Stochastic Demand and Revealed Preference

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- Objective is to uncover demand responses from consumer expenditure survey data.
- Inequality restrictions from revealed preference are used to improve the performance of nonparametric estimates of demand responses.
- Particular attention is given to nonseparable unobserved heterogeneity and endogeneity.

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- Derive welfare costs of relative price and tax changes.

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where demand functions $\mathbf{d}(x, \mathbf{p}, \mathbf{h}, \varepsilon) : \mathbb{R}^K_{++} \to \mathbb{R}^J_{++}$ satisfy adding-up: $\mathbf{p}'\mathbf{q} = x$ for all prices and total outlays $x \in \mathbb{R}$.

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- Will typically suppress observable heterogeneity $\mathbf{h}$ in what follows.

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- Here we consider the case of a small number of price regimes and use revealed preference inequalities applied to $\mathbf{d}(x, \mathbf{p}, \varepsilon)$ to improve demand predictions
- In other related work **Slutsky inequality** conditions have been shown to help in 'smoothing' demands for 'dense' or continuously distributed prices

Related Literature

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- **Inequality constraints and set identification:** Andrews (1999, 2001); Andrews and Guggenberger (2007), Andrews and Soares (2009); Bugni (2009); Chernozhukov, Hong and Tamer (2007)....

Revealed Preference and Expansion Paths

Suppose we have a **discrete price distribution,** $\{\mathbf{p}(1), \mathbf{p}(2), ...\mathbf{p}(T)\}$.

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- In this case **Revealed Preference** conditions, in general, only allow **set identification** of demands.

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- $\equiv$ if $\mathbf{q}^t$ is indirectly revealed preferred to $\mathbf{q}^s$ then $\mathbf{q}^s$ is not strictly preferred to $\mathbf{q}^t$
- $\exists$ a well behaved concave utility function $\equiv$ the data satisfy GARP

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- Blundell, Browning and Crawford (Ecta, 2003) develop a method for choosing a sequence of total expenditures that maximise the power of tests of RP (GARP).
- Define sequential maximum power (SMP) path

$$\{\tilde{x}_s, \tilde{x}_t, \tilde{x}_u, ...\tilde{x}_v, x_w\} = \{\mathbf{p}_s'\mathbf{q}_t(\tilde{x}_t), \mathbf{p}_t'\mathbf{q}_u(\tilde{x}_u), \mathbf{p}_v'\mathbf{q}_w(\tilde{x}_w), x_w\}$$

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- **Proposition (BBC, 2003)** Suppose that the sequence

$$\{\mathbf{q}_s(x_s), \mathbf{q}_t(x_t), \mathbf{q}_u(x_u) ..., \mathbf{q}_v(x_v), \mathbf{q}_w(x_w)\}$$

  rejects RP. Then SMP path also rejects RP. (Also define Revealed Worse and Revealed Best sets.)

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- use this information alone, together with revealed preference theory to assess consumer rationality and to place 'tight' bounds on demand responses and welfare measures.

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- **Fig 1b**

Support Sets and Bounds on Demand Responses:

- Suppose we observe a set of demands $\{\mathbf{q}_1, \mathbf{q}_2, ...\mathbf{q}_T\}$ which record the choices made by a particular consumer (ε) when faced by the set of prices $\{\mathbf{p}_1, \mathbf{p}_2, ...\mathbf{p}_T\}$.

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- **Figure 2(a)** - generating a support set: $S^V(\mathbf{p}_0, x_0, \varepsilon)$ for consumer of type ε

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- Blundell, Browning and Crawford (2008): The set of points that are consistent with observed expansion paths *and* revealed preference is given by the *support set*:

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- **Figure 2b, c** - $S(\mathbf{p}_0, x_0, \varepsilon)$ the identified *set* of demand responses for $\mathbf{p}_0, x_0, \varepsilon$ given $t = 1, ..., T$.

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- *Observed variables* (ignoring other observed characteristics of consumers):

$$\begin{aligned} \mathbf{p}(t) &= \text{prices that all consumers face,} \\ \mathbf{q}_i(t) &= (q_{1,i}(t), q_{2,i}(t)) = \text{consumer } i\text{'s demand,} \\ x_i(t) &= \text{consumer } i\text{'s income (total budget)} \end{aligned}$$

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- We first wish to recover demands for each of the observed price regimes t,

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- We will here only discuss the case of 2 goods with 1-dimensional error:

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$$\mathbf{d}(x(t), t, \varepsilon) = (d_1(x(t), t, \varepsilon), d_2(x(t), t, \varepsilon)).$$
- Given t, $d_1(x(t), t, \varepsilon)$ is exactly the quantile expansion path (Engel curve) for good 1 at prices $\mathbf{p}(t)$.

