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Consumer Demand and the Life-Cycle Allocation of Household Expenditures

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The purpose of this paper is to estimate the parameters of household preferences that determine the allocation of goods within the period and over the life cycle, using micro data. In doing so we are able to identify important effects of demographics, labour market status and other household characteristics on the intertemporal allocation of expenditure. We test the validity of the life-cycle model using excess sensitivity tests and find that controlling for demographics and labour market status variables can largely explain the excess sensitivity of consumption to anticipated changes in income.

1. INTRODUCTION

The purpose of this paper is to estimate the parameters of household preferences that determine the allocation of goods within the period and over the life-cycle, using micro data. In doing so we are able to identify important effects of demographics, labour market status and other household characteristics on the intertemporal allocation of expenditure. The distinctive feature of our approach is that it integrates traditional demand analysis with intertemporal substitution models in a coherent way.

The organizing idea behind our analysis is the life-cycle hypothesis. The principal implication of this hypothesis is that households will allocate consumption expenditures so as to try to keep the marginal utility of wealth (λ) constant over time (see Heckman (1974), Hall (1978), MaCurdy (1983) and Browning, Deaton and Irish (1985)). This is true both in the short run (for example, saving to capture the benefits of high real rates of interest) and in the long run (for example, saving for retirement or for a "rainy day"). Of course, λ is unobservable so that the empirical consequences of this "smoothing" have to be derived for expenditures on individual goods. Although nonparametric tests are available under certain conditions (see Browning (1989)) the usual procedure, which we follow here, is to specify a parametric utility function and then to derive the Euler equation that governs individual expenditures.

The relationship between λ and expenditures on individual goods depends on many things. Amongst these are the shape of Engel curves; the amount of substitution between goods; the demographic composition of the household and the labour market status of the household. Analyses that assume that agents buy only one good ("consumption") and that preferences over this good are additively separable from demographics and labour supply are potentially quite misleading. In this paper we specify a set of household preferences that allow us to model jointly within-period allocations (the demand system) and between-period allocations (the consumption function). Although our principal interest is in the latter we shall see that we also need to model the former.

We utilize a time series of U.K. cross-sections covering 70,292 households over a 17-year period to investigate the link between within-period preferences and intertemporal substitution. Micro-level data avoids the problem of aggregation bias and allows us to analyse the importance of demographic and labour supply variables.

Our empirical results are based on a structural model of demand and intertemporal substitution as well as on a simple iso-elastic specification which we extend to allow for household characteristics. To meet the great many reservations to the life-cycle model, documented recently by Deaton (1992), we carry out a number of specification tests including excess sensitivity tests, tests of overidentifying restrictions and a simple structural change test.

The main conclusion is that household characteristics are of fundamental importance in explaining the growth of consumption over a household's life cycle. We find that controlling for these characteristics is sufficient to eliminate excess sensitivity of consumption growth to predictable income growth. The issue of whether characteristics capture taste effects or are simply a proxy for the effects of liquidity constraints remains an open question.

2. A THEORETICAL FRAMEWORK

We assume the consumer maximizes the discounted sum of period-specific utilities. Our estimation procedure exploits the two-stage budgeting results of Gorman (1959). The within-period allocation of total consumption x to individual goods with prices p, is completely characterized by the indirect utility function $V(p, x)$ and is invariant to monotonic transformations of utility V.[1] Intertemporal allocations are then determined by the period-specific utility function $U = F[V(p, x)]$ where $F[\cdot]$ is a strictly increasing monotonic transformation such that U is strictly concave in x.

In what follows we partition goods into two groups. The first is the group of interest whose expenditure is x; let this have quantity and price vectors (q, p). The second group contains those goods that we do not model explicitly. For example, these include labour market status and demographic structure. Denote the vector of such goods z and let sub-vectors be given by z^1, z^2 and z^3, where z^i and z^j may have common elements. We represent period specific preferences by the conditional indirect utility function

$$U(p, z, x) = F[V(p, z^1, x), z^2] + H(z^3). \tag{2.1}$$

This function gives the maximum utility in the period for an agent who has total expenditure x on the first group of goods with prices p conditional on other goods and household characteristics z.

This treatment of conditioning factors has a natural interpretation for our discussion of intertemporal allocations. All those factors in z^3 but not in z^1 nor z^2 enter neither the

1. See Deaton and Muellbauer (1980*b*) for a detailed description of the properties of indirect utility functions.

demand system nor the consumption function. They are explicitly additive from all other commodities. Factors in z^2 but not in z^1 enter the consumption function but not the demand system. These are weakly separable from q. Those in z^1 influence the marginal rate of substitution between elements in q and are therefore not separable. Factors in z^1 but *not* in z^2 condition demand directly but do not affect intertemporal allocation except through their effect on the parameters of demand used in the consumption function.

2.1. *A model for within-period preferences*

The form of within-period preferences is independent of the normalization $F[\cdot]$ in (2.1). More precisely, the shape of Engel curves and the specific form of within-period substitution are independent of the parameters determining intertemporal substitution. Suppressing the conditioning variables z, the indirect utility representation of within-period preferences we use is

$$V(p,x)=\left(\frac{x}{a(p)}\right)^{\{-\theta\}}\frac{1}{b(p)},\qquad \theta\neq 0, \tag{2.2}$$

and

$$V(p,x)=\frac{\ln x-\ln a(p)}{b(p)},\qquad \theta=0, \tag{2.3}$$

where { } represents a Box-Cox transformation

$$y^{\{\lambda\}}=\frac{y^{\lambda}-1}{\lambda}.$$

These two are members of the PIGL and PIGLOG classes respectively (Muellbauer (1976)). To complete the specification we adopt the following standard specifications for the two price indices:

$$\ln a(p)=\alpha_0+\textstyle\sum_k \alpha_k \ln p_k+\frac{1}{2}\sum_k\sum_j \gamma_{kj}\ln p_k \ln p_j, \tag{2.4a}$$

$$\ln b(p)=\textstyle\sum_k \beta_k \ln p_k. \tag{2.4b}$$

To satisfy adding-up we require $\sum \alpha_k=1$, $\sum \beta_k=0$ and $\sum_k \gamma_{kj}=0$, while homogeneity implies $\sum_j \gamma_{kj}=0$. Thus $a(p)$ and $b(p)$ are homogeneous price indices of degree one and zero respectively.

Applying Roy's identity to (2.2) and using the above parameterizations we obtain the demand system

$$w_{it}=\alpha_i+\textstyle\sum_j \gamma_{ij}\ln p_{jt}+\beta_i(x/a(p))^{\{\theta\}}+v_{it} \tag{2.5}$$

where i denotes the i'th good, t is the observation index and v_{it} is assumed to represent unobservable components in demand.[2] The value of θ determines the shape of the Engel curve. Given our choice of $a(p)$ and $b(p)$, if $\theta=0$ we have the Almost Ideal Demand System of Deaton and Muellbauer (1980a). Alternatively, if $\theta=-1$ we have quasi-homothetic preferences whilst $\theta=+1$ gives a form of quadratic Engel curves in which

2. Note that the parameterization of the demand system implies that any stochastic variation of the parameters would either enter in a non-additive way (i.e. through $a(p)$) or would interact with the endogenous total expenditure term (if they enter through the βs). Moreover any random preferences would enter non-linearly in the utility index V. Hence, in order to be consistent we have to assume that the stochastic specification represents optimization errors in the allocation of budget shares. Adding-up implies that these errors sum to zero.

expenditure shares are linear in x. If all of the β_i's are zero we have homothetic preferences; in this case all Engel curves are linear through the origin and θ is not identified in the demand system.

One of our objectives is to assess the importance of demographic characteristics, labour market variables and other taste shifters on intertemporal substitution and consumption growth. Since $F[\cdot]$ and $V(\cdot)$ in (2.1) are separately identifiable when we use data on within-period and intertemporal allocations, omitting these characteristics from the within-period utility index $V(\cdot)$ may bias our conclusions relating to intertemporal allocations. Moreover, other studies on micro data sets have shown demographics to be very important in demand systems.

2.2. *The consumption function*

Our approach in deriving the consumption function is based on the Hall (1978) Euler equation formulation implemented on micro data by Heckman and MaCurdy (1980) and Altonji (1986); it follows closely the methodology of MaCurdy (1983). Defining $\lambda_t = \partial F_t/\partial x_t$ as the marginal utility of an extra unit of expenditure in period t, we can write the condition for optimal intertemporal behaviour under uncertainty as

$$E_t[(1+r_t)\lambda_{t+1}-\lambda_t]=0, \tag{2.6}$$

where r_t is the nominal interest rate earned on assets held between t and $t+1$ and $E_t\{\cdot\}$ represents the expectation conditional on information available at time t.

We take the following functional form for $F(\cdot)$

$$F(V_t)=(1+\delta)^{-t}\phi_t V_t^{\{1+\rho_t\}}, \tag{2.7}$$

where δ is the rate of time preference and where ϕ_t represents some scaling of utility which may depend on household characteristics and other conditioning factors and, as before, the superscript in { } denotes the Box-Cox transform. The parameter ρ will be allowed to vary across households and across time according to movements in demographic and other characteristics. Given this normalization we may write the log of the marginal utility of expenditure (in current terms) for any household in period t as:

$$\ln \lambda_t = \ln \phi_t - t\ln(1+\delta)+\rho_t \ln V_t + \ln V_t' \tag{2.8}$$

where V_t' is the derivative of V_t with respect to x_t.

