# Labour Supply and Taxation with Restricted Choices*

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**Abstract**

A model of labour supply is developed in which observed hours reflect both the distribution of preferences and restrictions on the choice of hours. In the absence of information on the choice set facing each individual, observed hours may appear not to satisfy the revealed preference conditions for 'rational' choice. We focus on the case where the choice set contains at most two offers and show that when the choice set distribution is known, preferences can be identified. We then show that, where preferences are known, the choice set distribution can be fully recovered. Conditions for identification of both preferences and the distribution of choice sets are also developed. We illustrate this approach in a labour supply setting with nonlinear budget constraints. Non-linearities in the budget constraint are used to directly reveal restrictions on the choice set. This framework is used to study the labour supply behaviour of a large sample of working age mothers in the UK, accounting for nonlinearities in the tax and welfare benefit system, fixed costs of work and restrictions on hours choices.

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# 1 Introduction

Observed hours of work among working age adults display considerable variation both over time and in the cross-section, especially among women with children (see, for example, Blundell, Bozio and Laroque, 2011). In this paper we ask under what conditions this variation can be used to identify preferences for labour supply. We start with the premise that it is unlikely that all workers are free to choose their working hours (see, for example, Blundell, Brewer and Francesconi, 2008). Different individuals at different times of their career may well face different sets of hours from which they can choose. Some may have no choice at all. Restrictions on the choice set become particularly noticeable when individuals face nonlinear tax and welfare benefit systems (Hausman, 1985). Restricted choices together with the nonlinear budget constraints give rise to ranges of hours that would never be rationally chosen if, that is, other hours choices had been available. Some of the choices appear to be dominated by feasible alternatives. For example, we may observe people working half time even though they would receive no smaller income if they were to work less, an observation inconsistent with the pure static neoclassical model of unrestricted hours choices. Similarly we may observe few observations at a tax kink even though optimal choice would imply a mass at that point, see Saez, 2010; and Chetty et al, 2011.

Observed hours may however be consistent with optimal choice given a restricted choice set. As empirical economists we typically do not know the complete set of alternatives available to individuals. This is similar to the idea of a 'consideration set' in the modern literature on bounded rationality, see Kfir and Spiegler (2011), for example. In that literature consumers make rational choices from a choice set that is limited by a combination of their own perception of the options and the strategy of firms. Our interpretation is one where rational choices are made from a set of hours restricted by the offers of employers. Nonetheless, there are many similarities between the two frameworks. In this paper we develop and estimate a structural model of labour supply choices that embeds restrictions on the set of available hours.

We are not the first to examine hours restrictions in labour supply models. There is a long history of incorporating such restrictions into labour supply models, including, Aaberge (1999, 2009), Altonji and Paxson (1992), Bloemen (2000, 2008), Dickens and Lundberg (1993), Van Soest et al (1990), Ham and Reilly (2002) and Dagsvik and Strom (2006); see also the recent discussion in Chetty (2012). The ideas we develop here place these models of hours retrictions in a constrained rational choice setting in which the set of alternative choices on offer is restricted. The framework is general and concerns the case where the econometrician does not directly observe the choice set from which the individual has chosen. We suppose that agents make their choices on a random subset of all possible hours. We analyze how this modified model works, and in particular the sets of assumptions under which it still allows to identify the parameters of the underlying structural model. We first consider the case where the econometrician knows the probability distribution of offered choices. In the more complete model we generalise this to the case of nonlinear budget sets and make the distribution of offers unknown but dependent on excluded covariates.

The labour supply model we consider is placed in a life-cycle setting in which hours of work, employment and savings decisions are made subject to a nonlinear tax and benefit system and fixed costs of work. We draw on the extensive existing literature on labour supply models with nonlinear budget sets (Burtless and Hausman (1978), Hausman (1979, 1985), Heckman (1974)), with fixed costs of work (Heckman (1974, 1979), Cogan (1981)), and with intertemporal choices (Heckman and MaCurdy (1981)). We further develop these models to the case in which individuals face constraints on hours choices.

Here we focus attention on developing a two-offer model in which each individual is assumed to face two independent hours offers - the one at which they are observed to work, if they are working positive hours, and one they turned down. We assume the option of not working is always available. The 'alternative' offer could include the observed hours point in which case the individual would be completely constrained and able to make no other hours choices apart from a zero hours option. As the number of offers increases the specification approaches the standard labour supply model at which observed choices coincide with the fully optimal choice over all hours options.

The policy environment we consider is the labour supply behaviour of women in the UK. We model their decisions in the face of non-linear budget constraints generated through the working of the tax, tax-credit and welfare system. We study the period 1997-2002 when there were a number of key changes to the budget constraint through reforms to the tax-credit and welfare system, see Adam, Browne and Heady (2010). We use data from the UK Family Expenditure Survey (FES) over this period. The FES is a detailed household survey that records hours worked, earnings and consumer expenditure. For every family in the data we have an accurate tax and benefit model (IFS-Taxben) that simulates the complete budget constraint incorporating all aspects of the tax, tax-credit and welfare systems. We use the consumption measure in the FES to ensure the hours of work model we develop is consistent with a life-cycle model (see Blundell and Walker, 1986).

The observed hours of work of women in our sample are concentrated at part-time and full-time. We use the nonlinearities in the budget set to provide direct evidence of hours restrictions by recording individuals working at hours of work that would be strictly dominated by other choices were a full range of hours choices to be available. We estimate a parametric specification of the two-offer model and show that women that appear to be subject to the choice constraints belong to significantly poorer households than average and work shorter hours. Simulating the economy with or without the hours restrictions, we find a smaller level of employment in the two-offer model than in the long run when restrictions on hours are lifted. Together with the estimated preferences for hours and employment we argue that the framework provide a compelling empirical framework for understanding observed hours and employment.

The remainder of the paper is as follows. Section 2 lays out the intertemporal labour supply model with nonlinear budget constraints and fixed costs. We then consider the interpretation of rejections of the standard model and develop a model of labour supply in which individuals face a two-offer distribution over possible hours choices. In section 3 we show that when the offer distribution is known preferences can be identified in the standard multinomial choice and random utility models. We are able to show that, where preferences are known, the offer distribution can be fully recovered. We also develop conditions for identification of both the parameters of preferences and of the offer distribution. In section 4 we develop the sample likelihood for the two-offer model and use this model to study the labour supply choices of a large sample of women in the UK, accounting for nonlinear budget constraints and fixed costs of work. In section 5 we present the estimates of the model and their implications for labour supply behaviour. Section 6 concludes.

# 2 A model of hours, employment and consumption

We begin by laying out a standard labour supply model in which there are non-linear taxes and fixed costs of work but otherwise workers are free to choose their hours of work. Decisions are made in an intertemporal setting with unrestricted hours choices at the extensive and intensive margins.

At date $t$, the typical individual chooses her consumption $c_t$ and labour supply $h_t$, maximizing

$$E_t \int_t^T u_t(c_\tau, h_\tau) d\tau$$

subject to an intertemporal budget constraint

$$\int_t^T \exp[-r(\tau - t)]\{c_\tau - R(w_\tau, h_\tau) + b_\tau \mathbf{1}_{h_\tau > 0}\} d\tau \leq S_t.$$

Here $u_t$ is the instantaneous utility index, a concave twice differentiable function of the vector $(c, h)$ of consumption and hours of work. It is increasing in consumption, decreasing in hours. The consumption good is the numeraire.

The function $R(w, h)$ denotes the income after taxes are deducted and benefits received for someone who works $h$ hours at wage $w$. This function will also depend on other characteristics that change tax rates and eligibility. The extensive margin comes from the fixed costs of being employed, i.e. having a positive $h$, costs $b$ units of consumption. Accumulated savings at date $t$ are equal to $S_t$. We denote by $\lambda_t$ the Lagrange multiplier associated with the budget constraint at date $t$.

Current consumption maximizes $u_t(c_t, h_t) - \lambda_t c_t$, and therefore satisfies the first-order condition

$$\frac{\partial u}{\partial c}(c_t, h_t) = \lambda_t.$$

Also, if the individual works, the optimal hours maximize $u_t(c_t, h_t) + \lambda_t R(w_t, h_t)$. Let $(c^e, h^e)$, the optimal choice of the working household, $c^o$ the consumption of the household with the worker out of the labour market. The household will be observed out of the labour market whenever the (revealed preference) inequality

$$u_t(c^e, h^e) - \lambda_t[c^e - R(w_t, h^e) + b_t] < u_t(c^o, 0) - \lambda_t[c^o - R(w_t, 0)]$$

is satisfied.

