

# An Exogeneity Test for a Simultaneous Equation Tobit Model with an Application to Labor Supply

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#### NOTES AND COMMENTS

#### AN EXOGENEITY TEST FOR A SIMULTANEOUS EQUATION TOBIT MODEL WITH AN APPLICATION TO LABOR SUPPLY

#### BY RICHARD J. SMITH AND RICHARD W. BLUNDELL'

#### 1. INTRODUCTION

THE PURPOSE OF THIS PAPER is to provide a simple test of weak exogeneity (WE) in limited information simultaneous limited dependent variable (LDV) models. This test can also be viewed as one for simultaneous equation bias. In addition our procedure generates a consistent estimation method which offers a computationally attractive alternative to those previously proposed; see Amemiya [1, 2], Heckman [7], Nelson and Olsen [10]. The asymptotic covariance matrix of the suggested estimator is a relatively straightforward extension of the standard tobit formula. Although a simultaneous tobit model is considered here, a similar procedure may be constructed for probit, truncated, and other LDV models with appropriate adjustments to the formulae.

In Section 2 the extended regression framework of Hausman [6] and Holly and Sargan [8] is adapted to give a conditional maximum likelihood estimation method for the simultaneous tobit model which provides a theoretical justification for the procedure adopted by MacKinnon and Olewiler [9, p. 202]. The asymptotic variance-covariance matrix for the parameter estimators obtained from our estimation scheme is derived in Section 3 following the methodology of Amemiya [2] and the test of WE consequently described. Section 4 demonstrates the asymptotic optimality of the test and as a by-product gives the score statistic. Section 5 provides an empirical illustration and Section 6 presents our conclusions and main results.

#### 2. EXOGENEITY AND THE SIMULTANEOUS TOBIT MODEL

In this paper we consider the fo<sup>1</sup>lowing two equation simultaneous model:

(2.1) 
$$y_{1i}^* = y'_{2i}\gamma_1 + x'_{1i}\beta_1 + u_{1i}$$
,

$$(2.2) y'_{2i} = x'_i \Pi_2 + v'_{2i},$$

with

$$\begin{bmatrix} u_{1i} \\ v_{2i} \end{bmatrix} \sim NI\left(0, \begin{pmatrix} \sigma_1^2 & \sigma_{12}' \\ \sigma_{21} & \Sigma_{22} \end{pmatrix}\right) \quad \text{for} \quad i = 1, \dots, H,$$

where  $x'_i = (x'_{1i}, x'_{2i})$  is a vector of observations on  $K (= K_1 + K_2)$  maintained weakly exogenous variables; the usual identification assumptions are made. However, although the endogenous variable *G*-vector  $y_{2i}$  is continuously observed, observations on the endogenous variable  $y^*_{1i}$  are censored; we only observe

(2.3) 
$$y_{1i} = \begin{cases} y_{1i}^* & \text{if } y_{1i}^* > 0, \\ 0 & \text{otherwise.} \end{cases}$$
  $(i = 1, ..., H)$ 

Notice that we allow the structural form for  $y_{2i}$  to depend directly on  $y_{1i}^*$  but not on the observed variable  $y_{1i}$ ; cf. Heckman [7].

Writing  $u_{1i}$  conditional on  $v_{2i}$  as

$$u_{1i} = v_{2i}' \alpha + \varepsilon_{1i}$$

<sup>1</sup> We should like to thank the editor and referees for comments which have improved the paper's presentation and for providing the MacKinnon and Olewiler [9] reference. Richard Blundell's research was supported by the ESRC under Project D00230004. The data for the empirical illustration were provided by the ESRC Data Archive at the University of Essex with permission from the Department of Employment. Excellent research assistance was provided by Elisabeth Symons.

where  $\varepsilon_{1i} \sim N(0, \sigma_{11,2})$ ,  $\alpha = \sum_{22}^{-1} \sigma_{21}$ , the conditional variance  $\sigma_{11,2} = \sigma_1^2 - \sigma_{12}' \sum_{22}^{-1} \sigma_{21}$  and  $\varepsilon_{1i}$  is independent of  $y_{2i}$  and  $v_{2i}$ . Substitution for  $u_{1i}$  in (2.1) generates the conditional model:

(2.4)  $y_{1i}^* = y_{2i}'\gamma_1 + x_{1i}'\beta_1 + v_{2i}'\alpha + \varepsilon_{1i}$  $= w_i'\delta + \varepsilon_{1i}$ 

with  $w'_i = (y'_{2i}, x'_{1i}, v'_{2i})$  and  $\delta' = (\gamma'_1, \beta'_1, \alpha')$  for observations i = 1, ..., H; cf. Hausman [6], Holly and Sargan [8]. This reformulation of (2.1), (2.2) into (2.4), (2.2) corresponds to the factorization of the joint density of  $(y^*_{1i}, y'_{2i})$  into the conditional and marginal densities of  $y^*_{1i}$  and  $y_{2i}$  respectively. Notice that the one-to-one reparameterization  $(\sigma^2_1, \sigma'_{12}) \rightarrow (\sigma_{11,2}, \alpha')$  induces the following conditional censoring rule

(2.5) 
$$y_{1i} = \begin{cases} y_{1i}^* & \text{if } \varepsilon_{1i} > -w_i'\delta, \\ 0 & \text{otherwise,} \end{cases} \qquad (i = 1, \dots, H)$$

which replaces (2.3) above. The parameters characterizing the marginal density for  $y_{2i}$ ,  $\Pi_2$ , enter  $y_{1i}$ 's conditional density only through  $v_{2i}$ . Thus a sufficient condition for  $y_{2i}$  WE for  $(\gamma'_1, \beta'_1, \sigma_{11,2})$  is  $\alpha = 0$ , mirroring Engle, Hendry, and Richard [5, Theorem 4.3(b)]. However, if  $\Pi_2$  and  $(\gamma'_1, \beta'_1)$  are linked by additional restrictions, our test, which is constructed ignoring such restrictions, is only one for the independence of  $y_{2i}$  and  $u_{1i}$ .

The idea underlying this paper is to substitute a consistent estimator (with known asymptotic normal distribution) for  $\Pi_2$  in  $v'_{2i} = y'_{2i} - x'_i \Pi_2$ , namely  $\hat{\Pi}_2 = (X'X)^{-1}X'Y_2$ ,  $X' = (x_1, \ldots, x_H)$ ,  $Y'_2 = (y_{21}, \ldots, y_{2H})$ , and derive an estimator for  $\alpha$  in (2.4), (2.5). Thus we estimate

$$(2.4') y_{1i} = y'_{2i}\gamma_1 + x'_{1i}\beta_1 + \hat{v}'_{2i}\alpha + e_{1i} = \hat{w}'_i\delta + e_{1i}, (2.5') y_{1i} = \begin{cases} y_{1i}^* & \text{if } e_{1i} > -\hat{w}'_i\delta, \\ 0 & \text{otherwise,} \end{cases}$$

by standard tobit, similar to suggestions by Amemiya [1, Section 6] and MacKinnon and Olewiler [9, p. 202]. It is interesting to rewrite (2.4') as

$$y_{1i}^* = \hat{y}_{2i}' \gamma_1 + x_{1i}' \beta_1 + \hat{v}_{2i}' (\alpha + \gamma_1) + e_{1i}$$

since Nelson and Olsen [10] estimate this equation but with  $\hat{v}'_{2i}(\alpha + \gamma_1) + e_{1i}$  as the error term; the two approaches are respectively conditional and marginal ML methods. Although in the uncensored problem both our and Nelson and Olsen's procedure produce identical estimators for  $\gamma_1$  and  $\beta_1$  (Hausman [6]) this is not the case when censoring is present.