To estimate ρ_t we shall exploit the Euler equation (2.6) which governs the evolution of λ_t over time. To proceed, we re-write (2.6) as:

$$(1+r_t)\lambda_{t+1}=\lambda_t u_{t+1}, \qquad E_t(u_{t+1})=1. \tag{2.9}$$

Taking logs through (2.9) we can then write (2.6) as:

$$\Delta \ln \lambda_{t+1}+\ln(1+r_t)+d_t=-\varepsilon_{t+1}, \tag{2.10}$$

where Δ refers to the first difference operator. Defining E_t $(\ln u_{t+1})=-d_t$, ε_{t+1} is such that $E_t(\varepsilon_{t+1})=0$. If u_{t+1} is lognormal then $d_t=\frac{1}{2}\sigma_t^2$, σ_t^2 being the variance of $\ln u_{t+1}$ conditional on information in time period t. In general d_t will also be a function of higher moments of $\ln u_{t+1}$.

Substituting (2.8) in (2.10) and re-arranging we have:

$$-\Delta \ln V_{t+1}'-\ln(1+r_t)=\Delta\rho_{t+1}\ln V_{t+1}+(d_t-\delta_t)+\varepsilon_{t+1} \tag{2.11}$$

in which all parameters are identifiable from within-period allocations and are summarised in the V and V' expressions. We have rewritten $\Delta \ln(\phi_{t+1}) - \ln(1+\delta)$ as $-\delta_t$ which could be a function of levels and changes in household characteristics. To interpret $(d_t - \delta_t)$ assume u_{t+1} in (2.9) is lognormally distributed; then the difference $(\frac{1}{2}\sigma_t^2 - \delta_t)$ can be interpreted as capturing the trade-off between impatience and caution. To see the relationship between this and intertemporal substitution, note that increases in future uncertainty, decreases in impatience and increases in the nominal rate r_t all act in the same direction. Thus d_t can be thought of as capturing the precautionary motive for saving.

Using our definition of indirect utility V from (2.2), the Euler equation (2.11) can be rewritten

$$[(1+\theta)\Delta \ln C_{t+1} - i_t + \Delta \ln b(p_{t+1})] = \Delta[\rho_{t+1}[\ln C_{t+1}^{\{-\theta\}} - \ln b(p_{t+1})]] + (d_t - \delta_t) + \varepsilon_{t+1}, \tag{2.12}$$

where we have used C_t to represent total real consumers' expenditure $x_t/a(p_t)$ and $i_t = \ln(1+r_t) - \Delta \ln a(p_{t+1})$ is the real interest rate measure implied by our specification of individual preferences.

In (2.12) the left-hand side resembles the iso-elastic specification; indeed if $b(p_t)$ is constant over time and if $\rho_t = 0$ then the difference between consumption growth scaled by $(1+\theta)$ and the real rate is just equal to $d_t - \delta_t$ and an innovation; this is precisely the log-linear version of the iso-elastic model. In general, given values for ρ_t and δ_t, intertemporal allocations will depend on the set of conditioning goods via the two price indices $a(p_t)$ and $b(p_t)$ which characterise within-period substitution. Nevertheless demographic variables may also affect intertemporal allocations directly since ρ_t (i.e. $F[\cdot]$ in (2.1)) may be a function of variables, such as the number and age of children and labour market status. We thus specify:

$$\rho_t = \rho_0 + \textstyle\sum_k \rho_k z_{kt}. \tag{2.13}$$

We discuss alternative interpretations of the results obtained from such a specification in the empirical section.

The relationships given by (2.5) and (2.12) constitute our description of the consumer's allocation scheme. This parameterization has a recursive nature: there are some parameters that enter both the intratemporal and intertemporal allocation decisions i.e. θ and the parameters of $a(p_t)$ and $b(p_t)$. There is also a set of parameters that characterize only intertemporal consumption allocations, namely ρ_t, d_t and δ_t. In particular, θ, $a(p_t)$ and $b(p_t)$ determine the shape of the underlying Engel curves while, for any given value of these, ρ_t determines intertemporal substitution. If we change the former then we shall usually change the latter.

The intertemporal substitution elasticity (ISE) $\partial \ln C_t/\partial \ln(1+i_t)$ implied by our model takes the form

$$\Phi_t = \frac{U_t'}{x_t U_t''} = \frac{C_t^{\{\theta\}}}{\rho_t - (1+\theta)C_t^{\{\theta\}}} \tag{2.14}$$

where U_t' is the partial derivative of $U(\cdot)$ in (2.1) with respect to x. If $\rho_t = 0$ then $\Phi_t = -1/(1+\theta)$ and (2.12) reduces to the standard iso-elastic Euler equation. Given our concavity assumption the intertemporal elasticity should be negative for all values of C_t. This requires $\rho < (1+\theta)C^{\{\theta\}}$ for all observations in our data.

The specification of the model implies various relationships between the ISE and consumption depending on the estimated parameters $\rho(z_t)$ and θ. As C tends to ∞, Φ tends to $-1/(1+\theta)$. (When $\theta = -1$, Φ tends to $1/\rho$). If $\rho < 0$ then the absolute value of Φ increases with consumption, while if $\rho > 0$ the reverse is true.

3. ESTIMATION AND EMPIRICAL EVIDENCE

3.1. *Data*

In estimating our demand system describing within-period preferences we have chosen a group of seven broad commodities for each household. These are: food, alcohol, fuel, clothing, transport, services and other goods. The results reported here refer to a sample of 70,292 households from 1970–1986 inclusive, whose eldest adult is more than 18 and less than 60 years of age and is not self-employed. These data are drawn from the annual U.K. Family Expenditure Survey and are more fully described in the data appendix at the end of the paper. Our choice of goods clearly excludes some non-durables like tobacco as well as most durables, leisure and public goods. As described in Section 2.1 we allow for the effect of some of these on our allocation scheme by entering them as conditioning variables in the demand system and consumption model (see Browning and Meghir (1991)).

We note first that our data, rather than being a panel following the same individuals across time, is a time series of repeated cross-sections. As a result we construct a pseudo-panel using cohort averages in order to estimate the model as suggested by Browning, Deaton and Irish (1985). The cohorts are chosen in five-year age bands. The methodology which uses exact aggregation, and the assumptions underlying it are described below.

In this paper our focus is on life-cycle patterns of consumption. We start by presenting some of the principal features of our data that are salient for consumption. In particular we look at co-movements in consumption, income, demographics and labour force participation. It is our strong belief that since these are all determined jointly by the same household, looking at any pair in isolation may be quite misleading. In Figure 1(a) we present the life-cycle path of consumption for married couples.[3] In each of these figures, each separate line represents the evolution over time of the relevant variable for one date of birth cohort defined over a five-year band. This has the familiar pattern: consumption rises initially and then falls after the mid-forties. Figure 1(b) shows the life-cycle path of household income. Once again, the shape and the correlation with consumption are familiar. If we identify the cause of the correlation seen here by assuming that income is exogenous then it looks like consumption tracks income very closely over the life-cycle. The next two figures (1(c) and 1(d)) look at the paths of two potentially important determinants of consumption and income respectively: children and female employment. As we expect, female employment drops during the child-bearing years and the number of children in the household peaks soon thereafter. What is particularly interesting here is that although female participation falls in the early years, household income does not. One obvious explanation for this is that households choose the timing of births relative to the husband's career profile but this once again implicitly assumes that the latter is exogenous. It will be clear, however, that these figures are also consistent with the hypothesis that husbands' income paths are chosen to facilitate particular paths of births.

In Figure 1(e) we present the path of consumption deflating by "equivalent" household size which equals the number of adults plus 0·4 times the number of children.[4] As can be seen, this removes most (if not all) of the "hump" shape in consumption.

3. In our analysis below we include single-adult households with appropriate controls; here we use just married couples to abstract from awkward composition effects.

4. Obviously we could use more sophisticated equivalence scales that allowed for economies of scale and for age differences in children; this does not change things very much.

