The Microeconometric Analysis of Consumption, Savings and Labour Supply

Richard Blundell (*UCL and IFS*)

Short Course

Lecture 'I'

Life-cycle labour supply decisions and human capital

Northwestern, November 2017

Outline of the Lectures

- The aim of the lectures is to explore empirical approaches to the analysis of consumption, savings and labour supply.
- The first introductory overview lecture (Lecture '0') is published on the Nemmers webpage:

  http://www.economics.northwestern.edu/events/nemmers/
- I used that lecture to run through the main ideas behind each of remaining lectures, to list some references and to pick out some of the key modelling challenges to be addressed in each case.
- The three 'main' lectures:
  - **I.** Life-cycle labour supply decisions and human capital,
  - **II.** Consumer behaviour and revealed preference,
  - **III.** Earnings dynamics and consumption inequality.

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The modeling approach and some of the main results are based on Blundell, Costa-Dias, Meghir and Shaw (*Ecta*, 2016), on my webpage

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with

- borrowing constraints,
- earnings/wage uncertainty.

The specific application also incorporates:

- uncertainty over partner and partner's income, and over children,
- and includes education decisions....

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- ... But what if education and work experience investments are complementary in the production of human capital?
- The research in this lecture aims to unravel the way these two aspects of human capital interact with labour supply decisions at the extensive and intensive margin.
- Draws on a long history of related research: see references in BCMS *Ecta* paper, all on my website.....

  http://www.ucl.ac.uk/ uctp39a/

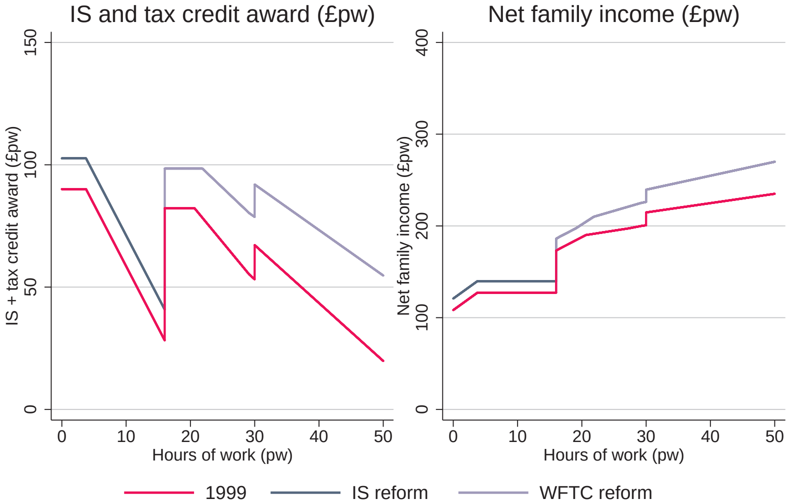
Policy Background

BCMS motivate the model through the evaluation and redesign of a Tax Credit and Welfare Reform in the UK:

- Focus on a specific reform - Working Families Tax Credit (WFTC) and Income Support (IS) in 1999/2000.
- Use this reform for reduced form comparisons with the structural model predictions. {Perhaps use this lecture to reflect on the pros and cons of structural models?}
- The reform involved an increase in the generosity of the welfare and earned income tax credit system for families with children.
- As in other countries, the motivation for these policies is that incentivising women into work, even when they have young children, will preserve labour market attachment and reduces skill depreciation.
- An additional peculiarity of the UK tax-credit system is the minimum hours eligibility rules that focus incentives on part-time work.

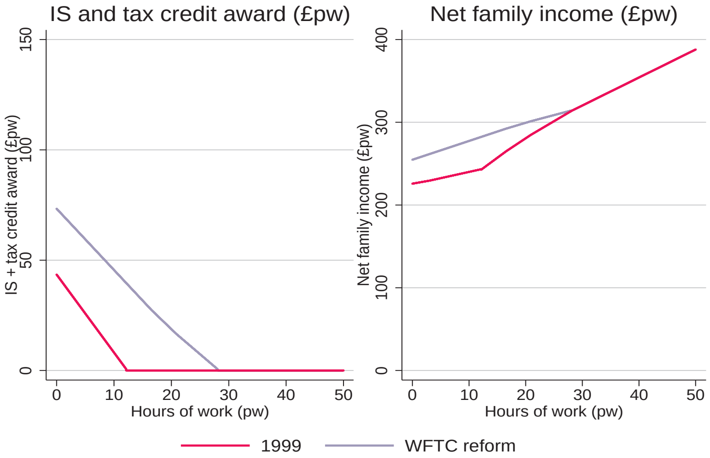
The UK (WFTC) Tax Credit and IS Reform

Figure 1: Income Support and Tax credit award for lone parent with 1 child



Impact on married women in couples

Figure 2: The budget constraint for second-earner parents



Notes: See background paper.

Hours rules

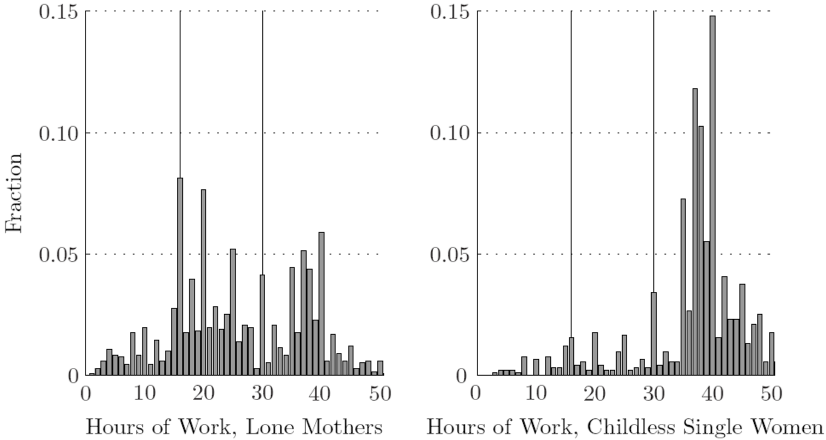
Hours rules

- In a static Mirrlees setting, Blundell and Shephard (RESud, 2012) have shown part-time hours rules of this type are unlikely to be optimal, even where there is clear justification for an earned income tax credit.

- Do the hours rules impact on observed behaviour?

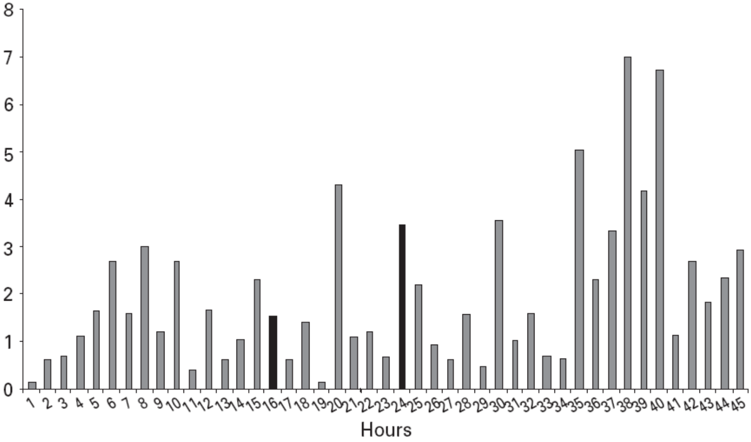
The Distribution of Weekly Hours of Work

Low Education Single Women with and without Kids in 1993



Single Mothers Hours

Before 16 Hour Rule (1990)



