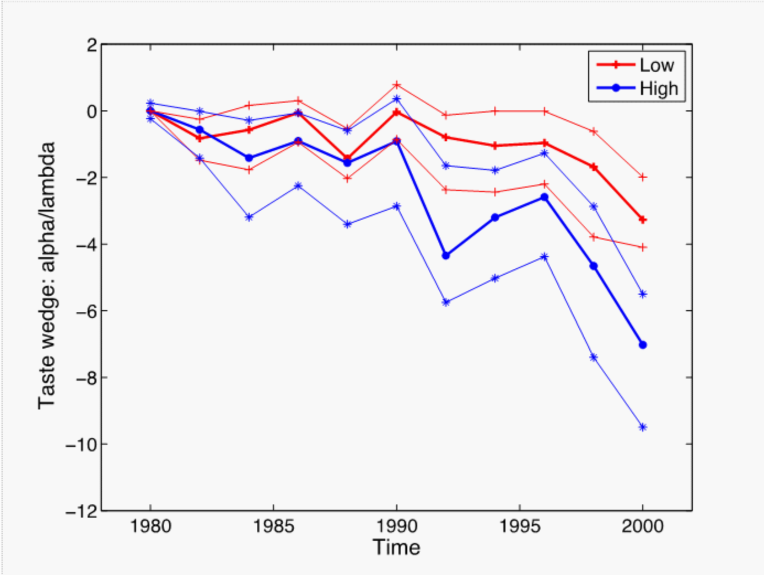
Taste wedges for light smokers



Taste wedges for medium smokers



Taste wedges for heavy smokers



Results: relaxing separability through conditional demands

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- We partition the set of observations into "light" and "heavy" drinkers depending on whether an individual is below or above the median budget share for alcohol.

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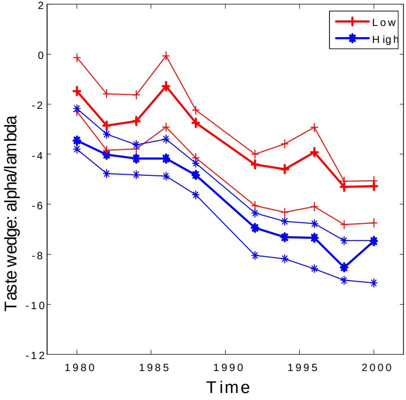
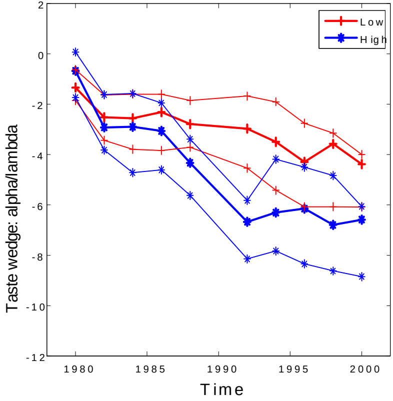
- We partition the set of observations into "light" and "heavy" drinkers depending on whether an individual is below or above the median budget share for alcohol.
- The significant difference by education group in the evolution taste change for light and moderate smokers is robust to non-separability.
- 95% confidence intervals on virtual prices and the taste wedge are disjoint across education groups for all cohorts except for the "heavy smoking"-"heavy drinking" group. Effective tastes for this group evolved very little for both education groups.

Taste Wedge Results: Conditional Quantiles (Moderate Smoker)

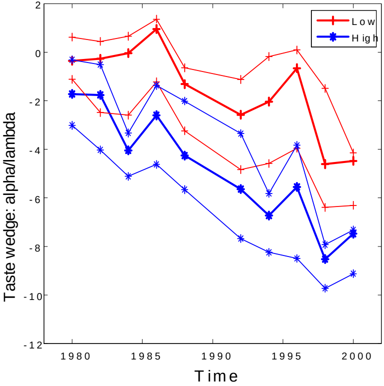


Light Drinker



Heavy Drinker

Taste Wedge Results: Conditional Quantiles (Heavy Smoker)



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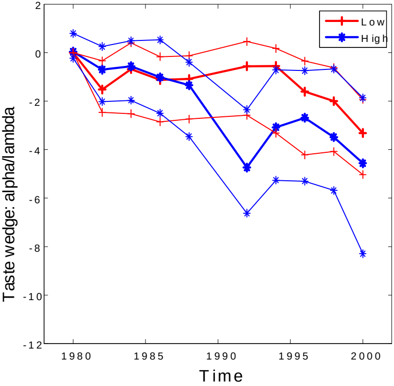