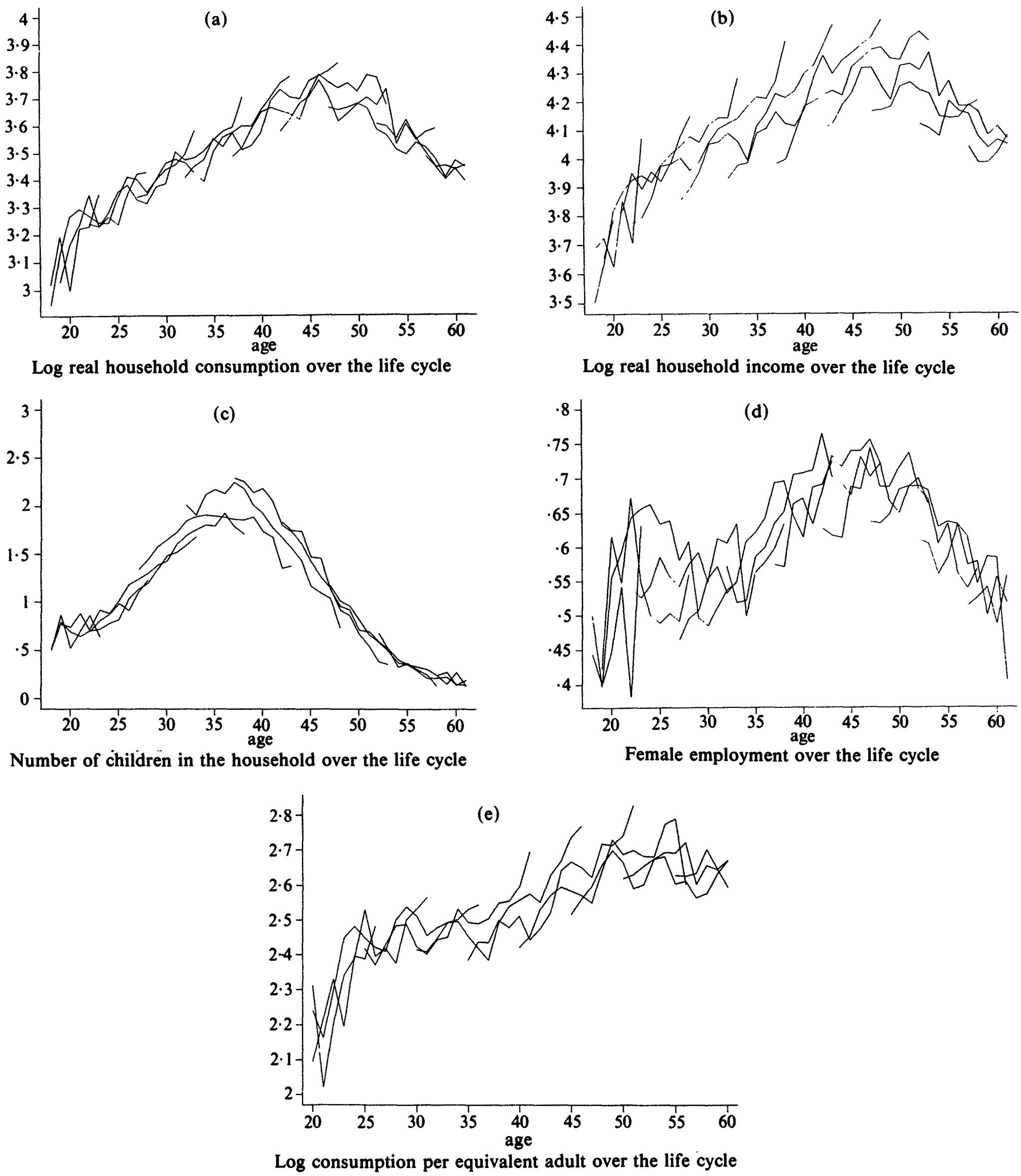

FIGURE 1

(a) Log real household consumption over the life cycle. (b) Log real household income over the life cycle. (c) Number of children in the household over the life cycle. (d) Female employment over the life cycle. (e) Log consumption per equivalent adult over the life cycle

The principal conclusion we draw from these figures is that it is very difficult to infer anything about life-cycle consumption from looking at simple descriptive graphs. Apart from not being able to distinguish between anticipated and unanticipated components in the series, we cannot properly take account of the possible endogeneity of other factors that affect consumption. Thus we must have recourse to an econometric analysis that addresses these issues by conditioning on demographics and labour market status and by using an instrumental variables approach.

Comparing Figures 1(a) and 1(e) also gives us some insight into how controlling for life-cycle events may change our perceptions of year to year consumption changes. As can be seen from 1(a), for younger (older) households, consumption growth is largely positive (respectively, negative). This life-cycle variation may mask a lot of year to year variations and the correlation of the latter with common variables such as the real rate. Thus not controlling for changes due to changes in "life-cycle" variables may make it difficult to pick up high frequency intertemporal substitution effects.

In Figure 2 we plot the time path of real consumption growth and the smoothed time path of the real interest rate.[5] As is well known the ex post real interest rate was negative throughout the 1970s. On the other hand consumption growth has been positive most of the time. In terms of the standard life-cycle model this must either imply that the ex ante real interest rate was positive or that precautionary savings is an important influence on consumption growth. In terms of our model this would mean that $d_t - \delta_t$ was a positive number in the 1970s. After 1981, the real interest rate jumps, becoming positive and higher than consumption growth. We investigate the implications of this real rate jump in the empirical section.

3.2. *Estimating within-period preferences*

To estimate the parameters of the expenditure share system we adopt an iterative moment estimator in which we allow for the endogeneity of total expenditure and iterate on the $a(p)$ price index and on the Engel curvature parameter θ. The estimation procedure and

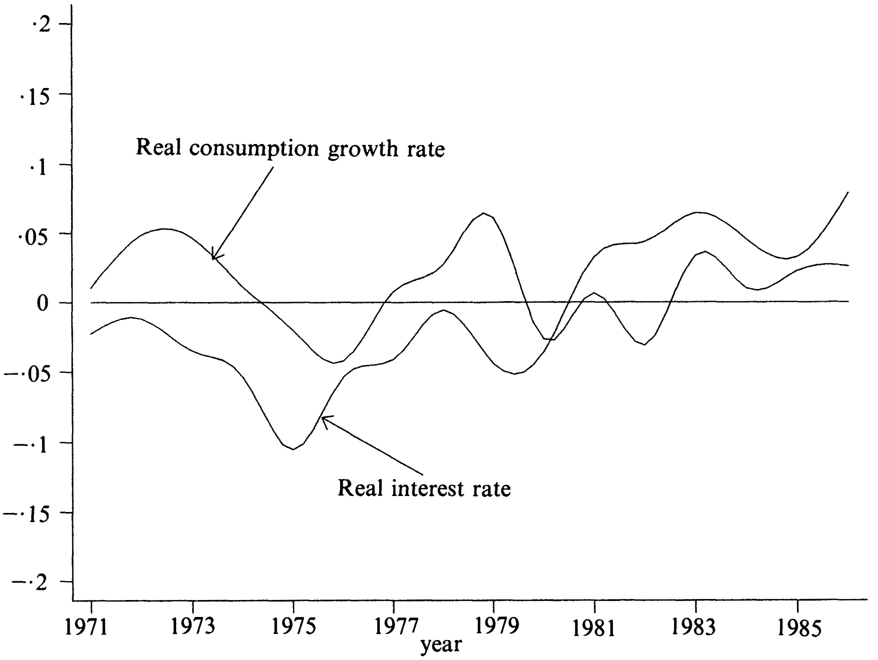

FIGURE 2

Real consumption growth and the "ex post" real interest rate

5. The consumption path in Figure 2 is for all cohorts averaged. In estimation we use the three-month treasury bill rate adjusted by the average tax rate. We use the Stone price index over our subset of commodities to construct the inflation rate with which the interest rate is deflated.

TABLE I

Price and income elasticities

Food	Alcohol	Clothing	Fuel	Transport	Services
Uncompensated price elasticities					
−0·619	−0·033	−0·068	0·019	−0·066	0·058
−0·229	−1·508	0·291	0·553	0·495	−0·215
−0·460	0·158	−0·454	−0·196	−0·331	−0·435
0·107	0·449	−0·159	−0·568	−0·672	−0·026
−0·110	0·203	−0·115	−0·329	−0·708	0·405
−0·311	−0·202	−0·455	−0·142	0·341	−0·901
Compensated price elasticities					
−0·368	0·015	0·006	0·081	0·062	0·143
0·082	−1·448	0·383	0·630	0·653	−0·110
0·023	0·252	−0·312	−0·076	−0·086	−0·271
0·330	0·492	−0·093	−0·513	−0·559	0·049
0·116	0·247	−0·048	−0·274	−0·593	0·481
0·429	−0·060	−0·237	0·040	0·717	−0·651
Average budget shares					
0·347	0·067	0·102	0·086	0·176	0·117
Income elasticities					
0·725	0·896	1·391	0·643	0·651	2·131

the computation of the asymptotic covariance matrix follows from Browning and Meghir (1991).

The complete specification, the parameter estimates and the list of instruments are all presented in Appendix A. There is a clear indication of the important role played by demographic and labour market variables in the allocation of within-period expenditures. It is also worth noting the significance of the β_i parameters which imply a rejection of homotheticity and hence the $b(p)$ index will vary with relative prices. The value of θ was estimated to be 0·54 with a standard error of 0·45. This would not reject the Almost Ideal model of Deaton and Muellbauer (1980*a*) but we have not imposed this restriction.

Table I presents the elasticities for our model with $\theta = 0{\cdot}54$. The overall results are sensible and conform with the usual findings in the literature: the budget shares for food, alcohol, fuel and transport decline with increasing total expenditure while clothing, services and other goods are luxuries. All own uncompensated and compensated price elasticities are negative and this is in fact true almost everywhere in the sample.[6]

3.3. *Cohort aggregation and the estimation of intertemporal substitution*

The estimation of the demand system provides estimates for the parameters that determine the two price indices $a(p_t)$ and $b(p_t)$ as well as the curvature parameter θ; in the estimation of the Euler equation which we now describe, we take these as given.[7] The main econometric problem here relates to the fact that we do not have repeated observations for any one individual: as noted above, our data consists of repeated cross-sections spanning the seventeen-year period 1970–1986. We thus use a grouping methodology used in Browning, Deaton and Irish (1985).