In this framework, the choice of hours and employment is made subject to fixed costs of work and nonlinear taxes with *all* hours alternatives available. But observed hours and employment may not be consistent with this choice model.

## 2.1 Rejections of the unrestricted choice model

To interpret inconsistencies of observed behaviour with rational choice within this framework it is useful to place some structure on preferences and individual heterogeneity.

Consider the following utility specification, separable in consumption and leisure

$$u(c,h) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{(L-h)^{1-\phi}}{1-\phi}a, \tag{1}$$

where $L$ (= 100, for example) is a physiological upper bound on the number of hours worked weekly, $\gamma$ and $\phi$ are non-negative parameters, and the strictly positive factor $a$, which governs the substitution between consumption and leisure.

From the analysis of the previous section of the optimizing household, we have the marginal utility of wealth given by

$$\lambda = c^{-\gamma}.$$

The optimal hours $h^e$ when working maximize the instantaneous current contribution to the Lagrangian

$$\frac{(L-h)^{1-\phi}}{1-\phi}a + c^{-\gamma}R(w,h), \tag{2}$$

and the household chooses to stay out of the labour market whenever the inequality

$$\frac{(L-h^e)^{1-\phi}}{1-\phi}a + c^{-\gamma}[R(w,h^e) - b] < \frac{L^{1-\phi}}{1-\phi}a + c^{-\gamma}R(w,0).$$

is satisfied. It will be useful when deriving analytic expressions for the likelihood function to let

$$v(h) = \frac{(L-h)^{1-\phi}}{1-\phi},$$

and

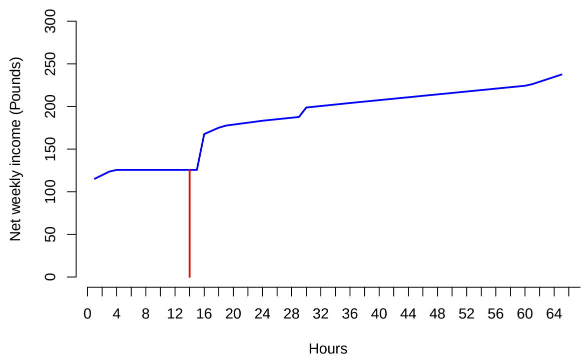
$$V(h,w,b,c,a) = av(h) + c^{-\gamma}[R(w,h) - b], \tag{3}$$

where we leave exogenous variables as mute arguments of $V$.

For a disposable income function $R$ which is not concave in $h$, some values of hours may never be chosen by an optimizing agent who behaves according to the above model. Figure 1, with weekly hours on the $x$ axis and disposable income on the $y$ axis, illustrates this point. In this Figure, the $R$ function has a flat horizontal portion between 4 and 16 hours per week, so that a choice of 14 hours say as indicated, can never be optimal, since it is strictly dominated by shorter positive hours, whatever the unobservable characteristics.

To check whether this phenomenon is important on real data, we write the revealed

Figure 1: A standard example of budget constraint




preference inequality as

$$\frac{(L-h^e)^{1-\phi}}{1-\phi}a + c^{-\gamma}R(w,h^e) - \frac{(L-h)^{1-\phi}}{1-\phi}a - c^{-\gamma}R(w,h) \geq 0,$$

where $h^e$ is the observed choice and $h$ is any other possible length of the workweek. Using the specification for $a$, we can separate the cases where $h$ is smaller than $h^e$ from those where $h$ is larger than $h^e$. That is

$$c^{\gamma}a \leq \left\{ \frac{R(w,h) - R(w,h^e)}{\frac{(L-h^e)^{1-\phi}}{1-\phi} - \frac{(L-h)^{1-\phi}}{1-\phi}} \right\}, \tag{4}$$

for all $h$ smaller than $h^e$, with the inequality in the other direction

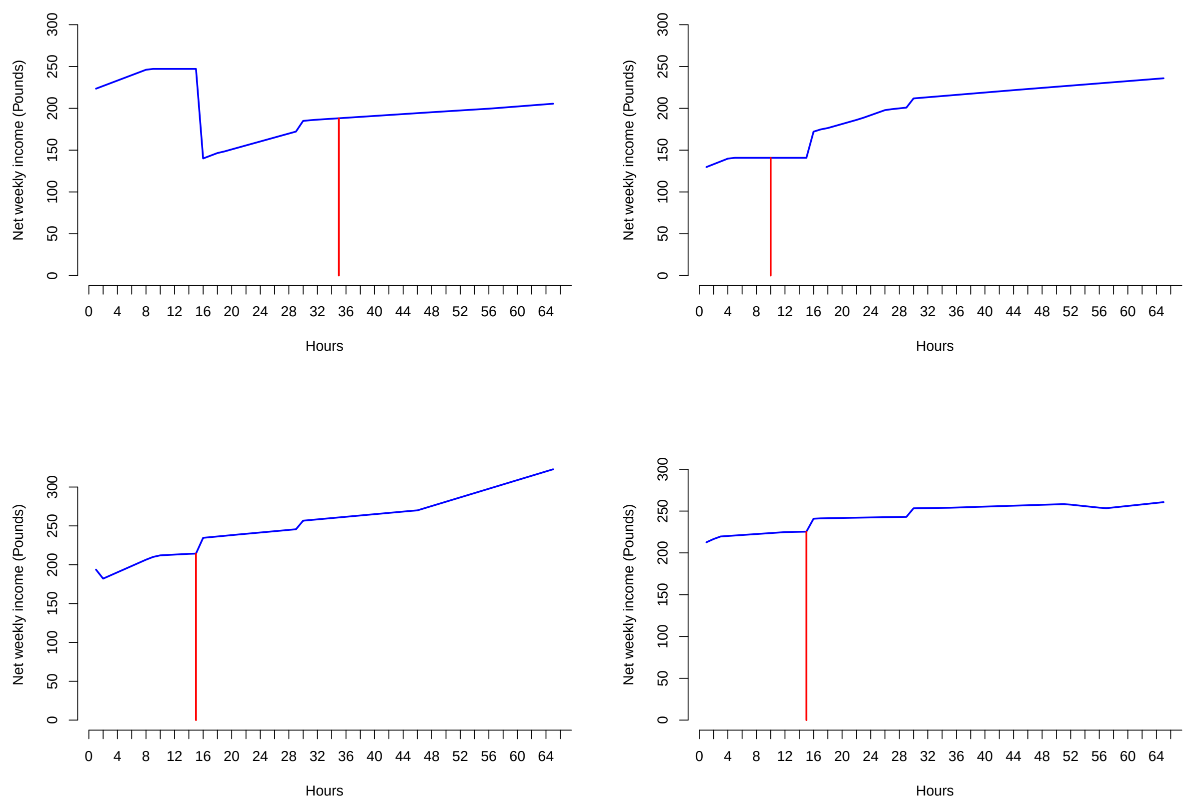
$$c^{\gamma}a \geq \left\{ \frac{R(w,h) - R(w,h^e)}{\frac{(L-h^e)^{1-\phi}}{1-\phi} - \frac{(L-h)^{1-\phi}}{1-\phi}} \right\} \tag{5}$$

for all $h$ larger than $h^e$.

The choice $h^e$ is compatible with optimization under our specification if and only if there is an $a$ satisfying the two above inequalities, i.e.

$$\min_{h \leq h^e}(1-\phi)\frac{R(w,h) - R(w,h^e)}{(L-h^e)^{1-\phi} - (L-h)^{1-\phi}} \geq \max\left[0, \max_{h \geq h^e}(1-\phi)\frac{R(w,h) - R(w,h^e)}{(L-h^e)^{1-\phi} - (L-h)^{1-\phi}}\right]. \tag{6}$$

Figure 2: Some examples of budget constraints for in work people




Inequality (10) can easily be checked on real data, since the only parameter that appears in it is $\phi$. In fact there are two ways of violating the condition: the positivity of the left hand side of (6) does not depend at all on the parametric specification, but only on the shape of the function $R$ and on the value of hours $h^e$ (see three graphs in the upper panel of Figure 2); the second inequality on the other hand does depend on $\phi$ and is illustrated in the two graphs in the lower panel of Figure 2. These are budget constraints for specific individual women drawn from the sample we use in estimation. We return to this discussion when we examine the details of the data and estimation below. First we develop a coherent model of restricted choice that can account for such observations.