Heavy Drinker

Characterising Taste Change

- In this final part of the lecture we have shown how to develop an empirical framework for characterising taste change that recovers the minimal intertemporal (and interpersonal) taste heterogeneity required to rationalise observed choices.
- A censored quantile approach was used to allow for unobserved heterogeneity and censoring.
- Non-separability between tobacco and alcohol consumption was incorporated using a conditional (quantile) demand analysis.
- Future work will use intertemporal RP conditions to recover the path of λ_t.
- *Systematic* taste change was required to rationalise the distribution of demands in our expenditure survey data. Statistically significant educational differences in the marginal willingness to pay for tobacco were recovered; more highly educated cohorts experienced a greater shift in their effective tastes away from tobacco.

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- Formalise the notion of taste change within the RP approach.

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- Formalise the notion of taste change within the RP approach.

  ▶ For example, evidence that tobacco consumption by low education households can be largely rationalised by relative prices whereas taste changes are key in the decline for higher educated households.
- Extend to a life-cycle model with habit formation.

Extra Slide 1: Life-cycle Planning and Habits

- Allow for short memory in tobacco consumption such that the base utility function depends on lagged quantity of good 1:

$$v^t = \psi\left(\mathbf{q}, q_1^{-1}\right) + \mu_t q_1$$

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$$v^t = \psi\left(\mathbf{q}, q_1^{-1}\right) + \mu_t q_1$$

- Following Browning (1989) and Crawford (2010), embed this felicity function in a standard lifecycle planning framework.

$$\max_{\{\mathbf{q}^t\}_{t=1,\ldots,T}} \sum_{t=1}^{T} \beta^{t-1} \left\{\psi\left(\mathbf{q}^t, q_1^{t-1}\right) + \mu_t q_1^t\right\}$$

s.t.

$$\sum_{t=1}^{T} \boldsymbol{\rho}_t' \mathbf{q}_t = A_0$$

for discounted prices $\boldsymbol{\rho}_t$.

Extra Slides 2: Stochastic Revealed Preference and RUM

The Random Utility Model framework:

- Let u be a random utility function (unobserved heterogeneity) for consumers facing prices $\mathbf{p}_t$ and budget set B_t. Assume repeated cross section data.
- We observe (a sample analog of) choice probabilities and ask: Can the observable choice probabilities be rationalized as an outcome of the RUM? As before this will then allow a prediction/ counterfactual analysis only using theoretical restrictions implied by the RUM?
- Follow recent work by Kitamura and Stoye (2017) who develop the theoretical background on the Axiom of Stochastic Revealed Preference (ASRP) from McFadden and Richter (1991) [MR], McFadden (2005).
- Also reference Manski (2012) who develops a nonparametric analysis of labor supply with revealed preference.

SRP and RUM

McFadden 2005 shows, without further assumptions on individual preferences (and unobserved heterogeneity) it is sufficient to consider finite partitions of the separate non-intersecting sections of the budget constraint. This is also used in Hoderlein and Stoye.

- Note that with discrete choice
  - Cannot use point demands and Engel curves.
  - How choices are made within a partition does not provide additional information.
  - The choice space is the collection of partitions.

SRP and RUM

To implement the MR theory of Stochastic Rationality, Kitamura and Stoye (2017)[KS] define u^* to be a realization of u and define the vector $\mathbf{a}(u^*)$ as a choice pattern over the choice set implied by u^*.

- Note that certain patterns are not allowed due to SARP, so create a matrix $\mathbf{A}$ of all (H) such valid vectors.
- The matrix $\mathbf{A}$ plays a key role, embodying the restrictions due to rationality. KS discuss algorithms to obtain the matrix A.
- If there exists a probability vector $\mathbf{v}$ for the H types such that

$$\mathbf{Av} = \pi$$

  then the choice probabilities are stochastically rational i.e. satisfy the Axiom of Revealed Stochastic Preference [ARSP].
- Various equivalent statements for ARSP have been noted by MR, KS choose to work directly with $\mathbf{Av} = \pi$. Use this to develop a nonparameteric approach to test the stochastic rationality hypothesis.