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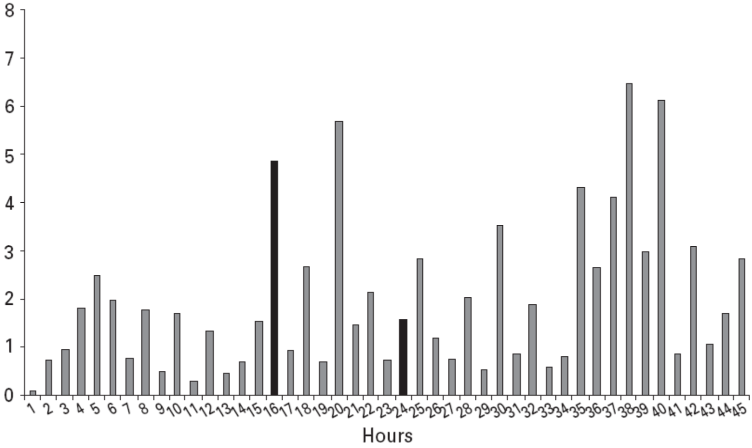

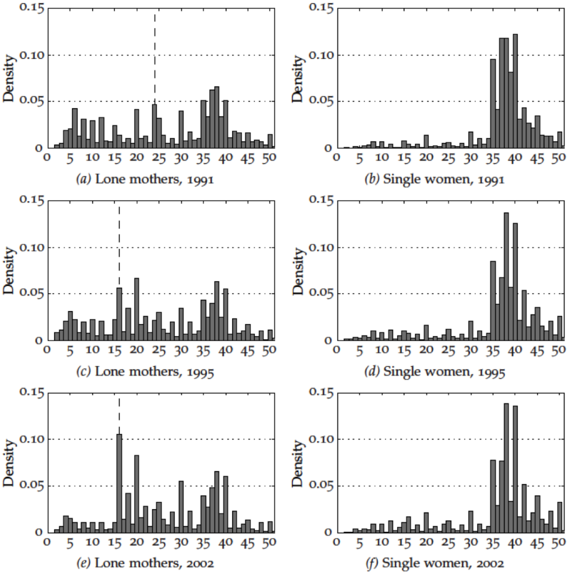
Density
0.15
0.10
0.05
0
0 5 10 15 20 25 30 35 40 45 50
(a) Lone mothers, 1991
(b) Single women, 1991
(c) Lone mothers, 1995
(d) Single women, 1995
(e) Lone mothers, 2002
(f) Single women, 2002

Policy Background

The key question we ask is:

- How do the features of this broad kind of tax, tax-credit and welfare benefit system affect education choices, experience capital accumulation, employment and hours of work over the life-cycle?

The approach we take:

- A structural dynamic approach: using the time series of tax, tax credit, welfare benefit and tuition reforms for new cohorts of women to identify parameters,
  - conditioning on *life-history family background* variables.
- Comparing with quasi-experimental contrasts where possible:
  - diff-in-diff contrasts for employment effects,
  - tilt in the trend of education investments by cohort.

Structural Dynamic Modelling Issues

- The estimation and simulation exercises require the solution of the life-cycle model under uncertainty.
- Since this stochastic life-cycle model has no analytical solution, we approximate numerically the policy functions for labor supply, consumption, and education choices conditionally on the woman's information at each period of her life (the state variables, 'X').
- We do this by backward recursion, starting from the end of life (age 70).
- A key feature of our model is that it studies the joint consumption and labor supply decisions over the working years of women, where the former is a continuous choice while the latter is discrete.

Structural Dynamic Modelling Issues (cont...)

- The numerical solution of problems with simultaneous discrete and continuous choices is considerably harder than that of problems with only continuous (or only discrete choices).
- The main difficulty in solving dynamic problems that combine discrete and continuous choices is that the smoothness and concavity of the value function that ensures the existence and uniqueness of a solution that is itself continuous and, if interior, is the root of the optimality condition (Euler equation) does not hold.
- Some studies (e.g., French and Jones (2011), Adda, Dustmann, and Stevens (2015)) have opted for discretizing the space of the continuous choice.
- More recently, solution methods to handle discrete and continuous choices have been proposed by Fella (2014) and Iskhakov, Jorgensen, Rust, and Schjerning (2015). Essentially we follow these approaches.

Aside: Learning from Structural Models (*AEA P&P* May)

Broad definition: A structural economic model is one where the *structure* of *decision making* is (fully) incorporated in the specification of the model.

Aim to identify:

1. structural 'deep' parameters: e.g. Frisch vs Marshallian elasticities.
2. underlying mechanisms: e.g. taxes vs self-insurance.
3. policy counterfactuals: e.g. ex-ante tax policy evaluation.

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The ability to provide counterfactual policy simulations set structural models apart from 'reduced-form'/treatment effect approaches.

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In these lectures I emphasize models that minimize assumptions on the *structural function* and on *unobserved heterogeneity*. And approaches that *align structural and 'reduced form'* moments/ *treatment effects*.

Structural Dynamic Labor Supply/Human Capital Models

Identification requires strong assumptions: e.g. on subjective discount rates and distribution of beliefs.

* Identify key life-cycle counterfactuals and mechanisms

-> e.g. ‘insurance value’ of redistributive policies - essential for tax design.

* Human capital investment breaks intertemporal separability

-> e.g. wage depends on experience capital in learning-by-doing model.

=>quasi-structural models and quasi-experimental contrasts more difficult.

* Estimation places strong demands on data too....

The Household Panel Data

British Household Panel Survey (BHPS), much like an enhanced PSID

Unbalanced panel of 4,200 females aged<50 over 18 waves, 1991-2008

- Measures of education, labour market outcomes, work-related and not-work-related training, childcare, detailed demographics, and assets.

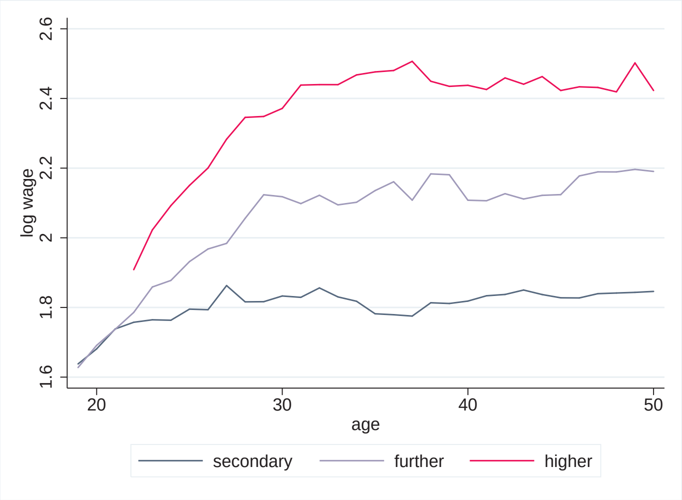
IFS taxben budget constraint simulation model working on every wave:

- Taxes: income tax, NI, council tax, tax credits
- Benefits: child benefit, maternity grant, income support, housing benefit, council tax benefit, free school meals.