6. For a more detailed analysis of the intratemporal allocations on this data (albeit on shorter time periods) see Browning and Meghir (1991) and Blundell, Pashardes and Weber (1993).

7. In Appendix B we show how the standard errors for the Euler equation can be adjusted for the fact that we are conditioning on pre-estimated parameters. This adjustment made very little difference here.

The method relies on obtaining a random sample of the same date of birth cohort of individuals over time and modelling the average behaviour of the group. Thus consider the expected value of the Euler equation for consumption (2.10), conditional on a cohort and on time period t. This is

$$\Delta E[\ln \lambda_{it} \mid i \in c, t] + \ln(1 + r_{t-1}) + d_t = -E[\varepsilon_{it} \mid i \in c, t], \tag{3.1}$$

where i stands for individual, t for time period and c for a cohort and where the conditional expectation of the log marginal utility of wealth is

$$E[\ln \lambda_{it} \mid i \in c, t] = -t \ln(1+\delta) + E[\rho(z_{it}) \ln V_{it} \mid i \in c, t] + E[\ln(V'_{it}) \mid i \in c, t]. \tag{3.2}$$

The estimation procedure models the behaviour of the cohort mean of the log of the marginal utility of wealth. Thus we estimate the grouped Euler equation analog of (2.12). Using (2.13) we have

$$\Delta[(1+\theta)\ln C_t + \ln b(p_t)]_c - [i_{t-1}]_c = \textstyle\sum_k \rho_k \Delta[z_{kt}(\ln C_t^{\{-\theta\}} - \ln b(p_t))]_c + (d-\delta) + [\varepsilon_t]_c. \tag{3.3}$$

In the above, the terms enclosed in square brackets with a subscript "c" denote sample averages of these quantities within a date of birth and time period cell. The left-hand side is the average growth rate of consumption minus the average real interest rate adjusted for the price index $b(p)$.[8] The right-hand side involves the growth rate of the average within-cohort cross-product of the characteristics z_{kt} with $[\ln C_t^{\{-\theta\}} - \ln b(p_t)]$. The next set of terms involve differences in the average of some household characteristics which may enter the discount rate or the conditional variance term d_t. Finally $[\varepsilon_t]_c$ is the within-cohort average of the innovations.

In estimation we use 5-year cohorts observed annually. At this level of aggregation the cell is probably sufficiently large so as to reduce the importance of measurement error (see Deaton (1985)); details on the grouped data are given in Table II.

TABLE II

The structure of the grouped data

Cohort	Year of birth	Period of observation	Average cell size
1	1916-1920	1970-1975	518
2	1921-1925	1970-1980	601
3	1926-1930	1970-1985	522
4	1931-1935	1970-1986	500
5	1936-1940	1970-1986	512
6	1941-1945	1970-1986	555
7	1946-1950	1970-1986	591
8	1951-1955	1974-1986	479
9	1956-1960	1979-1986	405

For consistent estimation of the parameters of interest we need to impose certain conditions. First, our estimation procedure (both for the demand system and the Euler equation) assumes asymptotics on both N (individuals) and T (time periods). Given this, the average innovations $[\varepsilon_t]_c$ tend to zero unless there are common shocks affecting

8. The real interest rate $i_{t-1} = \ln(1 + r_{t-1}) - \Delta \ln a(p_t)$ is individual specific for two reasons. First, because of the individual specific nature of the inflation rate $\Delta \ln a(p_t)$ which depends on household characteristics. In addition each household is observed at a different point in time. The resulting nominal interest rate is a weighted average of the interest rates for the grouping period with the proportion of individuals observed at each point in time as weights.

all members of the cohort. Moreover, any idiosyncratic random preference errors entering through the discount factor at the individual level average out at the cohort level. Thus our only source of stochastic variation is cohort level shocks; these are assumed to average out over our 17 years of data.

The nature of the model is such that $[\varepsilon_t]_c$ will be correlated with consumption, prices and most choice variables dated t (unless they are subject to relatively long decision lags). Moreover time-aggregation can induce serial correlation and hence $[\varepsilon_t]_c$ may also be correlated with past values of the decision variables and prices. Hence we must use an instrumental variables procedure using appropriate lags as instruments. The instruments, listed below, involve the second lag of the growth rate of income and consumption, the interactions of the latter with characteristics as well as the characteristics themselves. The latter are either dated $t-2$ (labour market status) or dated t (all other demographic variables).[9]

Consistency requires the grouped instruments to be orthogonal to the grouped error term over time. This will be true if each member of the cohort has in his information set the average cohort values of the variables we use as instruments. If the individuals' information sets contain only values relating to themselves then this orthogonality condition will not be satisfied and cohort aggregation will not lead to consistent parameter estimates despite our exact aggregation procedure (see Pischke (1991)).

Finally, sufficient conditions for consistency of our instrumental variables procedure are

(a) $\text{plim}_{T\to\infty} \sum_t [q_t]_c[\varepsilon_t]_c/T = 0$, where $[q_t]_c$ is the cell average of the instrumental variables,

(b) $\text{plim}_{T\to\infty} \sum_t [q_t]_c X'_{tc}/T = M_{qx}$ where rank M_{qx} = rows of X_t and where $X_t = [\Delta[z_t'(\ln C^{\{-\theta\}} - \ln b(p_t))], \Delta[z_t]_c']'$ is the set of right-hand side variables in (3.3) above, and

(c) $\text{plim}_{T\to\infty} \sum_t [q_t]_c[q_t]_c'/T = M_{zz}$ where M_{zz} is full column rank.

The stationarity assumptions implicit in the above do not relate directly to any of the individual series of interest rate or prices but to functions of these variables. Hansen (1982) and Hansen and Singleton (1982) discuss the sufficient conditions for consistency in the estimation of Euler equations with long time series.

Given these conditions, the estimator for the unknown parameters in the Euler equation (ρ_k, $k = 0, \ldots, K$ and $(d-\delta)$) is

$$\hat{\gamma} = (X'Q(Q'\Omega Q)^{-1}Q'X)^{-1}X'Q(Q'\Omega Q)^{-1}Q'Y, \tag{3.4}$$

where Y is the left-hand side of (3.3), X is the collection of the right-hand side regressors and Q is the matrix of instruments. We replace $Q'\Omega Q$ by its estimate $\sum_{ct} [q_t]_c\hat{\varepsilon}^2_{ct}[q_t]_c'$ where $[\hat{\varepsilon}]_{ct}$ is an estimate of $[\varepsilon_t]_c$ from a first round unweighted IV regression (see White (1980)). The covariance matrix of the estimator is presented in Appendix B.

3.4. *Empirical results for the intertemporal consumption model*

In Table III we present estimates of three variants of our base model. These three models illustrate how demographics and labour supply variables interact in the Euler equation. To interpret the results note that the coefficients presented are those in (2.13); that is, $\rho_t = \rho_0 + \sum_k \rho_k z_{kt}$ where the variables z_{kt} are those listed in Table III. Lower values of ρ

9. Precisely, we construct the average cross-product of the characteristics with log real consumption within a cohort and then we take first differences of this measure. The second lag of this variable is then our instrument.

TABLE III

The Euler equation for consumption

	(1)	(2)	(3)
ρ_0	−2·369	−2·233	0·570
	(3·923)	(3·785)	(3·273)
HWORK	2·863	3·096	
	(1·457)	(1·439)	
WWORK	2·212	2·884	
	(1·500)	(1·082)	
K1	−0·872		−2·377
	(0·989)		(0·724)
NCHILD	0·466	0·478	0·326
	(0·219)	(0·206)	(0·212)
OWNER	0·091	−0·291	2·242
	(1·027)	(0·938)	(0·762)
SINGLE	4·280	3·700	5·931
	(4·339)	(3·694)	(2·702)
MULTIPLE	1·432	1·204	1·947
	(1·140)	(1·078)	(0·865)
$d-\delta$	0·011	0·021	−0·028
	(0·018)	(0·013)	(0·010)
Sargan	22·5 (14)	24·72 (15)	28·7 (16)
p-value	7%	5·4%	2·6%
Φ	−0·75	−0·77	−0·75
	(0·12)	(0·12)	(0·11)

Notes: Asymptotic standard errors in parentheses. Φ: Estimate of the intertemporal elasticity of substitution for consumption at the sample mean. HWORK: Head of household in paid employment, WWORK: Wife in paid employment, K1: Number of children 0–2 years of age, NCHILD: Total number of children, OWNER: Owner occupier, SINGLE: Single adult household, MULTIPLE: Multiple (>2) adult household. Sargan: Test of over-identifying restrictions (degrees of freedom in parentheses). Sample: 95 grouped observations.
Test for the joint significance of cohort dummies (χ^2_8): 12·3 p-value 12%.
Instruments.
Age, Cohort Dummies, K1, NCHILD, real interest rate lagged two years, two year lags of the first difference in the interaction of real expenditure with: K1, NCHILD, HWORK, WWORK, SINGLE, MULTIPLE, OWNER and the second lags of OWNER, real income growth and real consumption growth.

imply less intertemporal substitution for the reference household which is a childless couple living in rented accommodation with neither spouse in employment. The value for the intertemporal substitution elasticity, Φ, presented at the bottom of the table is the value given by equation (2.14) evaluated at the sample mean. In all cases we use instruments lagged two periods; the Sargan statistic is a test of the orthogonality conditions (or over-identifying restrictions) for these instruments.