## 2.2 A model with restrictions on the choice set

To introduce our extension of the standard model, we consider choices over discrete hours. We focus on the determination of within period hours given the level of consumption. In this framework the typical worker is characterised by preferences, $a$, consumption, $c$, fixed cost $b$ and wage $w$. These are assumed to be a function of a set of parameters $\beta$, observed exogenous covariates $Z$ and unobserved heterogeneity $\varepsilon$. The value of hours $h$ for each individual is given by

$$V(h, Z, \beta, \varepsilon).$$

as in preference specification (3) above.

The possible choices $h$ belong to a finite set $\mathcal{H}$ made of $I$ elements $\{h_1, \ldots, h_I\}$. Given a subset of possible choices $H$ in $\mathcal{H}$, for each $\beta$ and $Z$, any distribution of $\varepsilon$ yields a probability distribution on $H$. We shall denote the probability of choosing $h_i$ in $H$ as $p_i(H, Z, \beta)$.

We assume that given $V$, the observation of the family of probabilities $p_i(H, Z, \beta)$ identifies the parameter $\beta$, when $Z$ varies in the population, and the union of the family of (non singleton) choice sets $H$ for which the probabilities are observed covers the whole of $\mathcal{H}$.

The standard choice model has $H$ equal to $\mathcal{H}$. For our application this model is not appropriate: because of underlying non-convexities in the budget constraint, for some $h_j$ alternative we have

$$p_j(\mathcal{H}, Z, \beta) = 0,$$

for all $(Z, \beta)$, while the data contains some observations of $h_j$.

To tackle this issue, we suppose that the agents do not make their choices over the whole set $\mathcal{H}$, but on a random subset of it. We analyze how this modified model works, and in particular the assumptions under which it still allows to identify the parameters $\beta$ of the underlying structural model.

### 2.2.1 The two-offer model

Suppose that there is a distribution of offers, the probability of being offered $h_i$ being equal to $g_i$, $g_i > 0$, $\sum_{i=1}^{I} g_i = 1$.[1] First consider the case where individuals draw

[1] Below we will allow the distribution of offers $g_i$, for any individual $i$, to depend on observable covariates $\{X_i\}$ and a finite of parameter vector $\gamma$.

independently two offers from $g$ and choose the one that yields the highest utility.[2] The distribution of the observed choices $\ell_{2i}(Z, \beta)$ (the first index '2' serves to mark that there are two offers) then takes the form

$$\ell_{2i}(Z, \beta) = g_i^2 + 2g_i \sum_{j \neq i} g_j p_i(\{i, j\}, Z, \beta), \tag{7}$$

with the first term on the right hand side corresponding to identical offers (leaving no choice to the decision maker), and the second reflecting choices among all possible couples of offers.

There are $I$ equations, of which only $I - 1$ are independent: the sum of all the equations is identically equal to 1 (on the right hand side, this follows from the observation that $p_i(\{i, j\}, Z, \beta) + p_j(\{i, j\}, Z, \beta) = 1$ for all $i$, $j$). On the right hand side, there are potentially $I(I-1)/2 + I - 1$ unknowns: the choice probabilities $p$ and the distribution of offers $g$. There is no possibility to identify all these unknown parameters from the mere observation of the choice distribution $\ell$. Below we explore alternative restrictions that deliver identification.

### 2.2.2 Increasing the number of offers

In the two-offer case, when the probability $g$ has full support, the choice sets are all the pairs made from elements of $\mathcal{H}$, allowing repetitions. More generally the number of offers $n$ determines the cardinality of the choice sets. If the draws are independent, for any finite $n$, there is a positive probability that there is no real choice: all the elements in the choice set are identical. However when $n$ increases, this probability goes to zero and more importantly the probability that the choice set contains all the elements of $\mathcal{H}$ goes to one. The $n$ offer model converges towards the standard unrestricted choice model as $n$ goes to infinity.

[2] Note that as we do not observe past choices we cannot distinguish between an offer that allows the individual to retain their previous hours work rather than choose among completely new offers. In principle we can therefore allow individuals to be offered, and to choose to keep, their existing hours worked. Our assumptions though will imply that the distribution of offers is independent of the past hours worked. This will be an important extension for future work.

# 3 Identification

## 3.1 Recovering choices, knowing the offer distribution

Even if the offer distribution $g$ is given, the number of unrestricted choice probabilities among pairs a priori is $I(I-1)/2$, larger than $I-1$ for $I$ greater than 2. We have to restrict the number of structural unknowns, imposing consistency requirements across pairs.

### 3.1.1 Independence of irrelevant alternatives

As a first step, consider the case of independence of irrelevant alternatives (IIA), where for all $i$, $j$

$$p_i(\{i,j\},Z,\beta)=\frac{p_i(\mathcal{H},Z,\beta)}{p_i(\mathcal{H},Z,\beta)+p_j(\mathcal{H},Z,\beta)},$$

or

$$p_i(\{i,j\})=\frac{p_i}{p_i+p_j},$$

where to alleviate notation we drop the arguments $Z$ and $\beta$, and denote by $p_i$ the probability of choosing $i$ among the whole set of alternatives. In this circumstance the number of unknowns is equal to the number of equations, and we may hope for exact identification. Indeed

**Lemma 1.** *Let $\ell$ and $g$ be two probability vectors in the simplex of $\mathcal{R}^I$, whose components are all positive. There exists at most a unique vector $p$ in the interior of the simplex of $\mathcal{R}^I$ that satisfies the system of equations*

$$\ell_i=g_i^2+2g_ip_i\sum_{j\neq i}\frac{g_j}{p_i+p_j}\ \textit{for}\ i=1,\ldots,I. \tag{8}$$

**Proof**: For all $i$, denote

$$P_i(p)=g_i^2+2g_ip_i\sum_{j\neq i}\frac{g_j}{p_i+p_j}$$

for $p$ in $\mathcal{R}_+^I$. For any $\lambda\neq 0$, observe that $P_i(\lambda p)=P_i(p)$. Suppose by contradiction that there are two solutions $p^0$ and $p^1$ to the system of equations both belonging to the interior of $\mathcal{R}_+^I$. Choose $\overline{p}_I$ such that

$$\overline{p}_I\geq\frac{p_I^0}{\min_i p_i^0}\ \text{and}\ \overline{p}_I\geq\frac{p_I^1}{\min_i p_i^1},$$

and define $\lambda^0$ and $\lambda^1$ through

$$\lambda^0 p_I^0 = \lambda^1 p_I^1 = \overline{p}^I.$$

This construction implies that the two vectors $\lambda^0 p^0$ and $\lambda^1 p^1$ are both solutions of

$$\ell_i = P_i(p) \text{ for } i = 1, \ldots, I-1,$$

have all their coordinates larger than 1, with $n$'th coordinate normalized at $\overline{p}_I$. We therefore study the reduced system of $I-1$ equations

$$\ell_i = P_i(p_1, \ldots, p_{I-1}, \overline{p}_I) \text{ for } i = 1, \ldots, I-1$$

with the unknowns $(p_1, \ldots, p_{I-1})$ in $[1, \infty)^{I-1}$. The fact that it has at most a unique root follows from Gale Nikaido, once it is shown that the Jacobian of $P$ is everywhere a dominant diagonal matrix. We have

$$\frac{\partial P_i}{\partial p_i} = 2g_i \sum_{j=1, j\neq i}^{I} \frac{g_j p_j}{(p_i + p_j)^2},$$

and for $j$ different from $i$

$$\frac{\partial P_i}{\partial p_j} = -2g_i p_i \frac{g_j}{(p_i + p_j)^2}.$$

The property of diagonal dominance with equal weights to all terms is equivalent to

$$\left|\frac{\partial P_i}{\partial p_i}\right| > \sum_{j=1, j\neq i}^{I-1} \left|\frac{\partial P_j}{\partial p_i}\right|,$$

that is

$$2g_i \sum_{j=1, j\neq i}^{I} \frac{g_j p_j}{(p_i + p_j)^2} > \sum_{j=1, j\neq i}^{I-1} 2g_i p_j \frac{g_j}{(p_i + p_j)^2}$$

or

$$\frac{g_I p_I}{(p_i + p_I)^2} > 0.$$

The inequality is satisfied, and the right hand side mapping is univalent on $[1\,, \infty)^{I-1}$, which completes the proof. $\square$