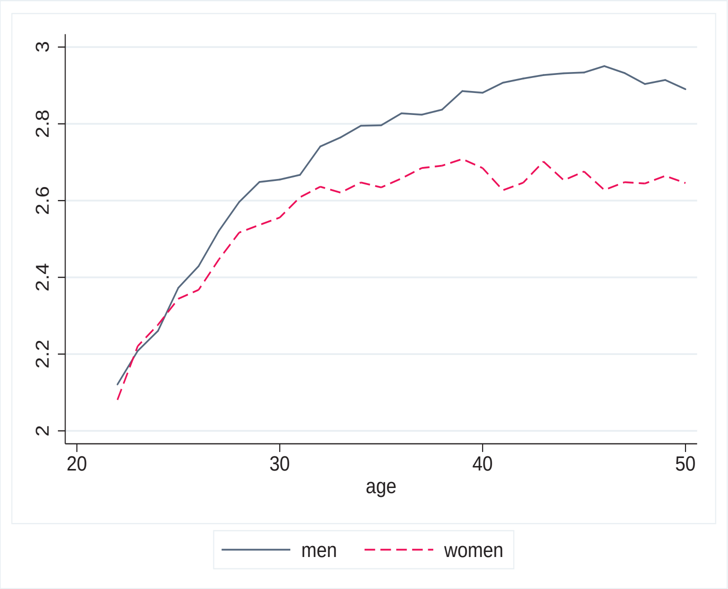
Linked life histories capture choices at age 16: educational qualifications; and detailed background measures, including

- parental education, number of siblings, sibling order, whether lived with parents when aged 16, books at home as a child, etc.

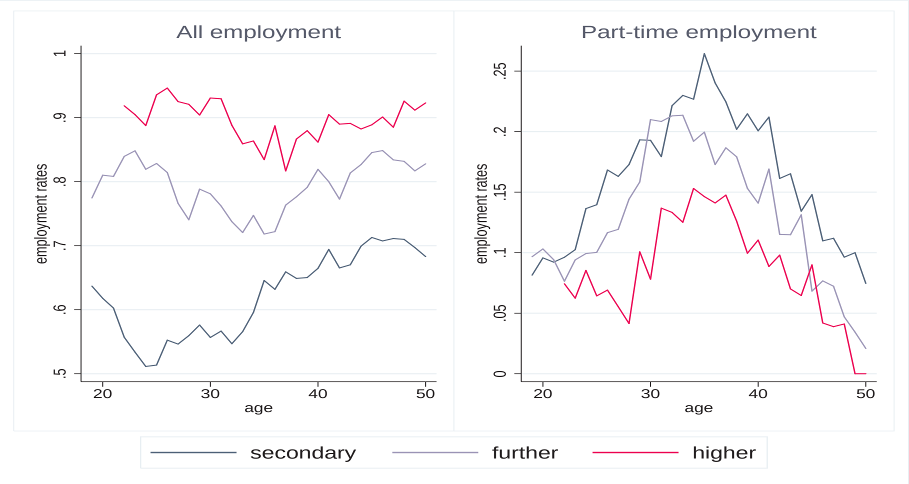
Wage Profiles by Education by Age



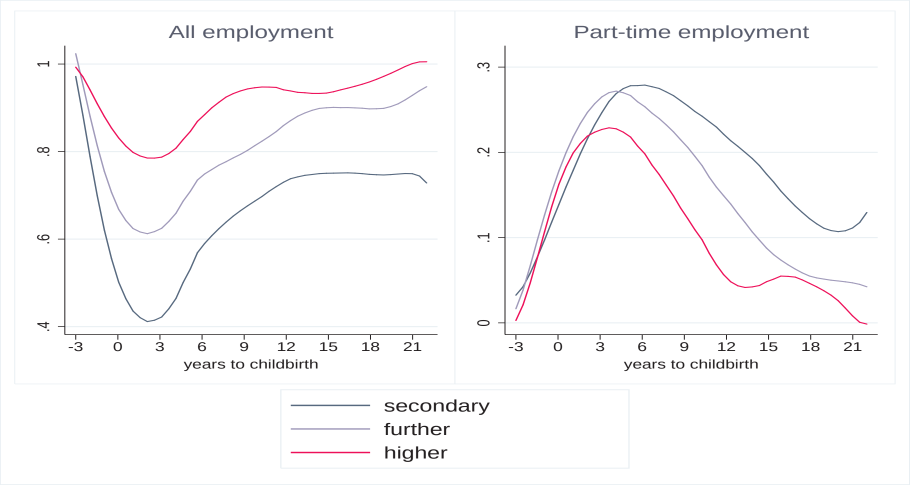
Male and Female Wage Age Profiles: University Graduates



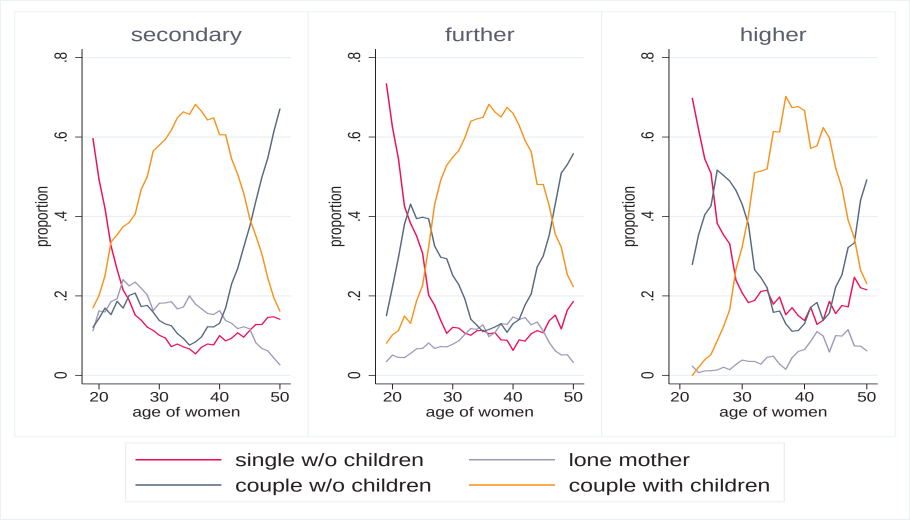
Employment over the life-cycle



Employment of mothers



Family Composition by Age



The Effects of the Reform - a reduced form approach

- We will first verify that the reforms did have an impact on labor supply and education using a reduced form approach.
- For labor supply we will compare the employment growth of single mothers to that of childless single women (by education).
- For education we need a different strategy, since in principle all are affected by the reform.
    - We exploit the idea that individuals with different family background will have a different educational attachment.
    - As a result more marginal individuals are more likely to react to changes in incentives.

The Effects of the Reform - Labor Supply

Table: Difference-in-differences employment regressions for lone mothers vs single women

	(1) Secondary	(2) High-School	(3) University
1999 compared to 2002 - Before and after all WFTC reforms			
Impact on employment	0.040***	0.055***	-0.005
Standard error	(0.012)	(0.015)	(0.016)
Pooled Sample 1995-2004			
Impact on Employment	0.0411**	0.0474*	-0.0095
	(0.0178)	(0.0266)	(0.0341)
lone-mothers	0.0015	-0.0086	-0.0105
x pre-reform linear trend	(0.0040)	(0.0067)	(0.0087)
N	24,648	8,113	5,088

The Effects of the Reform - Education

- A Simple reduced form model for the impact of the reforms on post-compulsory education

$$PC_{it} = \text{time} + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 \ln(EY_C) + \alpha_4 \ln(EY_{HS}) + \alpha_5 \ln(EY_U) + u_{it}$$

- f_1 and f_2 principal component factors summarizing family background.
- EY_C, EY_{HS}, EY_U are expected lifetime incomes under each education level
- These vary *only* by f_1 and f_2 and by tax/welfare regime.