## 3. A TEST FOR EXOGENEITY AND AN ESTIMATION METHOD FOR THE SIMULTANEOUS TOBIT MODEL

Given the multivariate normality and independence of  $\varepsilon_{1i}$  and  $v_{2i}$  the log likelihood becomes, after concentrating out  $\Sigma_{22}$ ,

(3.1) 
$$\ln L = \sum_{i} \left\{ \xi_{i} \left( -\frac{1}{2} \ln \sigma_{11,2} - \frac{\varepsilon_{1i}^{2}}{2\sigma_{11,2}} \right) + (1 - \xi_{i}) \ln (1 - F_{i}) - \frac{1}{2} \ln \left( \sum_{j} v_{2j} v_{2j}' / H \right) \right\}$$
$$= \sum_{i} \left\{ \xi_{i} \ln F_{i} + (1 - \xi_{i}) \ln (1 - F_{i}) + \xi_{i} \left( -\frac{1}{2} \ln \sigma_{11,2} - \frac{\varepsilon_{1i}^{2}}{2\sigma_{11,2}} - \ln F_{i} \right) - \frac{1}{2} \ln \left( \sum_{j} v_{2j} v_{2j}' / H \right) \right\}$$

where  $\xi_i$  is a binary variable corresponding to (2.5) and

$$F_i = \int_{-\infty}^{w_i^*\delta} \frac{1}{\sqrt{2\pi\sigma_{11,2}}} \exp\left\{-\frac{\theta^2}{2\sigma_{11,2}}\right\} d\theta.$$

The first 2 terms correspond to the probit log likelihood and the third term to the truncated log likelihood for the conditional model (2.4, 2.5). Thus the following analysis may be suitably adapted for either of these models.

To derive a simple test of the weak exogeneity hypothesis  $H_0: \alpha = 0$ , consider the distribution of the maximum likelihood estimator of  $\lambda_1 = (\delta', \sigma_{11.2})$  given the estimator  $\hat{H}_2$ . Denoting this estimator as  $\hat{\lambda}_1$  and using the result of Amemiya [2, equation 3.27] we have

(3.2) 
$$\hat{\lambda}_1 - \lambda_1 \stackrel{a}{=} -\left\{ E \frac{\partial^2 \ln L}{\partial \lambda_1 \partial \lambda_1'} \right\}^{-1} \left\{ \frac{\partial \ln L}{\partial \lambda_1} + E \frac{\partial^2 \ln L}{\partial \lambda_1 \partial \pi_2'} \cdot (\hat{\pi}_2 - \pi_2) \right\},$$

where  $\pi_2 = \text{vec } \Pi_2$ . As all expectations in (3.2) are taken conditional on  $v_{2i}$ , the first expression on the right-hand side is the standard covariance matrix for the tobit maximum likelihood estimator of  $\lambda_1$  in the model (2.4), (2.5). That is,

(3.3) 
$$-E\frac{\partial^2 \ln L}{\partial \lambda_1 \partial \lambda_1'} = \boldsymbol{W}' \boldsymbol{A} \boldsymbol{W}$$

with

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{W} & \boldsymbol{O} \\ \boldsymbol{0} & \boldsymbol{\ell} \end{bmatrix}$$

where  $W' = (w_1, \ldots, w_H)$ , O and  $\ell$  are column vectors of zeros and ones respectively, and 0 is a  $H \times (K_1 + 2G)$  matrix of zeros, and where A is defined as in Amemiya [2, equation 3.21]; (3.3) may be consistently estimated by replacing  $v_{2i}$  by  $\hat{v}_{2i}$  and  $\lambda_1$  by a consistent estimator.

In the Appendix we show that the covariance between the two expressions in the second set of parentheses of (3.2) is zero and in addition:

(3.4) 
$$E\frac{\partial^2 \ln L}{\partial \lambda_1 \partial \pi'_2} = \alpha' \otimes W' A \begin{bmatrix} I_H \\ \cdots \\ 0 \end{bmatrix} X.$$

From these results we conclude that:

(3.5) 
$$\hat{\lambda}_1 - \lambda_1 \stackrel{a}{\sim} N(0, V(\hat{\lambda}_1))$$

with

(3.6) 
$$V(\hat{\lambda}_1) \stackrel{a}{=} (W'AW)^{-1} + (W'AW)^{-1} \cdot \alpha' \otimes W'A \begin{bmatrix} I_H \\ \cdots \\ 0 \end{bmatrix} X \cdot V(\hat{\pi}_2) \cdot \alpha$$

$$\otimes X'(I_H: 0)AW(W'AW)^{-1}$$

$$\stackrel{a}{=} (\boldsymbol{W}'\boldsymbol{A}\boldsymbol{W})^{-1} + \alpha'\boldsymbol{\Sigma}_{22}\alpha(\boldsymbol{W}'\boldsymbol{A}\boldsymbol{W})^{-1}\boldsymbol{W}'\boldsymbol{A}\begin{bmatrix}\boldsymbol{I}_{H}\\ \vdots\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

$$\times X'(I_H: 0)AW(W'AW)^{-1}$$

replacing  $V(\hat{\pi}_2)$  by  $\Sigma_{22} \otimes (X'X)^{-1}$ .

Note that under the WE hypothesis  $H_0$ ,  $V(\hat{\lambda}_1)$  collapses to the standard tobit covariance matrix which is the inverse of (3.3), and thus the tobit estimator for  $\alpha$  in the estimated conditional model (2.4'), (2.5') provides the required test of  $H_0$ .

#### 4. ASYMPTOTIC OPTIMALITY OF THE TEST

Partitioning  $y'_i = (y_{1i}, y'_{2i})$ , we write the sample log likelihood for convenience as:

(4.1) 
$$\ln L = \ln L_{12}(y_1; \lambda_1, \Pi_2 | y_2) + \ln L_2(y_2; \Pi_2),$$

where  $L_{12}$  and  $L_2$  are the conditional and marginal likelihood functions for  $y_1$  and  $y_2$  respectively. As  $\alpha = 0$  is sufficient for  $y_2$  WE for  $(\gamma'_1, \beta'_1, \sigma_{11,2})$  and the structure of the information matrix under  $H_0$  is preserved under local alternatives, we may state the following proposition.

**PROPOSITION:** Under local alternatives  $H_{1H}$ :  $\alpha_H = H^{-1/2} \alpha$ ,

$$H^{1/2}(\hat{\lambda}_1 - \lambda_{1H}) \stackrel{a}{\sim} N(0, \mathscr{I}^{-1})$$

with  $\lambda'_{1H} = (\gamma'_1, \beta'_1, \alpha'_H, \sigma_{11.2})$  and  $\mathcal{I} = -\lim_{H \to \infty} (1/H) E\{\partial^2 \ln L_{12}/\partial \lambda_1 \partial \lambda'_1\}$  evaluated at  $\lambda'_1 = (\gamma'_1, \beta'_1, 0', \sigma_{11.2}).$ 

Partitioning  $\mathcal{I}$  conformably with  $(\gamma'_1, \beta'_1, \sigma_{11,2})$  and  $\alpha$ , equation (3.2) gives under  $H_{1H}$  with a suitable reordering of  $\lambda_1$ ,

(4.2) 
$$H^{1/2}\hat{\alpha} \stackrel{a}{=} H^{-1/2} \mathscr{I}^{22} (-\mathscr{I}_{21} \mathscr{I}_{11}^{-1}, I) \frac{\partial \ln L_{12}}{\partial \lambda_1}$$
$$\stackrel{a}{=} H^{-1/2} \mathscr{I}^{22} \frac{\partial \ln \tilde{L}_{12}}{\partial \alpha}$$

from standard maximum likelihood theory, where  $\tilde{.}$  denotes evaluation at the  $H_0$  MLE  $\lambda_1$ , i.e. the maximizer of  $\ln L_{12}(y_1; \lambda_1/y_2)$ . Note that the choice of  $\Pi_2$  estimator plays no role under  $H_{1H}$ ; cf. Smith [11]. We summarize in the following Theorem.

THEOREM: The test of Section 3 using  $\hat{\alpha}$  is: (i) asymptotically equivalent to the score test for  $y_2$  WE for  $(\gamma'_1, \beta'_1, \sigma_{11,2})$  under  $H_{1H}$ , (ii) an asymptotically optimal test for  $y_2$  WE for  $(\gamma'_1, \beta'_1, \sigma_{11,2})$ .