As we noted in Section 4, there may be cases which would never be rationally chosen. In these situations we can put zero weights on some of the decisions, that is $p_j = 0$ for some subset $J$ of the alternatives. A simple manipulation of the system of equations, using the equality $p_i(\{i,j\} + p_j(\{i,j\} = 1$ even when the marginal probabilities are zero, yield

$$\ell_J = \sum_{j \in J} \ell_j = \left( \sum_{j \in J} g_j \right)^2 = g_J^2,$$

and for all $i$ not in $J$

$$\ell_i = g_i(1 + 2g_J) + 2g_i p_i \sum_{k \notin J, k \neq i} \frac{g_k}{p_i + p_k},$$

where the notation $p_J$ denotes the sum of the components of the vector $p$ with indices in $J$. A minor adaptation of the proof of Lemma 1 then shows that the vector $p$ is uniquely determined. Using the first equation, a natural procedure is to compute the non-negative difference $\ell_J - g_J^2$ for all subsets $J$ of indices. The candidates $J$ for the solution are the ones for which the difference is zero. We do not know whether there can be multiple candidates.[3]

### 3.1.2 The random utility model

Consider now the random utility model where the agent has within period utility $V_i - \varepsilon_i$ for alternative $i$, $i = 1, \ldots, I$, and under full optimization, knowing the value of her utilities, chooses the alternative which gives the highest utility. This is close to our labour supply model with discrete hours. We are able to show that our identification results extend to this model.

The econometrician is supposed to know the joint distribution of the continuous variables $\varepsilon_i$ in the economy, and wants to infer from observed hours choices the values of $V_i$. We denote $F_{ij}$ the (assumed to be differentiable) cumulative distribution function of $\varepsilon_i - \varepsilon_j$ so that

$$p_i(\{i,j\}) = F_{ij}(V_i - V_j).$$

Since only the differences $V_i - V_j$ can be identified, we normalize $V_I$ to zero. As in the IIA case above, the number of unknowns is equal to the number of equations, and we may hope for exact identification. Indeed

[3] There cannot be two solutions with two disjoint sets $J_1$ and $J_2$. Indeed one would need to have

$$\ell_{J_1} = g_{J_1}^2 \quad \ell_{J_2} = g_{J_2}^2,$$

which implies

$$\ell_{J_1 \cup J_2} = g_{J_1}^2 + g_{J_2}^2 < g_{J_1 \cup J_2}^2,$$

which is impossible.

**Lemma 2.** *Let $\ell$ and $g$ be two probability vectors in the simplex of $\mathcal{R}^I$, whose components are all positive. There exists at most a unique vector $V_i$ with $V_I = 0$ that satisfies the system of equations*

$$\ell_i = g_i^2 + 2g_i \sum_{j \neq i} g_j F_{ij}(V_i - V_j) \text{ for } i = 1, \dots, I. \tag{9}$$

**Proof**: For all $i = 1, \dots, I-1$, denote

$$Q_i(V) = -\ell_i + g_i^2 + 2g_i \sum_{j \neq i} g_j F_{ij}(V_i - V_j),$$

and $Q(V)$ the $I-1$ vector obtained by stacking up the $Q_i$s. By construction, any $V$ such that $Q(V) = 0$ satisfies (9), since the $I$th equation follows from summing up the $I-1$ first ones.

The result then follows from Gale Nikaido since the Jacobian of $Q$ is everywhere a dominant diagonal matrix. Indeed

$$\frac{\partial Q_i}{\partial V_i} = 2g_i \sum_{j \neq i} g_j f_{ij}(V_i - V_j),$$

while for $j \neq i$, $j \neq I$,

$$\frac{\partial Q_i}{\partial V_j} = -2g_i g_j f_{ij}(V_i - V_j).$$

The diagonal terms are positive and the off-diagonal negative. The sum of the elements on line $i$ is positive equal to

$$2g_i g_I f_{iI}(V_i).$$

$\square$

### 3.2 Recovering the offer distribution, knowing choice probabilities

In contrast to the previous section, assume that we know the theoretical choice probabilities over all pairs of alternatives: $p_{ij}$ denotes the probability of choosing $i$ when both $i$ and $j$ are available for all $i$ different from $j$. We study whether the choices $\ell_i$ of agents getting two independent offers are constrained by the model, and whether the observation of $\ell$ allows to recover the probability of offers $g$. From (7), we have by definition

$$\ell_i = g_i^2 + 2g_i \sum_{j \neq i} g_j p_{ij} \tag{10}$$

where for all couples $(i, j)$, $i \neq j$,

$$p_{ij} + p_{ji} = 1. \tag{11}$$

**Lemma 3.** *Given the choice probabilities $p_{ij}$, $p_{ij} \geq 0$ satisfying* (11)*, for any observed probability $\ell_i$ in the simplex of $\mathcal{R}^I$, there exists a unique offer probability $g_i$ in the simplex of $\mathcal{R}^I$ which satisfies* (10)*.*

**Proof**: We first prove the existence of $g$, then its uniqueness. For all $i$, define

$$\overline{Q}_i(g) = g_i^2 + 2g_i \sum_{j \neq i} g_j p_{ij}.$$

By construction, for $g$ in the simplex of $\mathcal{R}^I$, under (11), $\overline{Q}(g)$ also belongs to the simplex of $\mathcal{R}^I$. Indeed

$$\sum_{i=1}^{I} \overline{Q}_i(g) = \sum_{i=1}^{I} \left[ g_i^2 + 2g_i \sum_{j<i} g_j \right] = 1.$$

Consider the mapping

$$\Gamma_i(g) = \frac{\max(0, g_i - \overline{Q}_i(g) + \ell_i)}{\sum_{j=1}^{I} \max(0, g_j - \overline{Q}_j(g) + \ell_j)}.$$

First note that $\Gamma$ is well defined: since $g$, $\overline{Q}$ and $\ell$ all belong to the simplex, the denominator is larger than 1. Therefore $\Gamma$ maps continuously the simplex into itself and it has a fixed point, say $g^*$. If $g_i^* = 0$, by definition $\overline{Q}_i(g^*) = 0$, so that

$$g_i^* - \overline{Q}_i(g^*) + \ell_i = \ell_i.$$

It follows that at the fixed point

$$\max(0, g_i^* - \overline{Q}_i(g^*) + \ell_i) = g_i^* - \overline{Q}_i(g^*) + \ell_i,$$

the denominator is equal to 1, and $\ell = \overline{Q}(g^*)$ as desired.

Uniqueness follows from the univalence of $\overline{Q}$. This is a consequence of the fact that the

Jacobian of $\overline{Q}$ is a dominant diagonal matrix, with weights $(g_i)$: for all $i$

$$g_i \frac{\partial \overline{Q}_i}{\partial g_i} > \sum_{j \neq i} g_j \frac{\partial \overline{Q}_i}{\partial g_j}.$$

Indeed

$$g_i \left[ 2g_i + 2 \sum_{j \neq i} g_j p_{ij} \right] > \sum_{j \neq i} 2 g_j g_i p_{ij}.$$

□

As we have seen in section 3, in the non-linear budget constraint cases that we are interested in, the choice probabilities can exclude some alternatives, say $i = 1$ to $k$. In the current setup, this means that, for all $i \leq k$, for all $j \neq i$

$$p_{ij} = 0.$$

In this case the two-offer model allows us to rationalize the data by letting for $i = 1$ to $k$

$$\ell_i = g_i^2 + 2g_i \sum_{j \neq i, j \leq k} g_j p_{ij}.$$

This nonlinear system can be shown, by an adaptation of the proof of the above lemma, to have a unique solution, satisfying $\sum_1^k g_i = \sqrt{\sum_1^k \ell_i}$.

### 3.3 Recovery of choice and offer probabilities

In general we will neither have prior knowledge of the theoretical choice probabilities $p_{ij}$ nor of the offer probabilities $g_i$. Note first, as we have illustrated in section 2.1, there will be some workers facing non-convex budget constraints which can rule out certain choices and help with identification. In the absence of such non-convexities the choice probabilities and the offer probabilities will not, without further assumptions, be separately identified.