The first column of Table III gives our most general base model; it includes both labour supply and demographic variables. As can be seen, the test of the over-identifying restrictions is borderline but the actual p-value is higher than for similar tests on aggregate data. The next two columns focus on the interaction between labour supply variables and the presence of an infant in the household. As is well known there is a strong correlation between the latter and female employment; the effect of this can be seen in columns 2 and 3. Excluding the labour supply variables increases the (absolute value) of the coefficient on infants a great deal but leaves the estimate of the ISE unchanged.

TABLE IV

Excess sensitivity tests

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ρ_0	−4·559	−2·185	−5·474	−2·360	8·479	7·563	10·070
	(4·123)	(3·741)	(3·562)	(2·832)	(1·776)	(2·137)	(1·29)
HWORK	2·066	2·182		3·430	0·462	0·331	
	(1·609)	(1·363)		(1·219)	(0·662)	(0·710)	
WWORK	2·252	2·253		3·660	1·501	1·510	
	(1·368)	(0·904)		(0·862)	(0·455)	(0·450)	
K1	−0·444		−1·616	0·740	0·007	0·044	
	(0·943)		(0·655)	(0·797)	(0·407)	(0·398)	
NCHILD	0·439		0·359	0·467	0·219	0·218	
	(0·198)		(0·157)	(0·188)	(0·089)	(0·087)	
OWNER	0·180	1·127	1·598	−0·353	0·160	0·229	
	(0·955)	(0·724)	(0·584)	(0·777)	(0·362)	(0·369)	
SINGLE	1·869	0·267	1·917	−0·723	0·577	0·122	
	(3·925)	(3·194)	(2·216)	(2·258)	(1·111)	(1·248)	
MULTIPLE	1·235	0·091	1·487	0·714	0·707	0·624	
	(1·124)	(0·760)	(0·779)	(0·898)	(0·401)	(0·402)	
INCG	0·545	0·276	1·115			0·110	0·320
	(0·368)	(0·397)	(0·305)			(0·172)	(0·102)
D1980					0·084	0·082	0·093
					(0·007)	(0·007)	(0·0056)
$d-\delta$	0·007	0·009	−0·025	0·041	−0·039	−0·039	−0·046
	(0·018)	(0·011)	(0·010)	(0·012)	(0·009)	(0·009)	(0·0039)
Sargan	25·6 (13)	29·0 (15)	28·4 (15)	25·7 (15)	9·2 (14)	9·6 (13)	26·4 (20)
p-value	2·6%	1·6%	1·9%	4%	82%	73%	15%
Φ	−0·64	−0·71	−0·55	−0·78	−1·21	−1·11	−1·17
	(0·11)	(0·14)	(0·07)	(0·09)	(0·13)	(0·17)	(0·12)

Notes: INCG: Growth rate of household income. D1980: Dummy equal to one pre 1981. See also notes for Table 3.
Instruments: Columns (1), (2) and (3) as in Table III. Columns (4), (5) and (6) include D1980.
Excess sensitivity test for model in column (4) 0·27 (0·48).

We prefer the model in the first column but since some will be sceptical of including labour supply variables in the Euler equation we present below estimates with and without them. The test statistic for the exclusion of the labour supply variables in column 1 has a p-value of 4·7%.

There are at least two good reasons why the labour supply variables in Table III might be significant. Within the maintained model the obvious explanation is that intertemporal allocation does indeed depend on labour force participation; for example, there are positive costs of going to work. An alternative explanation which lies outside the theoretical model is that consumption growth is correlated with anticipated income growth which is in turn correlated with labour force participation. To look at this issue of "excess sensitivity" we present some estimates in Table IV of models that include (instrumented) real income growth as an additional variable.[10]

Comparing the first column of Table IV with the first column of Table III we see that including anticipated income growth (the INCG variable) does not do very much; the extra variable is itself insignificant and the other parameter estimates are not much affected. The result that there is no evidence of excess sensitivity if we condition on demographics and labour supply has also been found by Attanasio and Browning (1991) and Attanasio and Weber (1993). Excluding the demographic variables as in column (2)

10. See Deaton (1987) and Hayashi (1987).

does little to change this result. If, however, we exclude the labour-supply variables as in column (3) then the coefficient on INCG does become "significant"; this reflects the strong correlation between these two sets of variables.

As shown in Figure 2, in 1981 the real interest rate jumped to a higher overall level at which it stayed for the remainder of the 1980s. Moreover, real consumption growth showed no corresponding increase, even allowing for a reasonable time lag. A possible interpretation is that the macroeconomic policies that triggered the increase in the interest rate also led to a fall in the conditional variance of income relative to the rate of time preference $(d-\delta)$. To investigate the effects of allowing for such an intercept shift we include a dummy (D1980) which takes the value 1 before 1981.[11] The results are presented in Table IV:[12] column (4) reproduces column (1) of Table III with the 1980 dummy included in the instruments. This does result in a shift of the parameters (with large effects on those most imprecisely estimated) but the main effect seems to be a large reduction in the standard error. However, the ISE at the mean point displays little change.

Including the 1980 dummy in the equation has a dramatic effect. First, the estimate of the ISE increases by about 50%; this is what we would expect given the differences before and after 1980 in Figure 2. Second, including the 1980 dummy changes the estimate of $(d-\delta)$; before it was constrained to be equal for the whole period and was estimated to be 0·041. Now it is 0·045 before 1981 and −0·039 after 1980. Third, the Sargan statistic is much lower; thus including the 1980 dummy clears up some of the correlation between consumption growth and lagged variables suggested by the Sargan statistics in Table III.

The inclusion of the 1980 dummy changes some of the other parameter estimates a good deal (compare columns 4 and 5 of Table IV). Having noted the impact of the 1980 dummy it is worth stressing that it has no effect on the significance of the anticipated income variable; compare the coefficient on INCG in columns 6 and 1 of Table IV. In view of the impact of the D1980 dummy, we present below elasticities derived from both specifications.

3.5. *Comparisons with a simple model*

This paper presents a number of innovations: we have allowed for intratemporal preferences when estimating preferences about allocation over time and we have allowed the latter to depend on demographics, labour supply variables and the level of consumption itself. We now consider the following question: how different would our conclusions be if we took an alternative simpler formulation and estimated them on our data?

We address this question in three steps. First, we look at how important it is to use the price indices derived from the demand system rather than some other *ad hoc* indices commonly used. In the second step we consider how much difference it makes to allow the ISE to vary with the level of consumption, given we allow it to vary with demographics and labour supply variables. Finally, we investigate how important it is to condition on the latter variables.

For the first step of our investigations we replace the $a(\cdot)$ price index in (2.12) by a weighted geometric mean of prices, where the weights are household-specific budget shares (that is, a household-specific Stone price index); this allows for some dependence of the deflator on family composition and relative prices (in so far as they affect budget

11. The asymptotics here require that the number of observations pre- and post-1980 is large enough to average out aggregate shocks within each sub period.

12. We postpone discussion of column 7 of Table IV until the next sub-section.

shares). To deal with the $b(\cdot)$ index we note that it depends only on relative prices so that it is likely to be of only second-order importance; we simply set it equal to unity for all households. Replacing the price indices in (2.12) in the way described above and setting θ to 0·54 leads to some minor changes in the parameter estimates (not reported) but the broad outlines of our results remain unchanged. The only change of note is that the coefficient on INCG (the expected income growth variable) increases and its standard error falls; not enough, however, to make the coefficient "significant".[13] From this we conclude that there is not much sensitivity to the precise form of the price indices. In particular $\Delta \ln b(p)$ is very small at the cohort level relative to the interest rate (see the consumption function (2.12)). Since $\Delta \ln b(p)$ is close to zero and, since the Stone price index at the cohort level is a good approximation of $\ln a(p)$, we conclude that all aspects of intertemporal substitution can be directly identified using total real consumption, at least for our data set.

Accepting that for our sample period we do not need to use more sophisticated price indices than those introduced in the last paragraph we now go on to investigate whether a simple iso-elastic form gives much the same results as our more complicated specification. To do this, we define consumption C_t as total nominal expenditure divided by our Stone price index and we estimate the following regression using the same instruments as before:

$$\Delta[\ln C_t]_c = \gamma_0[i_t]_c + \textstyle\sum_k \gamma_k \Delta[z_{tk} \ln C_t]_c + (d-\delta) + [u_t]_c$$

where the variables in square brackets are again cohort means. The implied ISE is $\Phi = -\gamma_0/(1-\sum_k \gamma_k z_k)$. The results of this regression are presented in Table V, columns 1 to 4. In all cases the estimated ISE is lower than the mean ISE for the corresponding specification; sometimes much lower (for example, compare columns 1 in Tables III and V). Thus it seems that if we constrain the ISE to be constant, the estimated value seriously underestimates the mean of the ISE when we use a flexible specification. Also of note is the fact that the 1980 dummy is less well-determined and there appears to be some significant excess sensitivity when we use this dummy. We conclude that constraining the ISE to be constant may lead to erroneous inference.