From the Proposition and (4.2), the score test for  $H_0$  is

$$rac{\partial \ln \tilde{L}_{12}}{\partial lpha'} S'_{lpha} ( ilde{m{W}}' ilde{m{A}} ilde{m{W}})^{-1} S_{lpha} rac{\partial \ln \tilde{L}_{12}}{\partial lpha}$$

as, under  $H_0$ ,  $H^{1/2}\hat{\alpha} \stackrel{a}{\sim} N(0, \lim_{H \to \infty} HS'_{\alpha}(WAW)^{-1}S_{\alpha})$ , where  $S_{\alpha}$  selects out  $\alpha$  from  $\lambda_1$ , the score  $\partial \ln L_{12}/\partial \alpha$  is given in the Appendix,  $v_{2i}$  is evaluated at the  $H_0$  MLE  $\hat{H}_2$ , and  $(\gamma'_1, \beta'_1, \sigma_{11,2})$  is estimated by standard tobit on (2.1). Alternatively the score test may be interpreted as an information matrix test; see Chesher [4].

#### 5. AN APPLICATION

This illustration of the procedure developed in previous sections considers the determination of female labor supply in a random sample of 1909 married couples from the 1981 UK Family Expenditure Survey. A standard female labor supply equation is specified which has, as one of its determinants, other household income. Although this income variable may be a choice variable for the household, this does not imply that it cannot be treated as (weakly) exogenous in the estimation of the parameters of the labor supply equation. For the purposes of this illustration, the other determinants of female labor supply are maintained (weakly) exogenous. In row (1) of Table I we present the estimates of the parameters  $\gamma_1$ ,  $\beta_1$  together with their standard errors under the restriction  $\alpha = 0$ . The variable  $\mu$  is a measure of other household income variable and contains both husband's earned income and household saving as described in Blundell and Walker [3]. The variables  $a_f$  and  $a_f^2$  refer to female age and age squared,  $e_f$  and  $e_f^2$  to female education and education squared, and  $D_1$ ,  $D_2$  and  $D_3$  to dummy variables indicating the presence of the youngest child in three age ranges (0-4), (5-10), (11-). Precise definitions are available from the authors on request.

Row (2) provides corresponding parameter estimates for  $\alpha \neq 0$  but where standard errors are calculated using the standard tobit expression in (3.3). Apart from the variables described above, male occupation dummies, housing tenure dummies, and regional unemployment rates were used in the reduced form for other household income. The parameter estimates have plausible signs and are generally well determined with the coefficient on the residual  $\hat{v}_2$ , an estimate of  $\alpha$ , being significantly different from zero. That is we can reject the null hypothesis of weak exogeneity of other income for this particular specification of female labor supply.

#### 6. SUMMARY AND CONCLUSIONS

We propose a simple asymptotically optimal test for weak exogeneity in the simultaneous equation tobit model, which may be calculated using any standard tobit computer package as a test for the exclusion of the residual vector obtained from an auxiliary regression for the hypothesized weakly exogenous variables. We also give the score statistic. The estimation procedure provides consistent estimators under the alternative, whose asymptotic variance-covariance matrix is a relatively simple extension of the usual tobit formula.

We illustrate our procedure using a female labor supply equation estimated from a random sample of married women from the 1981 Family Expenditure Survey for the UK; the hypothesis of weak exogeneity of other household income for the parameters of the female labor supply equation was rejected. In applications  $y_{2i}$  may appear nonlinearly in (2.1) as

$$y_{1i}^* = g(y_{2i}, x_{1i}, \theta) + u_{1i};$$

we would merely replace the estimated conditional model (2.4') by

$$y_{1i}^* = g(y_{2i}, x_{1i}, \theta) + \hat{v}_{2i}' \alpha + e_{1i}.$$

The test procedures are easily adapted for probit, truncated, and other limited dependent variable models such as considered by Chesher [4] by appropriate algebraic changes. The