Consider the setting for our empirical application. The utility from hours choice $h_i$ is given by $V(c, h_i, w, b, a)$ in (3), which can be written as a function of exogenous observed characteristics $Z$, a set of parameters $\beta$ and unobserved heterogeneity $\varepsilon$: $V(h, Z, \beta, \varepsilon)$.

The utility from choice $h_i$ is then given by $V_i(\beta) = V(h_i, Z, \beta, \varepsilon)$ and the probability that hours $h_i$ are chosen when the pair $(h_i, h_j)$ are available is given by

$$p_{ij} = \Pr[V_i(\beta) - V_j(\beta) > 0].$$

To make progress with identification in this case we assume that the offer probability $g_i$ is a smooth function of a finite parameter vector $\gamma$ and covariates $X$, and that

$$\dim[\beta : \gamma] \leq I - 1$$

and where $I$ is the number of possible choices.

From (9) we can write the system of equations

$$Q_i = -\ell_i + g_i(\gamma)^2 + 2g_i(\gamma)\sum_{j \neq i} g_j(\gamma)F_{ij}(V_i(\beta) - V_j(\beta)) \text{ for } i = 1, \ldots, I-1. \tag{12}$$

For identification we require full column rank of the matrix

$$\Pi = \left[\frac{\partial Q}{\partial V}\frac{\partial V}{\partial \beta}, \frac{\partial Q}{\partial g}\frac{\partial g}{\partial \gamma}\right] \tag{13}$$

where the matrix of derivatives relating to the $Q_i$ has elements of the form

$$\frac{\partial Q_i}{\partial V_i} = 2g_i \sum_{j \neq i} g_j f(V_i - V_j) \tag{14}$$

$$\frac{\partial Q_i}{\partial V_j} = -2g_i g_j f(V_i - V_j) \tag{15}$$

$$\frac{\partial Q_i}{\partial g_i} = 2g_i + 2\sum_{j \neq i} g_j F(V_i - V_j) \tag{16}$$

$$\frac{\partial Q_i}{\partial g_j} = 2g_i g_j F(V_i - V_j) \tag{17}$$

We note that $\frac{\partial Q_i}{\partial V_i} > 0$ and $\frac{\partial Q_i}{\partial V_j} < 0$ where the row sum is also positive. Inspection of the elements of $\Pi$, (14) .. (17), suggests no natural linear dependence and, in general, the rank condition should be satisfied.

To develop stronger identification results we make exclusion restrictions on the covariates $Z$ and $X$ that enter preferences $V$ and the offer probability $g$, respectively. With sufficient independent variation in $X$ and $Z$ the rank$[X : Z] \geqslant \dim[\beta : \gamma]$ and the full column rank of $\Pi$ will be satisfied.

# 4 Data and Sample Likelihood

## 4.1 The Data

The sample we use comprises women with children, either single or married mothers. We use years 1997 to 2002 of the UK Family Expenditure Survey (FES) as this covers the period of key reforms to the welfare and tax-credit system in the UK, see Adam, Browne and Heady (2010). The data provide detailed diary and face to face interview information on consumption expenditures, usual hours worked, gross wage earnings, education qualifications and household demographics. Tables 1 and 2 provide some basic descriptive statistics.

The overall sample contains some 10,543 women spread fairly evenly across the six years under study. A large group of women in this sample have relatively low education qualifications, meaning that they left formal schooling at the minimum school leaving age of 16. The majority of the rest have completed secondary school with less than 20% having a college or university degree. The modal number of children is two and a little more than 40% of the sample have a youngest child aged less than 5 (the formal school entry age in the UK). Almost 80% of the women in our sample are married or cohabiting (we label all these as 'cohabiting'), leaving just over 20% of the mothers in the sample as single parents. The median hours of work for this sample is 25 hours per week with a wide distribution. The median wage is £5.85 per hours and the average wage is £7.23 per hour.[4]

## 4.2 The sample likelihood for the two-offer model

In our sample we observe the employment status and the consumption expenditure of the women in the survey, their earnings and weekly hours when employed. We use the IFS TaxBen tax simulation model to recover income, net of tax and benefits, for each household.

To construct the likelihood for this sample we derive the distribution of employment, hours, consumption and wages from the model in section 2.2. We choose the following specification: the parameter governing the substitution between consumption and leisure

[4]All prices are given in 1997 prices.

Table 1: Some Descriptive Statistics

| | Sample | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
|---|---|---|---|---|---|---|---|
| Size | 10543 | 1701 | 1729 | 1807 | 1664 | 1910 | 1732 |
| | | | | | | | |
| Education | | | | | | | |
| Level 1 | 54.86 | 57.03 | 56.33 | 53.96 | 54.33 | 55.5 | 52.02 |
| Level 2 | 26.59 | 25.63 | 26.14 | 28.44 | 26.8 | 25.39 | 27.14 |
| Level 3 | 18.55 | 17.34 | 17.52 | 17.6 | 18.87 | 19.11 | 20.84 |
| | | | | | | | |
| Number of children | | | | | | | |
| 1 | 35.82 | 38.33 | 32.85 | 37.19 | 35.94 | 35.24 | 35.45 |
| 2 | 43.48 | 43.62 | 44.59 | 40.56 | 42.07 | 44.66 | 45.32 |
| 3 | 15.24 | 13.64 | 16.6 | 16.1 | 17.01 | 13.98 | 14.26 |
| 4 | 3.96 | 3.17 | 4.45 | 4.1 | 3.91 | 4.29 | 3.81 |
| 5+ | 1.49 | 1.23 | 1.5 | 2.05 | 1.08 | 1.83 | 1.15 |
| | | | | | | | |
| Age of the youngest child | | | | | | | |
| between 0 and 4 | 41.03 | 43.27 | 43.32 | 40.73 | 39.6 | 39.48 | 39.95 |
| between 5 and 10 | 33.79 | 32.86 | 32.39 | 33.37 | 33.83 | 34.61 | 35.57 |
| | | | | | | | |
| London | 9.34 | 10.11 | 9.31 | 9.57 | 8.71 | 9.63 | 8.66 |
| Cohabitant | 77.84 | 78.48 | 79.01 | 76.15 | 78.73 | 77.91 | 76.91 |
| | | | | | | | |
| Spouses Inwork Status | | | | | | | |
| Women In work | 63.12 | 62.02 | 60.56 | 61.7 | 63.58 | 64.87 | 65.88 |
| Spouse In Work | 69.49 | 70.02 | 69.35 | 67.68 | 70.19 | 70.05 | 69.69 |
| Both in work | 49.93 | 49.68 | 49.28 | 48.7 | 50.24 | 50.37 | 51.33 |
| Both out of work | 17.32 | 17.64 | 19.38 | 19.31 | 16.47 | 15.45 | 15.76 |

in the utility function (1) has the form

$$\ln(a) = Z^a\beta^a + \sigma^a\varepsilon^a, \tag{18}$$

where $Z^a$ contains observable characteristics, while $\varepsilon^a$ stands for unobservable preference heterogeneity. We also posit the following stochastic specification for the fixed cost of being employed

$$b = Z^b\beta^b + \sigma^b\varepsilon^b \tag{19}$$

where $\varepsilon^b$ reflects unobservable heterogeneity in work costs across individuals.