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- **Assumption A.1:** The variable $x(t)$ has bounded support, $x(t) \in \mathcal{X} = [a, b]$ for $-\infty < a < b < +\infty$, and is independent of $\varepsilon \sim U[0, 1]$.

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- **Identification Result:** $d_1(x, t, \tau)$ is identified as the τth quantile of $q_1 | x(t)$:

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- Thus, we can employ standard nonparametric quantile regression techniques to estimate d_1.

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be the check function used in quantile estimation.
- The budget constraint defines the path for d_2. We let $\mathcal{D}$ be the set of feasible demand functions,

$$\mathcal{D} = \left\{ \mathbf{d} \geq 0 : d_1 \in \mathcal{D}_1,\ d_2(x, t, \tau) = \frac{x - p_1(t) d_1(x, t, \varepsilon(t))}{p_2(t)} \right\}.$$

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- Let $(\mathbf{q}_i(t), x_i(t))$, $i = 1, ..., n$, $t = 1, ..., T$, be i.i.d. observations from a demand system, $\mathbf{q}_i(t) = (q_{1i}(t), q_{2i}(t))'$.

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- We then estimate $\mathbf{d}(t, \cdot, \tau)$ by

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- Then $\hat{d}_1(x, t, \tau) = \sum_{k \in \mathcal{K}_n} \hat{\pi}_k(t, \tau) B_k(x)$, where $\hat{\pi}_k(t, \tau)$ is a standard linear quantile regression estimator:

$$\hat{\pi}(t, \tau) = \arg\min_{\pi \in \mathbb{R}^{|\mathcal{K}_n|}} \frac{1}{n} \sum_{i=1}^{n} \rho_\tau (q_{1i}(t) - \pi' \mathbf{B}_i(t)), \quad t = 1, ..., T.$$

Unrestricted Demand Estimation

- Adapt results in Belloni, Chen, Chernozhukov and Liao (2010) for rates and asymptotic distribution of the linear sieve estimator:

$$||\hat{\mathbf{d}}(\cdot, t, \tau) - \mathbf{d}(\cdot, t, \tau)||_2 = O_P\left(n^{-m/(2m+1)}\right),$$

$$\sqrt{n}\Sigma_n^{-1/2}(x, \tau)\left(\hat{d}_1(x, t, \tau) - d_1(x, t, \tau)\right) \to^d N(0, 1),$$

where $\Sigma_n(x, \tau) \to \infty$ is an appropriate chosen weighting matrix.

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- Since RP imposes restrictions across t, the above estimation problem can no longer be split up into T individual sub problems as the unconstrained case.

RP-restricted Demand Estimation

- **Theoretical properties of restricted estimator:** In general, the RP restrictions will be binding. This means that $\hat{\mathbf{d}}_C$ will be on the boundary of $\mathcal{D}_{C,n}^T$. So the estimator will in general have non-standard distribution (estimation when parameter is on the boundary).

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- Redefine the constrained estimator to be the optimizer over $\mathcal{D}_{C,n}^T(\varepsilon) \supset \mathcal{D}_{C,n}^T$.

- Under assumptions **A1-A3** and that $\mathbf{d}_0 \in \mathcal{D}_C^T$, then for any $\epsilon > 0$:

$$||\hat{\mathbf{d}}_C^{\epsilon}(\cdot, t, \tau) - \mathbf{d}_0(\cdot, t, \tau)||_\infty = O_P(k_n/\sqrt{n}) + O_P\left(k_n^{-m}\right),$$

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- Also derive convergence rates and valid confidence sets for the support sets.
- In practice, use simulation methods or the modified bootstrap procedures developed in Bugni (2009, 2010) and Andrews and Soares (2010); alternatively, the subsampling procedure of CHT.

Demand Bounds Estimation

- Simulation Study: Cobb-Douglas demand function.

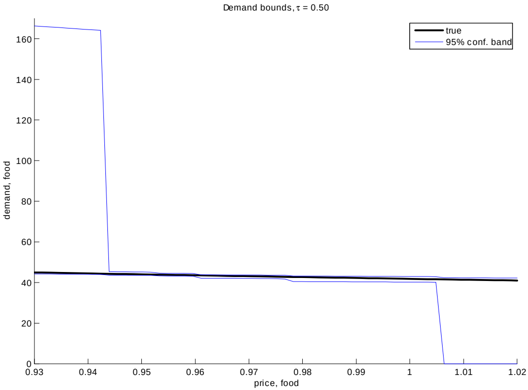

Figure: Performance of demand bound estimator.