Variable	Constant	μ	$a_f$	$a_f^2$	$e_f$	$e_f^2$
(1)	27.4202	-0.1643	-3.3362	-1.6332	2.2690	-0.1728
	(0.7404)	(0.0078)	(0.2432)	(0.2953)	(0.1434)	(0.0238)
(2)	25.0927	-0.0723	-4.3191	-1.5218	1.9761	-0.1537
	(0.8359)	(0.0094)	(0.3453)	(0.2781)	(0.1253)	(0.0210)
Variable	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	$\hat{v}_2$		
(1)	-30.8914	-11.1445	-2.1794			
	(1.2143)	(0.8089)	(0.8448)			
(2)	-34.5603	-14.1243	-4.7431	-0.0965		
	(0.9032)	(0.9521)	(0.8071)	(0.0143)		

TABLE I

1981 Parameter Estimates and Standard Errors

above results only apply to limited dependent variable models based on the normal distribution.

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#### APPENDIX

All expectations are taken conditional on  $v_{2i}$  so that

$$\operatorname{cov}\left(\hat{\pi}_2 - \pi_2, \frac{\partial \ln L}{\partial \lambda_1}\right) = 0$$

as  $E \partial \ln L/\partial \lambda_1 = 0$  and  $(\hat{\pi}_2 - \pi_2) = (I \otimes (X'X)^{-1}X')$  vec  $V_2$ . From the concentrated log likelihood we have

$$\frac{\partial \ln L}{\partial \pi_2} = \operatorname{vec} \left\{ X' V_2 (V_2' V_2 / H)^{-1} \right\} + \alpha \bigotimes_i \left\{ \frac{f_i}{1 - F_i} (1 - \xi_i) - \frac{1}{\sigma_{11,2}} \varepsilon_{1i} \xi_i \right\} x_i$$
$$\frac{\partial \ln L}{\partial \delta} = -\sum_i \left\{ \frac{f_i}{1 - F_i} (1 - \xi_i) w_i - \frac{1}{\sigma_{11,2}} \varepsilon_{1i} \xi_i w_i \right\}$$

and

$$\frac{\partial \ln L}{\partial \sigma_{11,2}} = \frac{1}{2\sigma_{11,2}} \sum_{i} \left\{ \frac{w_i' \delta f_i(1-\xi_i)}{1-F_i} - \xi_i + \frac{1}{\sigma_{11,2}} \varepsilon_{11}^2 \xi_i \right\}.$$

In particular

$$\frac{\partial \ln L_{12}}{\partial \alpha} = \sum_{i} v_{2i} \varepsilon_{1i}^{(1)}$$

where

$$\varepsilon_{1i}^{(1)} = E(\varepsilon_{1i}/y_{1i}) = \frac{1}{\sigma_{11,2}} \varepsilon_{1i}\xi_i - \frac{f_i}{1 - F_i} (1 - \xi_i).$$

So that the expected second derivatives are given by, writing  $\Pi_2 = (\Pi_{21}, \ldots, \Pi_{2G})$ ,

$$E \frac{\partial^2 \ln L}{\partial \delta \ \partial \Pi'_{2j}} = \alpha_j \frac{1}{\sigma_{11,2}} \sum_i \left\{ w'_i \delta f_i - \sigma_{11,2} \frac{f_i^2}{(1 - F_i)} - F_i \right\} w_i x'_i$$
  

$$= -\alpha_j W' A_{11} X \qquad (j = 1, \dots, G),$$
  

$$E \frac{\partial^2 \ln L}{\partial \sigma_{11,2}} = \frac{\alpha_j}{2\sigma_{11,2}^2} \sum_i \left\{ (w'_i \delta)^2 f_i + \sigma_{11,2} f_i - \sigma_{11,2} \frac{w'_i \delta f_i^2}{(1 - F_i)} \right\} x'_i$$
  

$$= -\alpha_j l' A_{12} X \qquad (j = 1, \dots, G),$$
  

$$E \frac{\partial^2 \ln L}{\partial \lambda_1 \partial \pi'_2} = -\alpha' \otimes W' A \begin{bmatrix} I_H \\ \cdots \\ 0 \end{bmatrix} X,$$

÷

as  $\pi'_2 = (\Pi'_{21}, \ldots, \Pi'_{2G}).$ 

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## <sup>5</sup>Exogeneity

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**NOTE:** The reference numbering from the original has been maintained in this citation list.



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