Since the wage process is external to hours choices we specify the reduced form speci-

Table 2: Consumption, Wages and Hours of Work

| | Mean | St. Dev. | p1 | p5 | p10 | p25 | p50 | p75 | p90 | p95 | p99 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Log of Consumption | | | | | | | | | | | |
| All | 5.3 | 0.6 | 3.85 | 4.25 | 4.49 | 4.93 | 5.33 | 5.68 | 6.02 | 6.23 | 6.74 |
| 1997 | 5.26 | 0.59 | 3.86 | 4.25 | 4.45 | 4.9 | 5.31 | 5.64 | 5.97 | 6.18 | 6.66 |
| 1998 | 5.24 | 0.58 | 3.88 | 4.21 | 4.44 | 4.88 | 5.26 | 5.63 | 5.95 | 6.15 | 6.56 |
| 1999 | 5.28 | 0.6 | 3.84 | 4.2 | 4.46 | 4.92 | 5.33 | 5.68 | 5.99 | 6.2 | 6.68 |
| 2000 | 5.33 | 0.6 | 3.87 | 4.3 | 4.54 | 4.96 | 5.36 | 5.71 | 6.05 | 6.28 | 6.8 |
| 2001 | 5.33 | 0.6 | 3.89 | 4.31 | 4.52 | 4.94 | 5.35 | 5.72 | 6.05 | 6.26 | 6.85 |
| 2002 | 5.33 | 0.6 | 3.81 | 4.3 | 4.54 | 4.96 | 5.35 | 5.72 | 6.1 | 6.29 | 6.78 |
| Hourly Wage | | | | | | | | | | | |
| All | 7.23 | 4.75 | 1.84 | 2.89 | 3.31 | 4.14 | 5.85 | 9 | 12.72 | 15.5 | 25.22 |
| 1997 | 6.66 | 4.6 | 1.83 | 2.78 | 3.12 | 3.84 | 5.21 | 8.03 | 11.59 | 14.55 | 26.19 |
| 1998 | 6.58 | 4.26 | 1.6 | 2.58 | 3.02 | 3.77 | 5.3 | 8.13 | 11.97 | 14.46 | 21.82 |
| 1999 | 7 | 4.42 | 1.65 | 2.78 | 3.32 | 4 | 5.62 | 8.73 | 12.51 | 15.1 | 23.31 |
| 2000 | 7.27 | 4.18 | 2.22 | 3.01 | 3.36 | 4.27 | 6.12 | 9.25 | 12.54 | 15.21 | 21.03 |
| 2001 | 7.88 | 5.17 | 2.33 | 3.15 | 3.56 | 4.45 | 6.35 | 10.05 | 13.53 | 16.9 | 27.62 |
| 2002 | 7.86 | 5.42 | 1.91 | 3.21 | 3.65 | 4.55 | 6.42 | 9.54 | 13.47 | 16.33 | 31.11 |
| Usual Hours of Work | | | | | | | | | | | |
| All | 26 | 11 | 3 | 7 | 11 | 18 | 25 | 37 | 40 | 41 | 50 |
| 1997 | 25 | 12 | 3 | 6 | 10 | 16 | 25 | 36 | 39 | 42 | 50 |
| 1998 | 25 | 11 | 3 | 6 | 10 | 16 | 25 | 36 | 39 | 40 | 49 |
| 1999 | 26 | 11 | 4 | 7 | 11 | 17 | 25 | 37 | 40 | 41 | 48 |
| 2000 | 27 | 11 | 3 | 7 | 12 | 18 | 27 | 37 | 40 | 43 | 50 |
| 2001 | 27 | 11 | 4 | 8 | 12 | 18 | 26 | 37 | 40 | 41 | 48 |
| 2002 | 27 | 11 | 4 | 9 | 13 | 19 | 26 | 37 | 40 | 41 | 48 |

fication of log market wages by

$$\ln(w) = Z^w \beta^w + \sigma^w \varepsilon^w \tag{20}$$

and use a reduced form for employment to adjust for selection into work.

The value of hours also depends consumption which is determined though the intertemporal saving decision, see specification (3), for example. We do not model savings directly and instead specify a reduced form for log consumption

$$\ln(c) = Z^c \beta^c + \sigma^c \varepsilon^c. \tag{21}$$

The residuals $(\varepsilon^a, \varepsilon^b, \varepsilon^c, \varepsilon^w)$ are assumed to be distributed joint normally distributed and independently from $Z^a$, $Z^b$, $Z^c$ and $Z^w$.

We use the distribution of hours and employment *conditional* on wages and consumption to derive the conditional likelihood for hours and employment. To allow for the endogeneity of wages and consumption in the determination of hours and employment we need to account for the dependence between the reduced form errors in (20) and (21) and the preference errors in (18) and (19). We adopt a control function approach, see Blundell and Powell (2003), in which the reduced form parameters are estimated in an initial step. At the second step the estimated error terms $\varepsilon^w$ and $\varepsilon^c$ are added as additional regressors in (18) and (19). The variance-covariance matrix of the estimated parameters in the conditional likelihood for hours and employment are then adjusted to account for this initial estimation.

To formulate the conditional likelihood we require an expression for the probability of being employed and choosing working hours conditional on $(c, w)$. Recall, the probability of being offered a job with working hours $h$ is given by $g(h)$.The probability of getting a couple of offers $(h, h')$, $h \neq h'$, is $2g(h)g(h')$, while that of getting $(h, h)$ is $g(h)^2$.[5]

First, consider the employment status. Assume $(\varepsilon^a, \varepsilon^c, \varepsilon^w)$ are known, i.e. consumption, wage and the parameter $a$. At weekly hours $h$ an individual is observed employed when

$$av(h) + c^{-\gamma}[R(w, h) - b] > av(0) + c^{-\gamma}R(w, 0),$$

or

$$b < R(w, h) - R(w, 0) + ac^{\gamma}[v(h) - v(0)].$$

From the expression for fixed costs of work $b$, the probability of this event knowing $a$ is easily computed from the cumulative distribution of $\varepsilon^b$:

$$F^b(\varepsilon^a, c, h, w) = \Phi\left[\frac{R(w, h) - R(w, 0) + c^{\gamma}[v(h) - v(0)]\exp(Z^a\beta^a + \sigma^a\varepsilon^a) - Z^b\beta^b}{\sigma^b}\right].$$

When the individual would like to work she can choose from two offers $h$ and $h'$. Offer $h$ is preferred to offer $h'$ when

[5] In the application, offers are modeled as a mixture of two independent normal distributions where mixture probability is allowed to depend on observable exogenous covariates $X$ that shift the hours offer distribution.

- either $h$ is larger than $h'$ and

$$a \leq c^{-\gamma} \frac{R(w,h) - R(w,h')}{v(h') - v(h)}$$

which can be written equivalently

$$\varepsilon^a \leq \alpha(c,h,h',w) = \frac{1}{\sigma^a} \left\{ -\gamma \ln c + \ln \left[ \frac{R(w,h) - R(w,h')}{v(h') - v(h)} \right] - Z^a \beta^a \right\},$$

- or $h$ is smaller than $h'$ and

$$c^{-\gamma} \frac{R(w,h') - R(w,h)}{v(h) - v(h')} \leq a,$$

which is also

$$\frac{1}{\sigma^a} \left\{ -\gamma \ln c + \ln \left[ \frac{R(w,h') - R(w,h)}{v(h) - v(h')} \right] - Z^a \beta^a \right\} = \alpha(c,h,h',w) \leq \varepsilon^a.$$

The probability of being employed and choosing $h$, conditional on $(c, w)$, is therefore

$$G(h|c,w) = g(h) \left\{ \sum_{h'<h} 2g(h') \int_{-\infty}^{\alpha(c,h,h',w)} F^b(\varepsilon,c,h,w)\phi(\varepsilon)\mathrm{d}\varepsilon \right.$$
$$\left. + g(h) \int_{-\infty}^{+\infty} F^b(\varepsilon,c,h,w)\phi(\varepsilon)\mathrm{d}\varepsilon + \sum_{h'>h} 2g(h') \int_{\alpha(c,h,h',w)}^{+\infty} F^b(\varepsilon,c,h,w)\phi(\varepsilon)\mathrm{d}\varepsilon \right\}.$$

Finally the probability of being out of employment at a given wage $w$ in the two offer model is obtained by summing over all the couples $(h, h')$, the probability of preferring not to work

$$\sum_h \sum_{h'} g(h)g(h') \int_{-\infty}^{\infty} \Phi[\frac{1}{\sigma^b} \big( R(w,0) + c^\gamma v(0) \exp(Z^a\beta^a + \sigma^a\varepsilon) + Z^b\beta^b$$
$$- \max(R(w,h) + c^\gamma v(h) \exp(Z^a\beta^a + \sigma^a\varepsilon), R(w,h') + c^\gamma v(h') \exp(Z^a\beta^a + \sigma^a\varepsilon))\big)]\phi(\varepsilon)\mathrm{d}\varepsilon.$$

Since the wage of non-workers is not known, $w$ has to be integrated out in the above expression.

# 5 Empirical results

## 5.1 Parameter Estimates for the Two-Offer Model

Table 3 presents the estimation results for the parameters of preferences and fixed costs. The offer distribution parameters are presented in Table 4. We estimated three different models. The first column presents the estimates of the baseline model (Model 1) in which we treat wages and consumption as exogenous in the determination of hours and employment. That is we set the correlation between the reduced form errors and the unobservable preference errors to zero. This baseline specification also excludes covariates from the specification of the offer distribution. Reduced form estimates for wages, consumption and employment are presented in Table 10 of Appendix A.