Our final experiment consists in dropping the conditioning variables. These results are reported in column 7 of Table IV for our specification and in columns 5 and 6 of Table V for the iso-elastic specification.

Comparing columns 6 and 7 of Table IV we see that dropping the labour supply variables and demographics does not change our estimate of the mean of the ISE very much. On the other hand, as expected, the coefficient on INCG is now significant. For the iso-elastic model we see that when we drop the conditioning variables the 1980 dummy becomes insignificant and there is strong evidence of excess sensitivity. Moreover for the first time in all our specifications the ISE is not well-determined (if we allow for the excess sensitivity). Referring back to the discussion at the end of Section 3.1 we see that this may explain why investigators using aggregate data or using micro data without controlling for demographics and labour supply have usually found only weak evidence of intertemporal substitution.

Our interpretation of these results is that in terms of estimating the ISE it may be misleading to use an iso-elastic specification. On the other hand, ignoring demographics and labour supply variables does not seem to lead to much bias in the estimate of the

13. A disadvantage of our Box-Cox transformation is that even when $\Delta \ln b(p)$ is zero the θ parameter remains identified from the Euler equation and hence its choice can affect even then the estimated ISE. In our experiments relating to the sensitivity to the choice of price indices we have just considered $\theta = 0{\cdot}54$ as implied by estimated curvature of the Engel curves. This retains comparability with our complete specification.

TABLE V

An isoelastic specification

	(1)	(2)	(3)	(4)	(5)	(6)
Comparison Table (col)	III (1)	IV (1)	IV (5)	IV (6)	IV (7)	—
Real rate	0·178	0·193	0·607	0·546	0·451	1·05
	(0·057)	(0·051)	(0·209)	(0·207)	(0·377)	(0·41)
HWORK	0·309	0·208	0·244	0·143		
	(0·070)	(0·096)	(0·070)	(0·081)		
WWORK	0·068	0·101	0·096	0·109		
	(0·094)	(0·084)	(0·070)	(0·064)		
K1	0·045	0·061	0·034	0·046		
	(0·061)	(0·056)	(0·056)	(0·050)		
NCHILD	0·034	0·023	0·038	0·024		
	(0·010)	(0·010)	(0·010)	(0·010)		
OWNER	−0·037	0·004	−0·026	0·013		
	(0·056)	(0·048)	(0·056)	(0·046)		
SINGLE	0·231	0·054	0·178	0·003		
	(0·262)	(0·283)	(0·188)	(0·182)		
MULTIPLE	0·024	0·067	0·140	0·068		
	(0·070)	(0·063)	(0·064)	(0·051)		
INCG		0·143		0·253	0·537	
		(0·142)		(0·120)	(0·196)	
D1980			0·051	0·042	0·035	0·102
			(0·023)	(0·023)	(0·041)	(0·043)
$d-\delta$	0·009	0·009	−0·017	−0·015	−0·019	0·046
	(0·007)	(0·001)	(0·018)	(0·017)	(0·021)	(0·023)
Sargan	14·6 (14)	11·4 (13)	13·8 (14)	10·0 (13)	0 (1)	4·2 (2)
p-value	40%	58%	46%	69%	99%	12%
Φ	−0·20	−0·21	−0·96	−0·73	−0·45	−1·05
	(0·065)	(0·056)	(0·36)	(0·21)	(0·38)	(0·41)

Notes: Dependent variable is real consumption growth deflated by a Stone price index defined on the included goods.
Instruments: Columns (1) and (2) as in Table IV including (INCG). Columns (3) and (4) include D1980, Columns (5) and (6) only include consumption growth, the real interest rate, real income growth all lagged two years and D1980.

mean of the ISE. Nevertheless, the ISE does vary systematically with these variables and the results of our excess sensitivity tests depend critically on whether we include such variables.

3.6. *The implications of our estimates for intertemporal substitution*

In Table VI we present the estimates of the ISE implied by three of our specifications above; the preferred model without the 1980 dummy (Table III, column 1) which we call model 1; the preferred model with the 1980 dummy (Table IV, column 5)—model 2—and the preferred iso-elastic model (Table V, column 4)—model 3.

Considering the top panel in Table VI we see that systematically the estimates from model 2 are higher (in absolute value) than those from model 1. For both models the important variable seems to be female participation: generally the ISE is higher if the wife is in employment.

The middle panel in Table VI gives the distribution of the ISE over our whole sample whereas the lower panel gives the distribution allowing consumption to vary but fixing all demographics and labour supply variables for each household at the sample mean.

TABLE VI*

Elasticities of intertemporal substitution (Φ)

(a) Elasticities at group means

WWORK	HWORK	OWNER	$C^{\{\theta\}}$	r(1)	(2)	Iso-elastic
0·67	0·89	0·58	14·65	−0·75 (0·12)	−1·21 (0·13)	−0·96 (0·36)
0	0	0	10·68	−0·61 (0·12)	−1·45 (0·32)	−0·70 (0·24)
0	0	1	12·75	−0·61 (0·11)	−1·22 (0·20)	−0·66 (0·22)
0	1	0	13·71	−0·69 (0·10)	−1·18 (0·14)	−0·94 (0·37)
0	1	1	15·49	−0·68 (0·09)	−1·08 (0·10)	−0·88 (0·33)
1	0	0	9·83	−0·82 (0·23)	−2·21 (0·82)	−0·84 (0·26)
1	0	1	12·74	−0·74 (0·16)	−1·43 (0·27)	−0·77 (0·24)
1	1	0	13·99	−0·81 (0·14)	−1·32 (0·16)	−1·10 (0·45)
1	1	1	15·75	−0·77 (0·12)	−1·19 (0·11)	−1·01 (0·39)

(b) The distribution of the elasticities over the entire sample

	M10	M25	M50	M75	M90	% positive
Φ (1)	−1·10	−0·81	−0·76	−0·70	−0·66	0%
Φ (2)	−2·8	−1·8	−1·4	−1·14	−0·96	5·0%

(c) The distribution of the elasticities at average household characteristics

	M10	M25	M50	M75	M90	% positive
Φ (1)	−0·85	−0·80	−0·77	−0·74	−0·72	0%
Φ(2)	−2·9	−1·9	−1·4	−1·13	−0·96	5·7%

Mi is the *i*-th percentile.

* *Notes*: Asymptotic standard errors in parentheses. Definitions of variables as in Table III. Column (1) and Φ (1) refer to the model in column (1) of Table III. Column (2) and Φ (2) refer to the model in column (5) of Table IV. "Iso-elastic" refers to the model in column (1) in Table V.

Three things stand out from these distributions. First, the estimates with the 1980 dummy are not only higher in mean but also much more spread out. Indeed, some of the estimates are positive which violates the concavity of the utility function. Second, the distributions are fairly similar across the two panels; this suggests that most of the variation in the ISE across the population is due to differences in consumption (which can loosely be thought of as a proxy for lifetime wealth) and not to differences in demographics and labour supply variables. This is consistent with the results reported in the last sub-section.

4. SUMMARY AND CONCLUSIONS

The life-cycle model plays an important part in our understanding of consumer behaviour. It provides a comprehensive framework for analyzing the relationship between intertemporal consumption and intratemporal expenditure allocations. This paper provides a rigorous analysis of the interaction between these two stages of the consumer's budgeting problem using a time-series of repeated cross-sections covering some 70,000 households in the U.K. over the 1970's and 1980's. We identify intertemporal behaviour using a pseudo-panel of grouped annual cohort data constructed from our repeated cross-section. Our principal conclusions can be summarized as follows:

(a) Although homotheticity is strongly rejected at the demand system level, a single price index is sufficient to describe intertemporal allocations in our sample. This is because the rate of change of the homogeneous-of-degree-zero price index $b(p)$, is in fact very small. Moreover using a Stone price index to deflate consumption gives very similar results to using the exact price index estimated from the within-period allocations.

(b) Allowing the intertemporal elasticity of substitution to vary with consumption, as opposed to using an iso-elastic specification, is important for the estimates. This extra degree of flexibility leads to higher estimates of the ISE. Moreover we find that the ISE is very well determined.

(c) An important innovation of our approach is the systematic analysis of the effects of demographic characteristics and labour market variables on intertemporal behaviour. We find that demographic characteristics and labour market variables have significant but relatively small effects on intertemporal allocations. This is true despite the fact that the price index deflating consumption is itself a function of these conditioning variables.

(d) In testing the validity of the life-cycle model we included the growth rate of income as an explanatory variable and performed standard excess sensitivity tests. Once we control for labour market status we find no excess sensitivity in our full specification although there is some evidence of excess sensitivity in the iso-elastic model.

The last of these findings is open to a variety of interpretations which cannot be distinguished convincingly within this framework. First, it is quite possible that the importance of labour market variables in the intertemporal model does in fact reflect shifting tastes as a function of labour market status; since labour market status and growth rate of income are obviously correlated ignoring the former makes the latter spuriously significant. For the same reasons the reverse is also possible: labour market status is a good predictor of income growth and thus labour market status may just capture excess sensitivity. Nevertheless, we should note that the labour market variables remain significant even in the presence of income growth.

DATA APPENDIX

A full description of the Family Expenditure Sata codes used to construct the variables is available from the authors on request.