The Effects of the Reform - Robustness

Table: The Effect of expected income on post-compulsory schooling

	(1)	(2)	(3)	(4)
$ln(EY_C)$	-0.8572^{**}	-0.8794^{**}	-0.8823^{**}	-1.0943^{**}
	(0.3758)	(0.3800)	(0.3839)	(0.5136)
$ln(EY_{HS})$				0.2616
				(0.6440)
$ln(EY_U)$				0.0362
				(0.4279)
f_1	0.1028^{***}	0.1042^{***}	0.1118^{***}	0.1138^{***}
	(0.0108)	(0.0123)	(0.0283)	(0.0289)
f_2	0.0119	-0.0030	-0.0031	-0.0040
	(0.0093)	(0.0102)	(0.0218)	(0.0209)
Includes pre and post '99 trends by f_1 and f_2 and time dummies				
Average Effect	-0.012^{**}	-0.012^{**}	-0.012^{**}	-0.012^{**}
St. Error	(0.0052)	(0.0052)	(0.0053)	(0.0054)
Changes in Expected income by Education group comparing 1999 to 2002				
$\Delta \ln \left(\overline{EY_C}\right) = 0.014$ $\Delta \ln \left(\overline{EY_{HS}}\right) = 0.010$ $\Delta \ln \left(\overline{EY_U}\right) = 0.004$				

Structural Model Features

A dynamic model of labour supply and human capital with the following features:

- Labour supply and consumption choices are heterogeneous and are made in an uncertain environment with credit constraints.
- Women can work part-time, full time, or not at all.
- Wages depend on accumulated part-time and full-time experience. They are stochastic and subject to potentially persistent shocks.
- The value of experience is allowed to differ by education and by part-time/full time work.
- Education choices are made reflecting uncertainty, risk aversion and credit constraints. We allow for a stochastic consumption value of education.
- Marriage/partner, partner income and children, while 'external' are stochastic and add further uncertainty.

Baseline Model: female life-cycle

Life in three stages:

- Education 's=0,1,2': three levels chosen sequentially up to age 18/21
    - secondary (GCSE-level at 16), further/high school (A-levels or vocational equivalent at 18), college (university at 21)
- Working life:
    - consumption 'c' and asset 'a' accumulation
    - labour supply 'l' (0, part-time and full-time)
    - experience accumulation
    - partnering
    - childbearing
- Retirement: pension incomes take effect exogenously at age 60 (see Fan/Sheshadri/Taber paper).

Back to the wage equation: for woman 'i', age 't', in each birth cohort; with school level 's', experience 'e', labour supply 'l'

$$\begin{aligned} lnw_{sit} &= \ln W_{sit} + \gamma_{si} \ln(e_{sit} + 1) + \upsilon_{sit} + \xi_{sit} \\ \upsilon_{sit} &= \rho_s \upsilon_{sit-1} + \mu_{sit} \\ e_{sit} &= e_{sit-1}(1 - \delta_s) + g_s(l_{sit}) \end{aligned}$$

- γ_{si} varies with schooling level s *and* background factors x_i.
- persistence of shocks - distinguish heterogeneity from state experience effects.
- ξ_{sit} is a transitory shock.
- correlation of initial permanent shock with preferences.
- concave profile of experience effects that differs by schooling level and background factors.
- $g_s(l_{sit})$ set to unity for full-time $g_s(FT) = 1$, the part-time experience value $g_s(PT)$ is estimated.
- δ_s depreciation of human capital - cost of not working.

Family formation dynamics

Children:

- Children are born with an (weakly) exogenous arrival rate,
- departure with certainty when child reaches age 18
- past employment(?).

$$\text{Prob}\left[t^k = 0 \,\middle|\, t, s, k_{t-1}, t^k_{t-1}, m_{t-1}\right]$$

Family Dynamics

Partner:

- Arrival rate depending on level of education and age,
  - characterised by education, employment status, prior marriage, children and earnings
  - arrival rate for male with given education depends on female age and education
  - departure probability depends on female age, presence of child and male education

$$\text{Prob}\left[s_t^m \,|\, t, s, m_{t-1}, s_{t-1}^m, k_{t-1}\right]$$

- Fertility and marriage behavior are 'weakly exogenous',
  - however, there is feedback: - individuals account for the implications of their choices on marriage and fertility.

Other family income - the Male

- Male employment depends on education and on whether he worked in the previous period or not.
- His earnings are uncertain:

male wage equation:

$$\begin{aligned} \ln w^m_{s^m it} &= \ln W^m_{s^m it} + \gamma^m_{s^m} \ln (t - 18) + \upsilon^m_{s^m it} + \xi^m_{it} \\ \upsilon^m_{s^m it} &= \rho^m_{s^m} \upsilon^m_{s^m it-1} + \mu^m_{s^m it} \end{aligned}$$

- Linked administrative, national insurance, earnings data are/will be key here.

Taxes and Assets

Detailed model of tax and benefit system - FORTAX (Shephard, 2011)

- Taxes: income tax, NI, council tax
- Benefits: child benefit, maternity grant, in-work tax credits, income support, housing benefit, council tax benefit, free school meals.

Assets:

- Initial period assets from the survey.
- Deal with the initial conditions problem by simulating from the start of life.
- Transfers implicit through funding of education.

Model: post education optimisation problem

$\{c_{it}, l_{it}\}_{t=\underline{t},\ldots,\bar{t}}$ are chosen over the life-cycle with preferences

$$V_{\underline{t}}\left(X_{i\underline{t}}\right) = E\left[\sum_{t=\underline{t}}^{\bar{t}} \beta^{t-\underline{t}} \frac{\left(c_{it}/n_{it}\right)^{\mu}}{\mu} \exp\left(U\left(l_{it}, l_{it}^{m}, X_{it}\right) + \theta_i l_{it}\right) \middle| X_{i\underline{t}}\right]$$

subject to the budget constraint (including taxes and childcare costs):

$$a_{it+1} = (1+r)a_{it} + l_{it}w_{sit} + d_{it}^{m} l_{it}^{m} w_{it}^{m} - T\left(X_{it}, l_{it}, l_{it}^{m}\right) - Q\left(t_{it}^{k}, l_{it}, l_{it}^{m}, X_{it}\right) - c_{it}$$

- net worth liquidity constraint: $a > a_s$.
- uncertainty: earnings (own and partner's) and family composition.
- $U\left(l_{it}, l_{it}^{m}, X_{it}\right)$ is a function of family composition, education, partner, partner labour supply, background factors, and unobserved heterogeneity.
- θ_i unobserved types.
- childcare costs (Q) and housing benefits (in T) vary by location and time=>

Childcare Costs

Q are childcare costs:

$$Q\left(t^k, h_t, \widetilde{h}_t, m_t\right) = \begin{cases} h_t * CC_h \text{ if } \ d_{cc} = 1 \\ \text{ and } t^k \leqslant 5 \text{ and } \left(\widetilde{h}_t = 40 \text{ or } m_t = 0\right) \\ 18 * CC_h \text{ if } \ d_{cc} = 1 \ \text{ and } 5 < t^k \leqslant 10 \\ \text{ and } h_t = 38 \text{ and } \left(\widetilde{h}_t = 40 \text{ or } m_t = 0\right) \\ 0 \text{ all other cases} \end{cases}$$

where CC_h is the per-hour rate, which is estimated externally from the data.

- Preschool children need childcare whenever no adult is staying at home, and school-age children only need childcare outside the school day as education is publicly provided.
- Childcare costs are zero for those with access to informal care ($d_{cc} = 0$), the probability of which is estimated from the data, and only depend on the age of the youngest child.

Model: education decisions

- Discrete choice model.
- Allow for borrowing constraints, tuition costs and student loans.
- Condition on factors formed of many family background variables at age 16, including
  - parental education/occupation, financial circumstances, siblings, region of birth, books in the home, whether lived with parents at 16, etc.
- Future earnings and family composition are uncertain.
- Use the fact that different cohorts enter under different tax/welfare systems as an additional source of exogenous variation for education.