The $\phi$ and $\gamma$ parameters refer to the exponents on hours (non-market time) and on consumption as described in the utility specification (1) of section 2. We let these parameters to vary according to the cohort of birth of women.

The next panel refers to the parameters that influence the marginal utility of hours through the specification of $\ln(a)$ in equation (18). We find that cohabiting women have a higher preference for non-market time, and that this preference is also higher when the youngest kid is younger. Following these are the parameters of fixed costs of equation (19). The fixed cost for the reference category of lone mothers who has one kid aged more than 10 years old, and who lives out of the London region is about £26 a week. Cohabitation lowers this cost by £16, and the cost increases with the number of children. Living in London also increases the cost of working by more than £25 per week.

For the two-offer specification of the restricted choice model, described in section (4.2) above, offers are modeled as a mixture of two independent normal distributions. The associated parameter estimates are presented in Table 4. These estimates suggest offers concentrated at full-time (around 38) hours and having a mode at part-time (around 20) hours.

The second column (Model 2) presents the results for a model in which we control for the potential correlation between unobserved heterogeneity terms in preferences, wages (20) and consumption (21). The correlation between consumption and preferences, $\rho(\varepsilon^a, \varepsilon^c)$ is significantly different from zero. Contrary to the Model 1 estimates in column 1, we see from the estimates of second model that there is a significant increase in the $\gamma$ parameter for the elder cohort of women born before 1963. Other parameters are qualitatively similar to the one obtained in the baseline model, although we can note that the reference group

Table 3: Estimation Results for 1997-2002 years: Preferences

| | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| $\phi$: yob $>$ 1963 | 10.09 (0.7056) | 10.04 (0.8612) | 12.698 (1.2845) |
| $\phi$: yob $\leq$ 1963 | 10.072 (0.6944) | 9.962 (0.8445) | 12.719 (1.2709) |
| $\gamma$: yob $>$ 1963 | 0.015 (0.0814) | 0.025 (0.1907) | 0.224 (0.2572) |
| $\gamma$: yob $\leq$ 1963 | 0.005 (0.0712) | 0.515 (0.1658) | 0.881 (0.2261) |
| $\beta^a$: Const | 42.86 (3.0156) | 42.945 (4.0551) | 52.61 (5.8824) |
| $\beta^a$: Cohab | 0.597 (0.0849) | 0.462 (0.0829) | 0.527 (0.1114) |
| $\beta^a$: Kid age 0-4 | 1.54 (0.1123) | 1.512 (0.1371) | 2.092 (0.2257) |
| $\beta^a$: Kid age 5-9 | 0.853 (0.0886) | 0.83 (0.097) | 1.156 (0.151) |
| $\beta^a$: yob $\leq$ 1963 | 0.414 (1.9794) | $-2.366$ (1.9465) | $-2.695$ (2.5787) |
| $\rho(\varepsilon^a, \varepsilon^c)$ | 0 (.) | $-0.24$ (0.0707) | $-0.304$ (0.0996) |
| $\rho(\varepsilon^a, \varepsilon^w)$ | 0 (.) | $-0.041$ (0.0422) | $-0.114$ (0.0624) |
| $\sigma_a$ | 1.521 (0.0833) | 1.573 (0.1255) | 2.052 (0.2022) |
| $\beta^b$: Const | 26.209 (3.6523) | 21.64 (3.7992) | 29.356 (3.5672) |
| $\beta^b$: Cohab | $-16.315$ (2.6547) | $-16.621$ (2.6864) | $-14.128$ (2.5278) |
| $\beta^b$: Kid age 0-4 | 7.169 (3.6202) | 3.884 (3.6587) | 5.004 (3.5055) |
| $\beta^b$: Kid age 5-9 | $-9.861$ (3.1489) | $-11.083$ (3.2013) | $-9.967$ (3.0021) |
| $\beta^b$: yob $\leq$ 1963 | $-5.337$ (3.0258) | $-4.471$ (3.1138) | $-3.62$ (2.8662) |
| $\beta^b$: 2 kids | 2.62 (2.3147) | 2.822 (2.3943) | 2.22 (2.2979) |
| $\beta^b$: 3 kids | 8.44 (2.9414) | 9.3 (3.0094) | 8.551 (2.9081) |
| $\beta^b$: 4 kids or more | 25.848 (4.0176) | 26.663 (3.9866) | 26.069 (3.9383) |
| $\beta^b$: London | 25.136 (3.2027) | 23.425 (3.2361) | 21.761 (3.3442) |
| $\sigma_b$ | 44.759 (1.7037) | 40.649 (1.4824) | 43.768 (1.63) |
| Lik | -2.87189 | -2.86823 | -2.86507 |
| N | 10543 | 10543 | 10543 |

fixed cost decreased from £26 to £21 a week.

The last column of Tables 3 and 4 shows the results for a model in which, we let the distribution of offers to depend on three additional covariates: education, living in London

Table 4: Estimation Results for 1997-2002 years: Offer Distribution

| | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| $m_1$ | 20.417 (1.1108) | 22.804 (1.0054) | 18.834 (1.172) |
| $\sigma_1$ | 19.674 (1.1329) | 19.185 (1.0824) | 19.71 (1.1793) |
| $m_2$ | 38.08 (0.0679) | 38.104 (0.0686) | 38.001 (0.0673) |
| $\sigma_2$ | 1.687 (0.0622) | 1.684 (0.0625) | 1.689 (0.0603) |
| $p_1$: Ref | 0.79 (0.009) | 0.781 (0.0094) | 0.863 (0.0106) |
| Odd-ratios | | | |
| $p_1$: Edu = 2 | 1 (.) | 1 (.) | 0.542 (0.0534) |
| $p_1$: Edu = 3 | 1 (.) | 1 (.) | 0.334 (0.034) |
| $p_1$: London | 1 (.) | 1 (.) | 0.903 (0.1244) |
| $p_1$: year 99-00 | 1 (.) | 1 (.) | 1.008 (0.1081) |
| $p_1$: year 01-02 | 1 (.) | 1 (.) | 0.938 (0.0963) |

and years dummies. These variables enter the mixture parameter of the offer distribution. The last panel of Table 4 shows the odd-ratios of the mixture parameter with respect to each of the variables. The more educated women are, the more likely they are to receive an offer from the higher (full-time) mixture, but we find no statistically significant difference along location and years. Accounting for this heterogeneity also affects the estimates of preferences and fixed cost parameters. From that specification, we see a higher income effect in particular for the older cohort. Moreover, the results show a stronger negative correlation between unobserved heterogeneity terms, and a stronger preference for leisure for women who have young kids.

## 5.2 Model Fit

Table 5 summarizes employment and distribution of hours obtained from simulation of the two-offer model. All three models predict employment with accuracy and do particularly well in replicating the twin peaks of the actual hours distribution. Figure 3 plots the simulated hours distributions against the actual hours distribution. As expected, we see that Model 3, allowing for observable heterogeneity in the distribution of offers, fits the distribution of hours better than the first two specifications.

Table 5 also compares the mean and variance of the log-wage distribution of the data to the one simulated by the model. The lower panel of Table 5 gives a description of the joint distribution of hours and wages. The fit is less precise than the one of unconditional moments but it shows that our model is able to reproduce the positive correlation between hours and wages that is observed in the data.