TABLE D.1

Descriptive statistics for demographic characteristics and expenditures

Year	1970–1974	1975–1979	1980–1984	1985–1986
Female works	0·597	0·699	0·682	0·696
	(0·490)	(0·458)	(0·465)	(0·459)
Male works	0·931	0·900	0·832	0·797
	(0·252)	(0·299)	(0·373)	(0·402)
Married	0·827	0·792	0·757	0·711
	(0·378)	(0·405)	(0·428)	(0·452)
Age of head	40·843	40·308	40·273	39·732
	(11·00	(11·00)	(10·80)	(10·60)
Child 0–2	0·218	0·174	0·173	0·170
	(0·470)	(0·428)	(0·433)	(0·424)
Child 2–5	0·152	0·128	0·108	0·114
	(0·380)	(0·353)	(0·320)	(0·337)
Child 5–10	0·457	0·413	0·346	0·317
	(0·780)	(0·726)	(0·658)	(0·640)
Child 11–16	0·328	0·339	0·339	0·283
	(0·650)	(0·664)	(0·651)	(0·585)
Child 17–18	0·040	0·044	0·060	0·054
	(0·206)	(0·215)	(0·253)	(0·239)
Number of adults	2·083	2·054	2·042	1·997
	(0·648)	(0·680)	(0·710)	(0·740)
Expenditure (x)	31·436	63·655	113·643	142·404
	(20·40)	(42·40)	(76·40)	(103·0)
Share of food	0·364	0·352	0·321	0·307
	(0·122)	(0·126)	(0·120)	(0·117)
Share of alcohol	0·061	0·068	0·067	0·068
	(0·074)	(0·078)	(0·077)	(0·078)
Share of clothing	0·109	0·100	0·093	0·096
	(0·105)	(0·102)	(0·097)	(0·100)
Share of fuel	0·081	0·081	0·091	0·096
	(0·059)	(0·058)	(0·065)	(0·069)
Share of transport	0·169	0·170	0·185	0·173
	(0·142)	(0·140)	(0·148)	(0·140)
Share of services	0·108	0·113	0·126	0·139
	(0·096)	(0·102)	(0·109)	(0·119)

Sample standard deviations in parentheses

APPENDIX A. THE PARAMETERS OF THE ESTIMATED DEMAND SYSTEM (×100)

The demand system has the form

$$w_{ih} = \alpha_{ih} + \sum_j \gamma_{ij} \ln p_{jt} + \beta_{ih}(x/a(p))^{\{\theta\}} + v_{ih}$$

where

$$\alpha_{ih} = \alpha_{i0} + \sum_k z_{kh}\alpha_{ik}, \qquad \beta_{ih} = \beta_{i0} + \sum_k z_{kh}\beta_{ik}, \qquad \gamma_{ij} = \gamma_{ji}.$$

TABLE A

Parameters and standard errors (×100)

	Food	Alcohol	Clothing	Fuel	Transport	Services
γ parameters						
P Food	8·65					
	(1·31)					
P Alcohol	−1·87	−3.45				
	(0·81)	(1·00)				
P Clothing	−2·79	1·92	5·77			
	(1·20)	(0·89)	(1·49)			
P Fuel	−0·55	3·62	−1·50	3·32		
	(0·71)	(0·68)	(0·80)	(0·81)		
P Transport	−4·88	3·13	−2·31	−6·58	3·49	
	(1·43)	(1·22)	(1·51)	(1·04)	(3·04)	
P Services	2·68	−1·39	−4·72	−0·01	7·57	0·25
	(1·02)	(0·95)	(1·06)	(0·76)	(1·63)	(1·58)
α parameters						
Constant	46·39	4·60	8·64	18·33	6·08	3·93
	(3·95)	(3·29)	(4·30)	(2·35)	(2·26)	(4·27)
Quarter 1	9·89	−2·32	−2.81	1·57	−14·07	7·14
	(3·40)	(2·77)	(3·64)	(1·98)	(4·14)	(3·83)
Quarter 2	4·93	−2·02	0·14	−0.17	−6.32	1·46
	(3·46)	(2·81)	(3·69)	(2·01)	(4·18)	(3·87)
Quarter 3	12·13	1·22	−4·93	−0·52	−6.29	−1·00
	(3·46)	(2·82)	(3·70)	(2·01)	(3·98)	(3·85)
Oct and Nov	7·12	−3·55	−0·70	1·85	−11·26	6·81
	(3·53)	(2·88)	(3·78)	(2·05)	(4·10)	(3·93)
HWORK	−1.34	1·30	−1·47	0·35	5·82	−2·03
	(1·96)	(1·57)	(2·05)	(1·15)	(2·50)	(2·15)
WWORK	10·74	4·20	2·70	−9·25	15·83	−0·47
	(1·99)	(1·57)	(2·09)	(1·14)	(2·93)	(2·20)
Married	4·84	−5.14	−1.18	−1.34	4·86	−2·20
	(1·93)	(1·53)	(2·01)	(1·12)	(2·64)	(2·13)
Age	0·66	−0·73	0·02	0·02	−0·65	0·68
	(0·27)	(0·21)	(0·29)	(0·15)	(0·40)	(0·30)
Age^2	−0·27	−0·02	0·07	0·03	0·19	0·10
	(0·06)	(0·05)	(0·06)	(0·03)	(0·09)	(0·07)
K1	2·19	−1·09	0·31	0·94	−1·88	−0·59
	(0·10)	(0·08)	(0·10)	(0·06)	(0·15)	(0·11)
K2	2·22	−1·03	0·15	0·56	−1·48	−0·29
	(0·11)	(0·09)	(0·12)	(0·07)	(0·17)	(0·13)
K3	2·72	−0·95	0·20	0·25	−1·21	−0·91
	(0·07)	(0·05)	(0·07)	(0·04)	(0·10)	(0·07)
K4	2·76	−0·78	0·51	0·06	−0·86	−1·79
	(0·10)	(0·08)	(0·11)	(0·06)	(0·15)	(0·11)
K5	2·74	−1·10	0·44	0·16	−0·37	−1·77
	(0·23)	(0·18)	(0·24)	(0·14)	(0·33)	(0·26)
Number of Adults	1·92	2·50	0·26	−0·56	1·30	−4·55
	(0·28)	(0·22)	(0·29)	(0·17)	(0·40)	(0·31)

TABLE A—continued

	Food	Alcohol	Clothing	Fuel	Transport	Services
NORTH	−0·30	1·18	−0·14	−0·57	−0·56	0·17
	(0·20)	(0·16)	(0·21)	(0·11)	(0·30)	(0·22)
YORKSHIRE	0·30	0·34	−0·53	−0·51	−0·95	0·52
	(0·19)	(0·15)	(0·19)	(0·11)	(0·28)	(0·21)
EAST MIDLANDS	0·02	0·11	−0·80	−0·22	−0·83	0·59
	(0·19)	(0·15)	(0·20)	(0·11)	(0·29)	(0·21)
EAST ANGLIA	0·37	−1·10	−0·97	0·01	−0·61	0·43
	(0·23)	(0·18)	(0·24)	(0·13)	(0·35)	(0·26)
LONDON	0·84	−1·03	−0·80	−0·59	0·11	0·52
	(0·17)	(0·13)	(0·18)	(0·10)	(0·25)	(0·19)
SOUTH EAST	0·11	−1·47	−1·18	0·02	−0·09	0·54
	(0·17)	(0·14)	(0·18)	(0·10)	(0·26)	(0·19)
SOUTH WEST	0·63	−1·10	−0·90	−0.30	0·06	0·55
	(0·18)	(0·14)	(0·19)	(0·10)	(0·28)	(0·21)
WALES	0·76	−0·86	−0·56	−0·41	−0·51	0·20
	(0·18)	(0·14)	(0·19)	(0·10)	(0·27)	(0·20)
WEST MIDLANDS	0·04	−0·41	−0·66	−0·30	−0·48	0·52
	(0·19)	(0·15)	(0·20)	(0·11)	(0·29)	(0·21)
NORTH WEST	−0·13	0·28	−0·72	−0·02	−0·37	0·35
	(0·18)	(0·14)	(0·19)	(0·10)	(0·27)	(0·20)
COUNCIL HOUSE	0·88	1·38	−0·06	0·86	−1·63	−0·11
	(0·25)	(0·19)	(0·26)	(0·14)	(0·36)	(0·27)
RENT UNFURN	0·73	1·40	−0·00	0·01	−1·64	0·46
	(0·27)	(0·22)	(0·29)	(0·15)	(0·41)	(0·30)
RENT FURN	−0·54	2·78	0·94	−2·32	0·46	0·31
	(0·32)	(0·25)	(0·33)	(0·18)	(0·48)	(0·36)
OWNER MORT	0·10	0·55	0·38	0·55	−1·31	0·51
	(0·24)	(0·19)	(0·26)	(0·14)	(0·36)	(0·27)
OWNER NO MORT	0·56	0·17	0·56	0·32	−1·63	0·82
	(0·27)	(0·21)	(0·28)	(0·15)	(0·40)	(0·30)
CENT HEAT	−0·76	−0·39	0·21	0·13	0·54	0·34
	(0·10)	(0·08)	(0·10)	(0·06)	(0·15)	(0·11)
TREND	−3·70	0·68	1·26	0·50	−0·21	−0·45
	(0·56)	(0·45)	(0·66)	(0·36)	(0·75)	(0·55)
COH 1911-15	1·90	0·62	−3·39	3·29	−2·20	−0·17
	(1·40)	(1·10)	(1·46)	(0·79)	(2·06)	(1·56)
COH 1916-20	1·46	0·55	−3·02	2·80	−1·56	0·14
	(1·27)	(1·00)	(1·33)	(0·72)	(1·88)	(1·42)
COH 1921-25	1·46	0·32	−2·88	2·64	−1·07	0·07
	(1·16)	(0·91)	(1·21)	(0·66)	(1·70)	(1·29)
COH 1926-30	1·33	0·52	−2·70	2·46	−0·99	−0·24
	(1·04)	(0·82)	(1·09)	(0·59)	(1·54)	(1·16)
COH 1931-35	1·12	0·39	−2·60	2·33	−0·90	0·16
	(0·93)	(0·74)	(0·97)	(0·53)	(1·37)	(1·04)
COH 1936-40	0·94	0·50	−2·49	2·23	−0·97	0·24
	(0·83)	(0·65)	(0·86)	(0·47)	(1·22)	(0·92)
COH 1941-45	0·39	0·60	−2·24	2·06	−0·87	0·17
	(0·72)	(0·57)	(0·76)	(0·41)	(1·07)	(0·81)
COH 1946-50	0·34	0·44	−2·25	1·92	−0·91	0·40
	(0·64)	(0·50)	(0·66)	(0·36)	(0·95)	(0·71)
COH 1951-55	0·21	0·73	−2·01	1·39	−0·51	0·24
	(0·56)	(0·45)	(0·59)	(0·32)	(0·84)	(0·63)
COH 1956-60	−0·15	0·36	−1·29	1·14	−0·17	0·55
	(0·53)	(0·41)	(0·55)	(0·30)	(0·79)	(0·58)
WHITE COLLAR	−1·13	−0·80	0·26	0·06	−0·63	1·61
	(0·12)	(0·09)	(0·12)	(0·07)	(0·17)	(0·13)
HOUSE DURS	−1·43	−0·83	0·72	0·25	0·82	−0·28
	(0·22)	(0·18)	(0·24)	(0·13)	(0·33)	(0·25)
CARS	−3·25	−0·88	−1·74	−0·61	10·04	−2.28
	(0·17)	(0·13)	(0·17)	(0·10)	(0·24)	(0·18)