Model: schooling decisions

$$s = \underset{s \in \{1,2,3\}}{\operatorname{argmax}} \left\{ W_s\left(X_{17}\right) - B_s\left(X_{17}\right) \right\}$$

$$B_s\left(X_{17}\right) = \pi_{1s} f_1 + \pi_{2s} f_2 + \pi_{5s} y_p + \varpi_s .$$

$$W_s\left(X_{17}\right) = \begin{cases} \mathrm{E}\left[V_{19}\left(X_{19}\right) \middle| X_{17}, s\right] \text{ if } s = 1, 2 \\ \mathrm{E}\left[\max\limits_{c_{19}, c20, c_{21}} \left\{ \sum_{t=19}^{21} \beta^{t-19} u\left(c_t, F; \theta, Z_{17}\right) \right.\right. \\ \left.\left. + \beta^{22-19} V_{22}\left(X_{22}\right) \middle| X_{17}, s \right] \right\} \quad \text{if } s = 3 \end{cases}$$

$$\begin{aligned} a_{19} &= a_{17} = 0 \\ a_{22} &= -(1+r)^2 c_{19} - (1+r) c_{20} - c_{21} - D \quad \text{if } s = 3 \end{aligned}$$

Structural Dynamic Modelling Issues

- As noted earlier, a key feature of our model is that it models the joint consumption and labor supply decisions over the working years of women, where the former is a continuous choice while the latter is discrete.
- The smoothness and concavity of the value function that is typical of continuous problems and that ensures the existence and uniqueness of a solution that is itself continuous and, if interior, is the root of the optimality condition (Euler equation) does not hold.
- The addition of a discrete choice makes the value function piecewise concave, with kinks falling at the points where the agent is indifferent between any two possible alternatives along the discrete choice domain,
- \- these then translate into discontinuities in the optimal choice of the continuous variable (consumption or savings)

Structural Dynamic Modelling Issues

- Kinks created by present choices at time t are what Iskhakov et al. (2015) called primary kinks do not pose difficulties. They can be dealt with by conditioning the continuous choice on the discrete choice in a first step, followed by the choice of the alternative with highest value in the second step.

- This is computationally more demanding than the purely continuous problem because the root of the Euler equation must be calculated for each point in the domain of the discrete choice, but the solution method is a trivial extension of that for a purely continuous problem.

- However, kinks propagate backwards through the (expected) continuation values - the secondary kinks. These are caused by indifference points in future choices, from $t+1$ onwards, and hence cannot be easily conditioned on. The further back one moves, the more kinks there will be.

Structural Dynamic Modelling Issues

- Associated with secondary kinks are discontinuities in future choices, which need to be accounted for in the Euler equation, as they affect the marginal utility of the continuous choice variable at both time t and t +1. This implies that the Euler equation is no longer a sufficient optimality condition, even after conditioning on the discrete choice at time t.
- As noticed by Iskhakov et al. and others kinks can be eliminated and the expected continuation value can be concavified by uncertainty. This is the approach we explore given the rich characterization of uncertainty we account for in the model.
- Using a fine grid of 50 points in assets, we inspect the concavity of our numerical approximation of the expected value function. This is a finer grid than we use to solve and estimate the model; -> find no evidence at the estimated parameterization, that the expected value function is not globally concave.

Structural Estimation

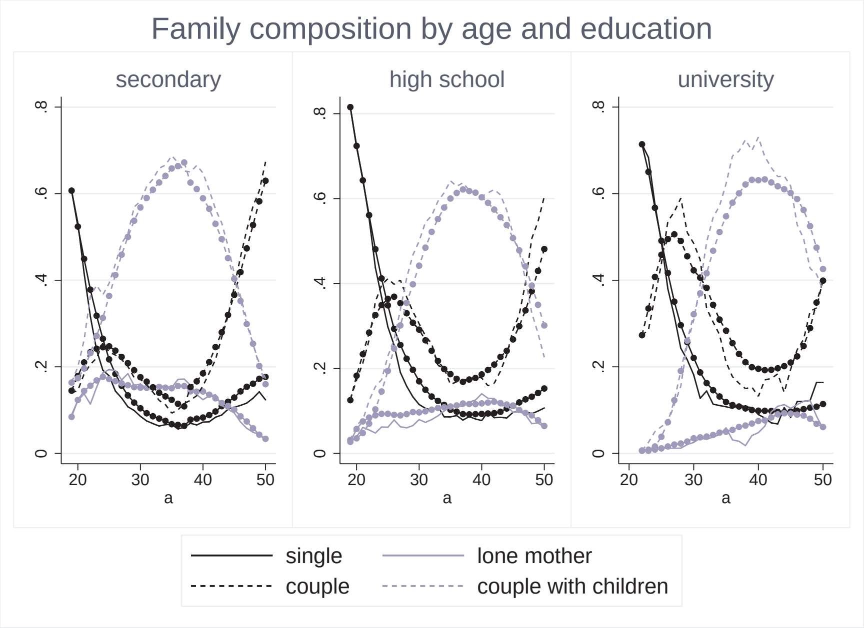
1. Estimate processes for male earnings and employment, family dynamics and childcare costs, recursively 'outside' the model.
2. Method of Simulated Moments for the remaining parameters: Simulate individuals under different tax regimes; Compute overall moment to match with those in the data.

$$\hat{\Theta} = argmin_{\Theta}\{\Sigma_{k=1}^{K}[(M_{kN}^{d} - M_{ks}^{m}(\Theta))^2 / Var(M_{kN}^{d})]\} \qquad (1)$$

- Matched moments include employment rates by family type, employment and hours transition rates, means, variances and percentiles of earnings distribution, earnings at entrance in working life, change in earnings by past hours, education achievement,...

The second step in more detail

- The second step implements an iterative procedure to estimate the preferences and wages of women within the structural model.
- In each iteration, we start by solving the female life-cycle problem for a particular set of the estimating parameters, given the economic environment and the exogenously set parameters.
- We then simulate five (try higher number for sensitivity too) replications of the life-cycle choices of (all 3,901) women observed in the data, conditional on observed family background and parental income.
- The same sequences of lifetime shocks are used in all iterations of the estimation procedure to avoid changes in the criterion function due to changes in the random draws.



Notes: Distribution of family types by age of woman. Data in solid lines, simulations in dashed

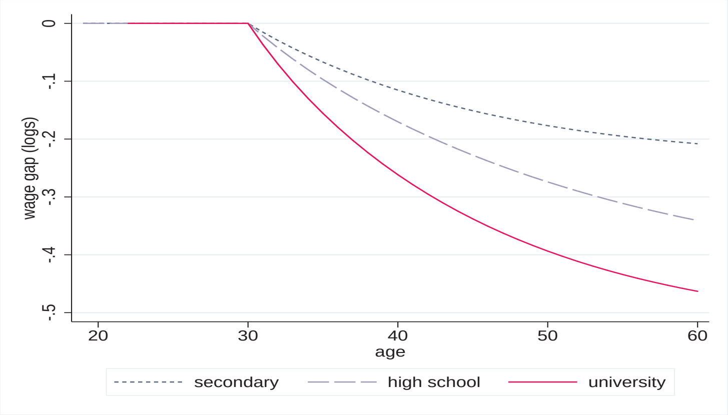
Structural Parameter Estimates

Table: Female wage equation estimates

	Secondary		Further		Higher	
baseline at age 25	7.19	(.050)	8.64	(.067)	10.55	(.31)
returns to experience	.15	(.01)	.23	(.01)	.31	(.02)
autocorrelation coef	.92	(.01)	.92	(.01)	.88	(.02)
se innovation	.12	(.01)	.15	(.01)	.14	(.01)
initial prod	.14	(.01)	.13	(.01)	.31	(.03)
initial productivity: se	.14	(.02)	.20	(.02)	.23	(.03)
depreciation rate	.08	(.01)	.06	(.01)	.07	(.01)
accumulation of HC in PTE	.15	(.02)	.10	(.02)	.12	(.02)