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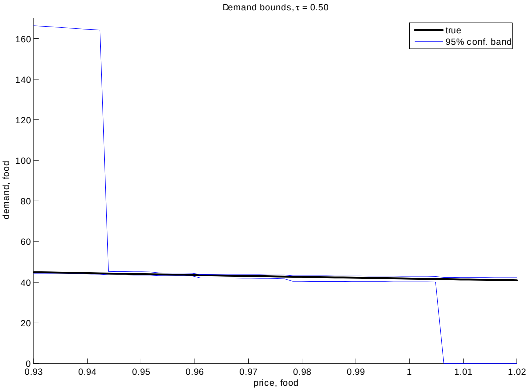
- 95% confidence bands of demand bounds.



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- We wish to **test the null of consumer rationality**.
- Let $\mathbb{S}_{\mathbf{p}_0,x_0}$ denote the set of demand sequences that are rational given prices and income:

$$\mathbb{S}_{\mathbf{p}_0,x_0} = \left\{ \mathbf{q} \in \mathcal{B}^T_{\mathbf{p}_0,x_0} : \begin{array}{c} \exists V > 0, \lambda \geq 1 : \\ V(t) - V(s) \geq \lambda(t)\,\mathbf{p}(t)'(\mathbf{q}(s) - \mathbf{q}(t)) \end{array} \right\}.$$

Testing for Rationality

- **Test statistic:** Given the vector of *unrestricted* estimated intersection demands, $\widehat{\mathbf{q}}$, we compute its distance from $\mathbb{S}_{\mathbf{p}_0,x_0}$:

$$\rho_n\left(\widehat{\mathbf{q}},\mathbb{S}_{\mathbf{p}_0,x_0}\right) := \inf_{\mathbf{q}\in\mathbb{S}_{\mathbf{p}_0,x_0}} \left\|\widehat{\mathbf{q}}-\mathbf{q}\right\|^2_{\hat{W}_n^{\text{test}}},$$

where $\|\cdot\|_{\hat{W}_n^{\text{test}}}$ is a weighted Euclidean norm,

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- **Distribution under null:** Using Andrews (1999,2001),

$$\rho_n\left(\widehat{\mathbf{q}},\mathbb{S}_{\mathbf{p}_0,x_0}\right) \to^d \rho\left(Z,\Lambda_{\mathbf{p}_0,x_0}\right) := \inf_{\lambda\in\Lambda_{\mathbf{p}_0,x_0}} \left\|\lambda - Z\right\|^2,$$

where $\Lambda_{\mathbf{p}_0,x_0}$ is a cone that locally approximates $\mathbb{S}_{\mathbf{p}_0,x_0}$ and $Z \sim N(0, I_T)$.

Estimating e-Bounds on Local Consumer Responses

- For each household defined by (x, ε), the parameter of interest is the consumer response at some new relative price $\mathbf{p}_0$ and income x or at some sequence of relative prices. The later defines the demand curve for (x, ε).

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Estimation

- In the estimation, we use log-transforms and polynomial splines

$$
\begin{aligned}
\log d_{1,n}(\log x, t, \tau) &= \sum_{j=0}^{q_n} \pi_j (t, \tau) (\log x)^j \\
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- In the implementation of the quantile sieve estimator with a small penalization term was added to the objective function, as in BCK (2007).

Unrestricted Engel Curves

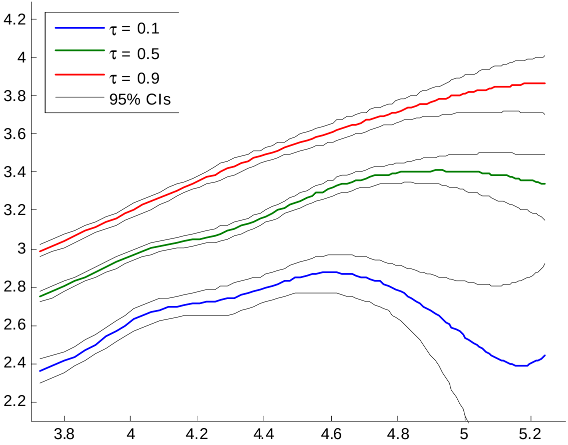
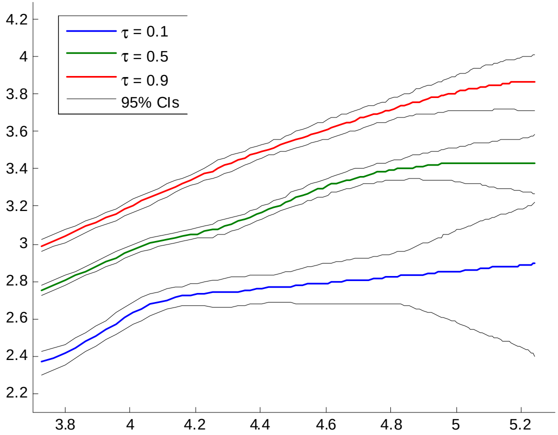


Figure: Unconstrained demand function estimates, $t = 1983$.