TABLE A—continued

	Food	Alcohol	Clothing	Fuel	Transport	Services
β parameters $(C = x/a(p))$						
C	−1·42	−0·18	0·61	−1·11	0·28	1·54
	(0·36)	(0·30)	(0·40)	(0·22)	(0·29)	(0·40)
CxQUAR1	−0·67	−0·03	−0·09	0·04	1·42	−0·28
	(0·30)	(0·25)	(0·33)	(0·18)	(0·37)	(0·34)
CxQUAR2	−0·26	−0·02	−0·27	0·15	0·70	0·25
	(0·31)	(0·25)	(0·33)	(0·18)	(0·37)	(0·34)
CxQUAR3	−0·93	−0·31	0·20	0·04	0·74	0·52
	(0·31)	(0·25)	(0·33)	(0·18)	(0·36)	(0·34)
Cx(OCT-NOV)	−0·49	0·13	−0·08	−0·18	1·13	−0·37
	(0·32)	(0·26)	(0·34)	(0·18)	(0·37)	(0·35)
CxWWORK	−0·01	−0·06	0·22	−0·11	−0·45	0·17
	(0·18)	(0·14)	(0·19)	(0·10)	(0·23)	(0·20)
CxHWORK	1·00	−0·23	−0·21	0·75	−1·52	−0·01
	(0·23)	(0·18)	(0·24)	(0·13)	(0·34)	(0·25)
CxMARRIED	−0·23	0·49	0·03	0·11	−0·50	0·10
	(0·20)	(0·16)	(0·21)	(0·12)	(0·28)	(0·22)
θ			0·54 (0·45)			

Notes: Asymptotic standard errors in parentheses. All parameters and standard errors are multiplied by 100. The parameters of the seventh commodity (OTHER) are implied by adding up. AGE = (AGE − 40)/10. Default region: Scotland. Default season: Christmas. Default housing: Rent Free.

Instruments: Seasonal dummies, regional dummies, cohort dummies, asset income, asset income squared, married, age and age squared of head of household, male and female employment status, child dummies, housing tenure dummies, number of adults, central heating dummy, occupation, durables and car ownership dummy, trend, trend squared and interactions of asset income with child dummies, employment status, married and seasonal dummies.

APPENDIX B. THE VARIANCE COVARIANCE MATRIX OF THE ESTIMATOR

We represent the Euler equation by the regression function

$$y(\zeta)_{it} = \rho' X(\zeta)_{it} + u_{it}$$

where we denote the parameter vector of the demand system by ζ and where

$$y(\zeta)_t = \Delta\left(\frac{1}{N_{ct}}\Sigma_{j\in c}[(1+\theta)(\ln x_{tj} - (\ln a(p|\zeta))_{tj}) + (\ln b(p|\zeta))_{tj}]\right) - [\ln(1+r_{t-1}) - \Delta(\ln a(p|\zeta))_{tj}], \tag{B.1}$$

$$X(\zeta)_{it} = \Delta\left(\frac{1}{N_{ct}}\Sigma_{j\in c}\, z^2_{itj}\{[\ln(x_{tj}/a(p|\zeta)_{tj})]^{\{-\theta\}} - \ln b(p|\zeta)_{tj}\}\right). \tag{B.2}$$

In the above c denotes a cohort, j an individual and t a time period. N_{ct} is the size of the cohort in time period t and z^2_{itj} is the i-th characteristic included in the Euler equation, for the j-th individual in the t-th time period.

To compute the asymptotic standard errors we need the derivatives of $y(\zeta)$ and $X(\zeta)$ with respect to ζ. We denote the conditioning characteristics in the demand system by some vector z^1. The non-zero elements of these derivatives have the following form:

$$\partial y(\zeta)_t/\partial\alpha_k = \theta\Delta\left(\frac{1}{N_{ct}}\Sigma_{j\in c}\, z^1_{tj}\ln p_{tk}\right), \tag{B.3}$$

$$\partial y(\zeta)_t/\partial\gamma_{sk} = \theta\Delta(\ln p_{ts}\ln p_{tk})\frac{1}{1+\delta_{ks}}, \tag{B.4}$$

$$\partial y(\zeta)_t/\partial\beta_k = \Delta\left(\frac{1}{N_{ct}}\Sigma_{j\in c}\, z^1_{tj}\ln p_{tk}\right), \tag{B.5}$$

$$\partial y(\zeta)_t/\partial\theta = \Delta\left(\frac{1}{N_{ct}}\sum_{j\in c}\ln C_{tk}\right), \tag{B.6}$$

$$\partial X(\zeta)_{it}/\partial\beta_k = -\Delta\left(\frac{1}{N_{ct}}\sum_{j\in c} z_{itj}^2 z_{tj}^1 \ln p_{tk}\right), \tag{B.7}$$

$$\partial X(\zeta)_{it}/\partial\alpha_k = -\Delta\left(\frac{1}{N_{ct}}\sum_{j\in c} z_{itj}^2 \frac{z_{tj}^1 \ln p_{tk}}{C_{tj}^{\{-\theta\}} a(p\,|\,\zeta)_{tj}(x_{tj}/a(p\,|\,\zeta)_{tj})^{\theta}}\right), \tag{B.8}$$

$$\partial X(\zeta)_{it}/\partial\gamma_{ks} = -\Delta\left(\frac{1}{N_{ct}}\sum_{j\in c} z_{tj}^2 \frac{\ln p_{ts}\ln p_{tk}}{C_{tj}^{\{-\theta\}} a(p\,|\,\zeta)_{tj}(x_{tj}/a(p\,|\,\zeta)_{tj})^{\theta}}\right)\frac{1}{1+\delta_{ks}}, \tag{B.9}$$

where δ_{ks} is the Kronecker-δ and k is the goods index. The vector of derivatives is completed by placing zeros at the correct positions.

The estimator of ζ has an asymptotic covariance matrix V_{ζ}. The error term of the model when we condition on consistent parameter estimates $\hat{\zeta}$ can be approximated to the first order by

$$u^* = u - [\partial X/\partial\zeta](\hat{\zeta}-\zeta) + [\partial y/\partial\zeta](\hat{\zeta}-\zeta) = u + Q(\hat{\zeta}-\zeta). \tag{B.10}$$

Hence $Eu^*(u^*)' = \Sigma + QV_{\zeta}Q'$.

Finally the asymptotic covariance matrix of the GMM estimator of ρ is

$$V_{\beta} = (X'P_{\Sigma}X)^{-1} + (X'P_{\Sigma}X)^{-1}X'P_{\Sigma}[QV_{\zeta}Q']P_{\Sigma}X(X'P_{\Sigma}X)^{-1}, \tag{B.11}$$

where $P_{\Sigma} = Z(Z'\Sigma Z)^{-1}Z'$, where Z is the matrix of instruments used for the estimation of the Euler equation. This covariance matrix is computed by replacing Σ by a diagonal matrix whose diagonal elements are the squared residuals from the second step GMM regression. (See White (1980)).

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