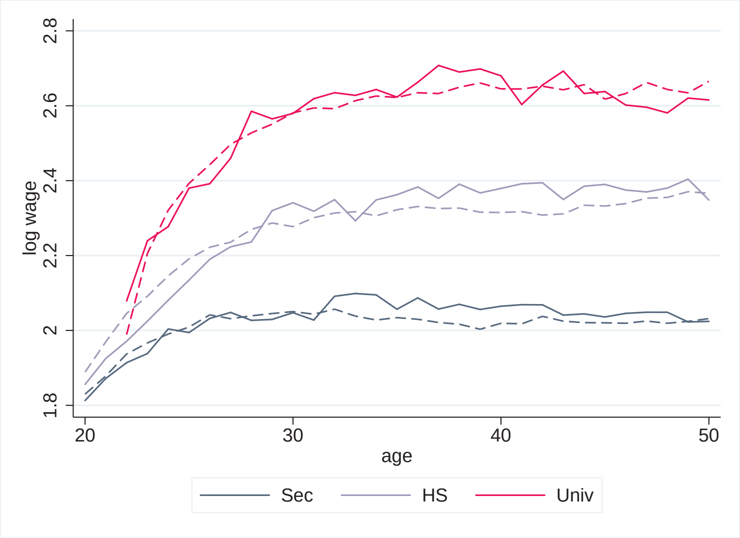
Notes: Interactions with background factors are included - for example, see experience coefficient in Table 8 in background paper.

Part-time Experience Penalty



Model fit

Life-cycle profiles of wages

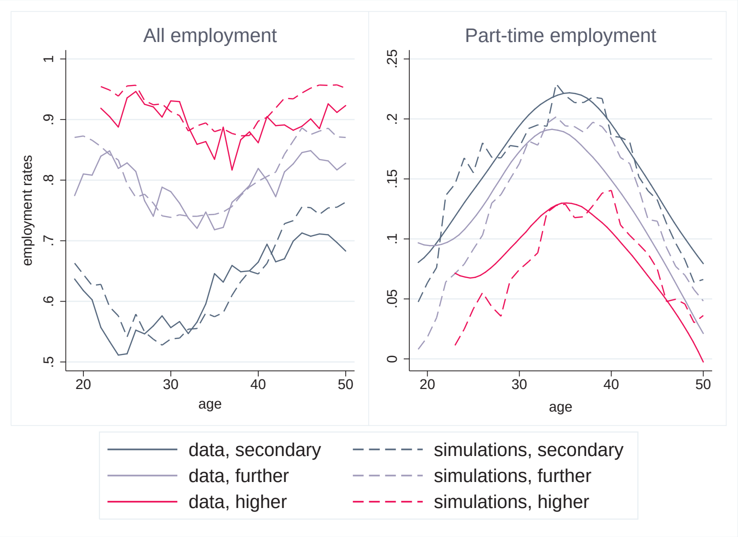


Model fit

Distribution of female wage rates by age

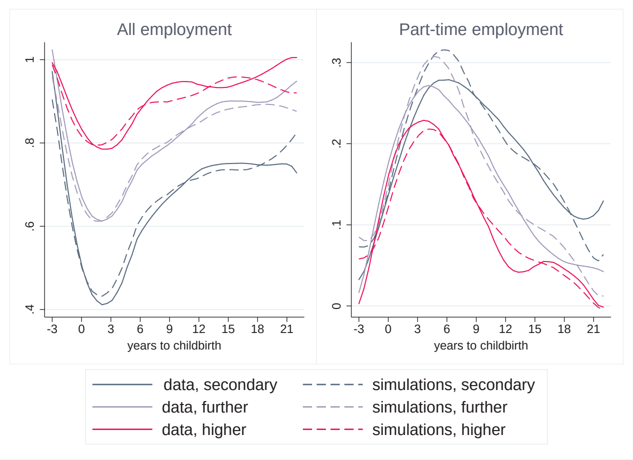
Model fit

Employment over life-cycle



Model fit

Employment of mothers



Comparison with Diff-in-Diff

WFTC and Income Support Reforms for Lone Mothers

Table 5: % Point employment impact and matched diff-in-diff for low educated lone parents.

1999 - 2002	Average Impact
Simulations	+4.4 (0.09)
Matched Diff-in-diff	+4.2 (0.11)

[Even though diff-in-diff assumptions strictly invalidated with forward looking behaviour.]

- Placebo effects on pre-reform data in Table 6 of background paper.

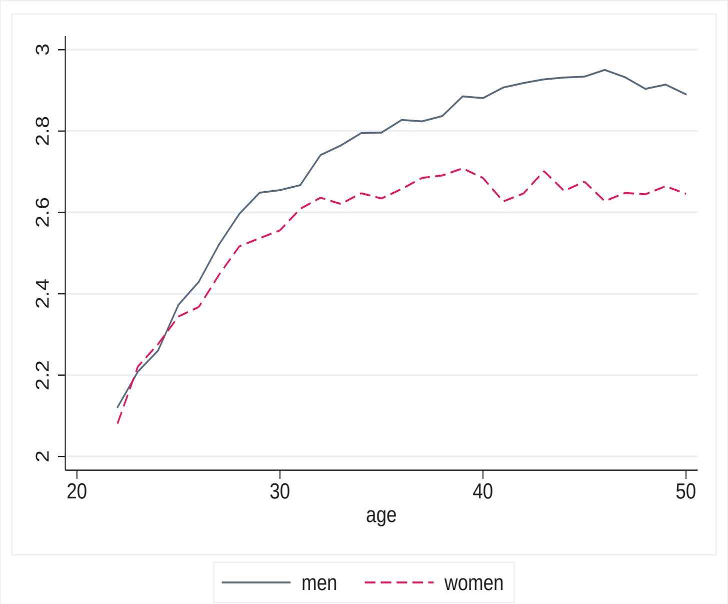
- Graphical comparison of the labor force participation of single women without children to single mother in Figure 3.

Table: The effect of part-time and non-employment on female wages at age 50

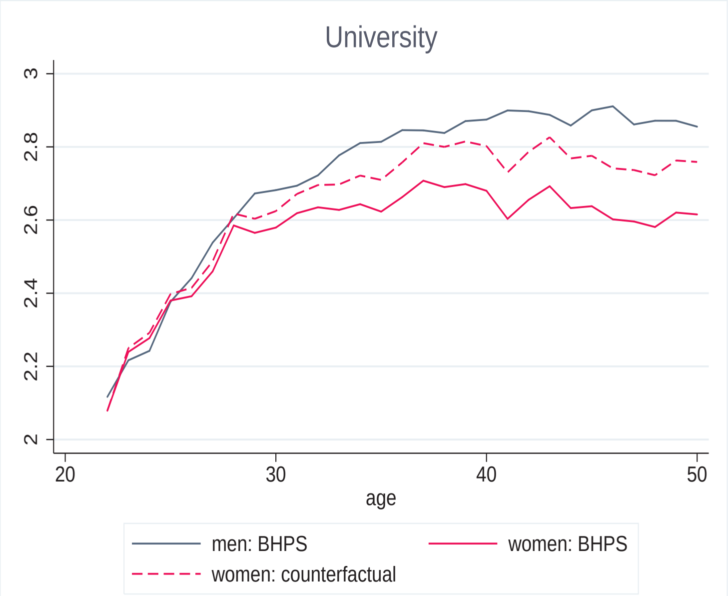
	No part-time penalty	No penalty for not working *and* no part-time penalty
Secondary (%)	5.3	10.5
High School (%)	7.0	12.5
University (%)	7.7	14.3

Notes: The first column shows the effect on wages at 50 if the amount of experience gained from part-time work is the same as that of full time work. The second column cancels, in addition, the experience cost of not working.