RP Restricted Engel Curves



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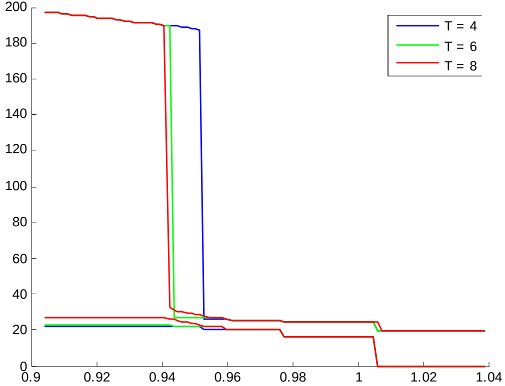

Figure: Demand bounds at median income, $\tau = 0.1$.

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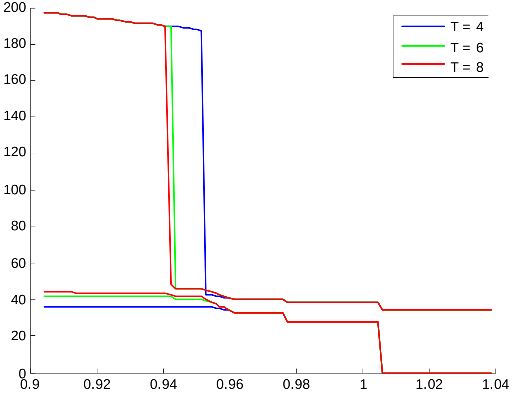

Figure: Demand bounds at median income, $\tau = 0.5$.

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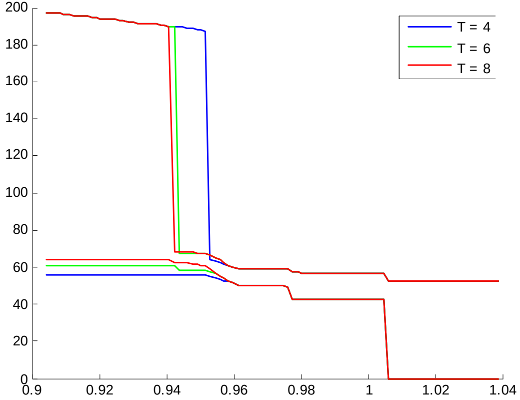

Figure: Demand bounds at median income, $\tau = 0.9$.

Demand Bounds Estimation

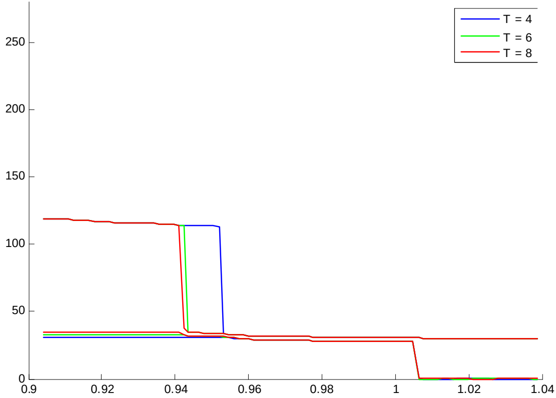

Figure: Demand bounds at 25th percentile income, $\tau = 0.5$.

Demand Bounds Estimation

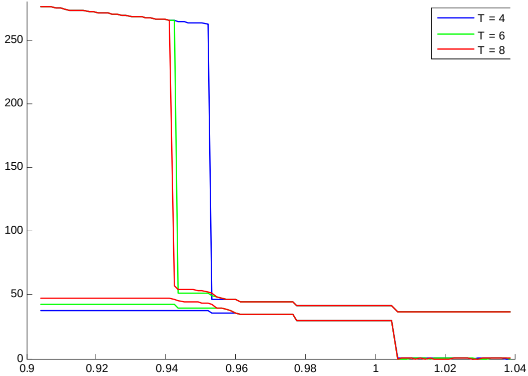

Figure: Demand bounds at 75th percentile income, $\tau = 0.5$.

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- Alternatively, the **control function** approach taken in Imbens and Newey (2009) can be used. Again they estimate using the exact same data and instrument. Specify

$$\ln x = \pi(\mathbf{z}, v)$$

where π is monotonic in v, $\mathbf{z}$ are a set of instrumental variables.

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- Derive welfare costs of relative price and tax changes across the distribution of demands by income and taste heterogeneity.