Male and Female Wage Age Profiles: University Graduates



Male and Female Wage Age Profiles: University Graduates



Estimates: preferences for work

	all employment			part-time employment		
	secondary	further	university	secondary	further	university
intercept	0.41 (.00)	0.41 (.00)	0.47 (.01)	-0.15 (.01)	-0.16 (.01)	-0.20 (.02)
children		0.05 (.01)			-0.06 (.01)	
child aged 0-2		0.15 (.01)			-0.05 (.01)	
child aged 3-5		0.07 (.01)			-0.06 (.01)	
child aged 6-10		-0.02 (.01)			0.03 (.01)	
child aged 11-18		-0.07 (.01)			0.06 (.01)	
male		-0.06 (.01)			-0.02 (.02)	
male working		-0.17 (.01)			0.09 (.01)	

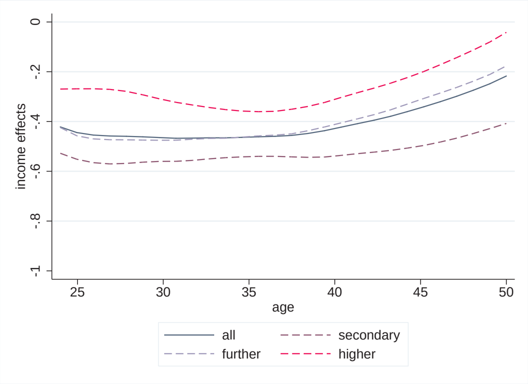
Notes: Full interactions in Table 9 of background paper.

Overall Marshallian Labour Supply Elasticities

	extensive	intensive
All	0.47	0.23
Secondary	0.71	0.31
High School	0.43	0.23
University	0.28	0.15
Lone mother	1.65	0.41
Mothers in couples	0.53	0.29
Childless women	0.22	0.22

- Note: Find key role for saving and note implications for elasticities, see section 7.4 in background paper.

Income Effects at Extensive Margin by Age



Results: Frisch Wage Elasticities of Labour Supply

	extensive	intensive
All	0.66	0.28
Secondary	1.01	0.40
High School	0.61	0.27
University	0.36	0.15
Lone mother	1.65	0.45
Mothers in couples	0.69	0.30
Childless women	0.29	0.22

Costs of Education

Table: Estimates of preferences for education - forward looking sequential discrete choice with background factors and parental income shock.

		High School		University	
		coeff	st. error	coeff	st. error
(1)	intercept	-0.053	(0.025)	0.682	(0.015)
(2)	background factor 1	0.227	(0.012)	0.363	(0.014)
(3)	background factor 2	0.009	(0.022)	0.299	(0.011)
(4)	parental income shock y_p	0.305	(0.158)	0.695	(0.036)
(5)	cost of ed ($\sqrt{V\varpi_s}$)	1.579	(0.093)	1.015	(0.183)

Notes: See Table 10 in background paper.

Effects of the reform on Education

Table: The impact of the reforms on education attainment – model simulations versus 'reduced form' data estimates.

		High school	University
(1)	Estimates based on BHPS data	-.012	-.005
	St. Error	(.005)	(.005)
(2)	Model simulation	-.008	-.005

Counterfactual Policy Simulations

Table: 15: Counterfactual Effects of Tax Credit on Employment

		Pre-reform education choice					
		Secondary		High School		University	
		Impact on Employment: Mothers of Dependent Children (0-18)					
		Single	Married	Single	Married	Single	Married
(1)	All (pp)	20.4	-6.6	14.9	-3.6	5.5	-1.0
(2)	Full-time(pp)	9.3	-3.6	6.5	-2.4	-2.1	-1.1
(3)	Part-time(pp)	11.1	-3.0	8.4	-1.2	6.6	0.1
		Impact on Employment: Mothers of Adult Children (19+)					
(4)	All (pp)	0.4		0.3		0.0	
(5)	Full-time(pp)	0.4		-0.0		-0.2	
(6)	Part-time(pp)	-0.0		0.3		0.2	

Notes: Reform is revenue neutral by adjusting the income tax rate.
See Table 15 in background paper.

Table: 15 (cont.): Counterfactual Effects of Tax Credit Policy on Education and Lifetime Welfare

		Pre-reform education choice		
		Secondary	High School	University
			Impact on Education	
(7)	Education (pp)	0.54	-0.11	-0.25
		Impact on Lifetime Disp. Income and Welfare		
(12)	Disp. Income (%)	-1.09	-0.25	-0.87
(13)	Cons. equiv. (%)	2.49	0.89	-0.27
(14)	Adjustment in the basic rate of Income Tax to fund reform: +0.9pp			

Reform is revenue neutral by adjusting the income tax rate. Education is allowed to adjust.

Computing Consumption Equivalent Welfare Measure

- Use a money metric measure of the welfare effects of certain policies.
- Derive the amount of consumption individuals are willing to give up (positive or negative) to move to new regime.

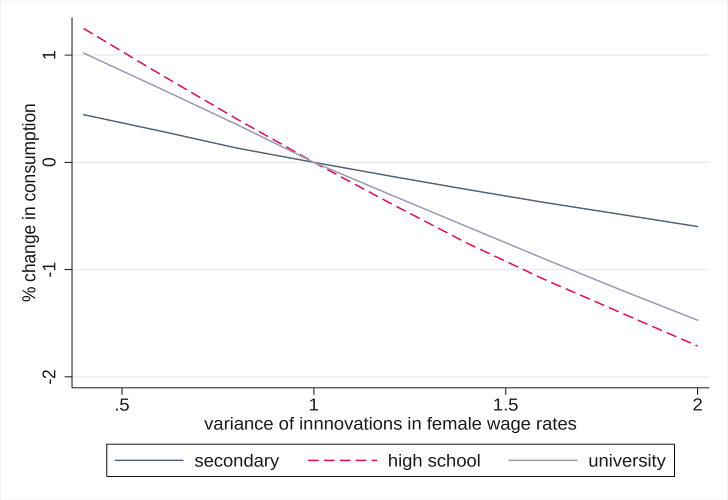
$$EV_0 \quad = \quad E\sum_t \beta^{a-\underline{A}} \frac{((1-r)c_{1a}/n_{1a})^{\mu}}{\mu} \exp\left\{U(l_{1a}, X_{1a}) + \theta(l_{1a})\right\}$$

where the index 0/1 stands for the pre/post-reform solutions and the value function is evaluated at different stages in life for different rows. The equation can be solved for r, yielding:

$$r \quad = \quad 1 - \left(\frac{EV_0}{EV_1}\right)^{\frac{1}{\mu}}.$$

Risk Aversion and the Value of Insurance

Willingness to pay in consumption units



Alternative policies: lifetime income and welfare, % gains

	Sec	HS	Uni	Pre-Ed
Tax Rate Adjustment				
Lifetime Disp. Income	.68	.77	.88	
Welfare	.48	.63	.80	.80
Tax Credit Adjustment				
Lifetime Disp. Income	.84	.36	-.24	
Welfare	1.38	.78	.77	1.09
Income Support Adjustment				
Lifetime Disp. Income	-.15	-.48	-.54	
Welfare	.72	.32	.30	.51

- Notes: Impacts of an exogenous increase in public spending distributed through alternative routes.
- Welfare Effects of increasing Expenditure by 0.5% of total Gross Earnings.
- Tax rate decreases by 0.95pp or Max Tax Credit increases by 22.2 pounds or Income Support increases by 10 per week.

Summary of main results:

- Experience effects display strong dynamic complementarity with education.
- Lower experience effects for *low educated* and those in part-time work.
- Lower education women with children have more elastic labour supply and larger income effects.
- There is a significant but small effect of tax credit/welfare reform on education choice, attenuating some of the employment gains.
- The insurance value of the tax/welfare system is substantial.
- The results explain previous structural and quasi-experimental results. Omitting experience effects for low educated, also recall the Canadian SSP RCT.
- Counterfactuals show experience and part-time penalty can explain a large part (70%) of the gender gap in wages.

In terms of the 'Structural - Reduced Form' discussion

(i) the recovery of structural 'deep' parameters: e.g. experience effects display **strong dynamic complementarity,** with lower experience effects for low educated and those in part-time work.

(ii) the identification of underlying mechanisms: e.g. the **insurance value of tax-credits is substantial** part of welfare gain.

(iii) counterfactuals and policy design: e.g.
- lower education women with young kids have **larger supply** responses => tax credits are an optimal design (but little earnings progression).
- significant, but small, effect of **tax credits on education choice**, => attenuating some of the employment gains.the ex-ante simulation of a reform to the tax-credit and welfare system.

• Reconciliation of past results (learning from data and structural models):
- Can **explain past (static) structural and quasi-experimental results** - human capital depreciation and part-time work imply negligible experience capital wage dynamics/non-separability for low educated women.

But full counterfactuals require modelling of preferences and constraints.

Solving Dynamic Life-cycle Models

The simple life-cycle consumption model

$$\max_{(c_1,\ldots,c_T)\in\mathbb{C}^T} \quad \sum_{t=1}^{T} \beta^{t-1} u(c_t)$$

$$\text{s.t} \quad a_{t+1} = k(a_t - c_t) \text{ for } t = 1, \ldots, T$$

$$a_{T+1} \geq 0$$

$$a_1 \, (\in \mathbb{A}) \text{ given}$$

- Per-period wellbeing u: increasing in consumption
- Consumption: choice variable, with domain $\mathbb{C}$ (here $\mathbb{R}_0^+$ or $\mathbb{R}^+$, depending on u)
- Assets is the state variable, with domain $\mathbb{A}$ (here $\mathbb{R}_0^+$ or $\mathbb{R}^+$)
- k: law of motion for assets, a positive and increasing function in $\mathbb{A}$

$$k\left(a_t - c_t\right) = R\left(a_t - c_t\right) \quad \text{where } R = 1 + r \text{ is the interest factor}$$

Classical solution

- Objective function is continuously differentiable: interior optimum satisfies foc.
- Classical solution: attack problem directly by solving all its foc's.
- Lagrangian for this problem

$$\mathcal{L} = \sum_{t=1}^{T} \beta^{t-1} u(c_t) - \lambda \left(\sum_{t=1}^{T} R^{1-t} c_t - a_1 \right)$$

- With necessary foc's with respect to c_t, for $t = 1, \ldots, T$:

$$\beta^{t-1} u'(c_t) = \lambda R^{1-t}$$

- Putting together two subsequent conditions yields

$$u'(c_t) = \beta R u'(c_{t+1}) \quad \text{for } t = 1, \ldots, T-1 \tag{2}$$

- These are the Euler equations for this problem.

- The Euler equation establishes relationship between consumption in subsequent periods, but not the consumption level.
- For that we need the budget constraint, and the Kuhn-Tucker conditions do just that.
- The Euler conditions allowing for corner solutions are

$$\begin{array}{rrcll} & u'(c_t) & \leq & \beta R u'(c_{t+1}) & \text{for the possibility of } c_t = 0 \\ \text{or} & u'(c_t) & \geq & \beta R u'(c_{t+1}) & \text{for the possibility of } c_t = a_t \end{array}$$

- Typical choices of utility functions are continuously differentiable and monotonically increasing in $\mathbb{R}^+$, with the additional following property: $\lim_{c_t \to 0^+} u(c_t) = -\infty$ and $\lim_{c_t \to 0^+} u'(c_t) = +\infty$
- In this case a solution, if it exists, is interior.

Dynamic programming

- Dynamic programming splits the big problem into smaller problems that are of similar structure *and* easier to solve
- The solution determines a set of policy functions, one for each period t: optimal consumption (policy function) is $h_t(a_t)$
- Specific solution to the simple consumer problem can be constructed recursively, by iterating

$$
\begin{aligned}
c_t &= h_t(a_t) \\
a_{t+1} &= R(a_t - c_t)
\end{aligned}
$$

- The problem for a consumer with assets a_t at time t is

$$
V_t(a_t) = \max_{(c_t,\ldots,c_T)\in\mathcal{C}_{t:T}(a_t)} \sum_{\tau=t,\ldots,T} \beta^{\tau-t} u(c_\tau)
$$

- V_t is the value function: max lifetime (from t onwards) utility assets a_t can deliver.

The Bellman equation

$$V_t(a_t) \quad = \quad \max_{c_t \in \mathcal{C}_t(a_t)} \left\{ u(c_t) + \beta V_{t+1}\left(R[a_t - c_t]\right) \right\}$$

- Breaks the large life cycle problem in smaller problems - depends only on the value of state variables at the time of decision,
- V_{t+1} exists (by recursion) but is unknown!

An insight of dynamic programming: the unknown V can be pinned down by backward induction, $=>$ inspires the numerical strategy to solve models with no closed-form solution.

Solution strategy: start from period T and move backwards as the future value function, the continuation value, is determined.

If the solution is unique and V is differentiable *then* the first order conditions together with the initial and terminal conditions are necessary and sufficient for an interior optimum.

Algorithm for recursive solution

1. Parameterise model and select grid in assets: $\{a^i\}_{i=1,\dots,n_a}$
2. Store $V_{T+1}(a^i)$ for all $i = 1, \dots, n_a$
3. Loop over t backwards: $t = T, \dots, 1$

   For each $i = 1, \dots, n_a$

   1. Compute $g_t^i = \arg\max_{a_{t+1} \in \mathcal{D}_t(a^i)} \left\{ u\left(a^i - \frac{a_{t+1}}{R}\right) + \beta \widetilde{V}_{t+1}(a_{t+1}) \right\}$
   2. Compute $V_t^i = u\left(a^i - \frac{g_t^i}{R}\right) + \beta \widetilde{V}_{t+1}(g_t^i)$
   3. Approximate V_t over its entire domain to get $\widetilde{V}_t$ and store it. This step can be done in step 4.1 or skipped depending on the solution method

- Step 4.1 is the (computationally) heavy part of the solution algorithm. There are two main ways of finding the optimum g_t^i
  - Use a search algorithm to look for the value of savings a_{t+1} that maximise $V_t(a_t)$
  - Or look for the root of the Euler equation $u'(c_t) = \beta R V'(a_{t+1})$

Solution at each point using the foc

Generally, the consumption policy function h is typically not very non-linear. So all we need is to:

1. Store $h_t(a^i)$ after solving consumers problem at time t
2. "Connect the points" to obtain the solution over the entire domain as needed: Linear Interpolation

Notice that V is not needed to solve the problem using the foc, but one may still need the value function:

- to study the value of different policy interventions
- or attitudes towards risk once uncertainty is considered.

Adding an income process does not much change the problem

- If credit is rationed, there are T inequality restrictions in assets now, so we have T first order and Kuhn Tucker conditions.

More interesting problems in economics involve some sort of uninsurable risk - a Markov process for income. But typically concave and can still use the root of the Euler equation, function of (a_{t+1}, y_t).