Table 5: Model fit: hours, employment and wages

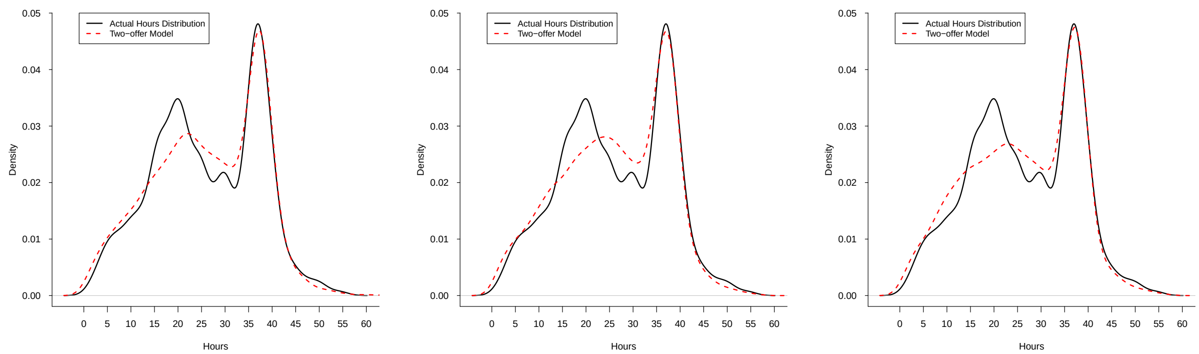
| | Data | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|
| $\overline{E}$ | 0.63 | 0.62 | 0.62 | 0.63 |
| Hours | | | | |
| $\overline{h}\|E = 1$ | 26.23 | 26.20 | 26.12 | 25.92 |
| $sd(h\|E = 1)$ | 11.14 | 11.28 | 11.14 | 11.34 |
| $h\|E = 1$, p 25 % | 18.00 | 18.00 | 18.00 | 17.00 |
| $h\|E = 1$, p 50 % | 25.00 | 27.00 | 27.00 | 27.00 |
| $h\|E = 1$, p 75 % | 37.00 | 36.00 | 36.00 | 36.00 |
| Wages | | | | |
| $\overline{\log(w)}\|E = 1$ | 1.82 | 1.86 | 1.85 | 1.85 |
| $sd(\log(w)\|E = 1)$ | 0.54 | 0.55 | 0.55 | 0.54 |
| $\log(w)\|E = 1$, p 25 % | 1.42 | 1.48 | 1.48 | 1.48 |
| $\log(w)\|E = 1$, p 50 % | 1.77 | 1.85 | 1.83 | 1.84 |
| $\log(w)\|E = 1$, p 75 % | 2.20 | 2.22 | 2.21 | 2.21 |
| Joint distribution | | | | |
| $\overline{h}\|\log(w) \leq 1.5$ | 21.50 | 23.84 | 23.64 | 23.31 |
| $sd(h\|\log(w) \leq 1.5)$ | 10.80 | 11.35 | 11.37 | 11.30 |
| $\overline{h}\|\log(w) \in ]1.5, 2]$ | 27.46 | 25.95 | 25.65 | 25.60 |
| $sd(h\|\log(w) \in ]1.5, 2])$ | 10.71 | 11.26 | 10.92 | 11.10 |
| $\overline{h}\|\log(w) > 2$ | 29.21 | 28.00 | 28.24 | 27.99 |
| $sd(h\|\log(w) > 2)$ | 10.48 | 10.94 | 10.79 | 11.18 |
| $corr(\log(w), h)$ | 0.23 | 0.16 | 0.17 | 0.17 |

**Note:** Simulation obtained from the estimates of the respective models.

## 5.3 Observations rejecting the unrestricted model

From the data we find that about 2.6% (see Table 6) of working women are observed working at hours that do not comply with the left hand side of standard revealed preference inequality (6). For this group we can reject the unrestricted choice model as there are alternative hours of work that strictly dominate the observed choices. This is a nonparametric rejection of the unrestricted choice model in the sense that the rejection does not depend on the specification, provided the utility function is increasing in consumption and

Figure 3: Hours distributions: observed and two offers model




(a) Model 1 (b) Model 2 (c) Model 3

leisure. The actual budget constraints for some of the individuals in this rejection group were used in the upper panel of Figure 2 above.

Table 6: Estimation Results for 1997-2002 years: Rejection of the Unconstrained Model

| | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| NP-Reject (%) | 2.57 | 2.57 | 2.57 |
| $\phi$-reject (%) | 10.01 | 10.01 | 8.69 |

In addition to these 2.6%, we observe 0.4% of working women who would earn more by staying out of employment. Again these observations reject the model whenever the utility function has the usual monotonicity properties and the fixed cost of work is positive.

From equation (6), conditional on the model specification and the data, parametric rejection of the standard model with unrestricted hours choices only depends on the $\phi$ parameter. Thus, the extent of the rejection is a function of this parameter. In order to assess the extent of the rejection, we use the value of $\phi$ obtained from the estimation of the third model: 8.69% of working women violate the revealed preference inequality. Table 7 contrasts the characteristics of these observations with the rest of the sample. The 'non optimizers' are more often lone mothers than married ones, their wage is lower than average and, as Figure 4 shows, their distribution of hours worked is shifted to the left.

## 5.4 Elasticities with Linear Budget Constraints

To further describe the preferences underlying the model, we compute the distribution of Frisch and Marshallian elasticities at the intensive margin assuming no hours restrictions

Table 7: Observations rejecting the unrestricted choice model

| | Observations... | |
|---|---|---|
| | ...not rejecting the Unrestricted Model | ...rejecting the Unrestricted Model |
| Proportion among 'in work' women | 0.91 | 0.09 |
| Age at end of studies | 17.52 | 16.62 |
| Age | 37.43 | 36.24 |
| Hourly wage | 7.42 | 5.33 |
| Marginal tax rate | 2.35 | -1.02 |
| Usual weekly hours | 26.79 | 20.42 |
| Log of consumption | 5.48 | 5.00 |
| Number of kids | 1.78 | 1.92 |
| A kid younger than 4 | 0.35 | 0.29 |
| The youngest kid between 5 and 10 | 0.35 | 0.41 |
| London | 0.08 | 0.06 |
| Cohabitant | 0.87 | 0.46 |
| Spouse in work | 0.84 | 0.29 |
| Out of work income | 358.32 | 205.31 |
| In work income | 510.86 | 246.12 |

and a linear budget constraint.

Frisch elasticities hold the marginal utility of consumption constant and, in our additive utility specification (1), the labour supply elasticity just depends on $\phi$ and $L$. The Marshallian elasticities account for the change in consumption that is induced by the within period change in labour earnings, holding non-labour income constant. The distribution of estimated elasticities are displayed in Figure 5. The estimated Frisch labour supply elasticities are positive across the distribution and are moderately sized. The Marshallian elasticities account for the income effect. The results in Figure5 show these are smaller, as expected, and can be negative. These estimates are relatively modest in size but lie in the range of estimates of intensive labour supply elasticities found in the literature, for example see Blundell and MaCurdy (1999).

Figure 4: Hours distributions and rejection of the unrestricted model

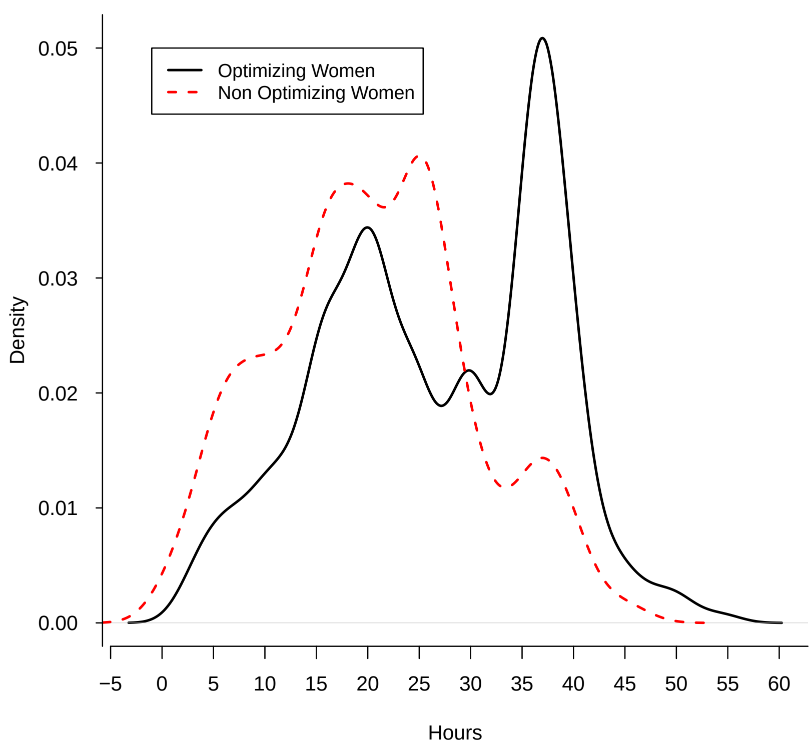


Figure 5: Intensive Margin Elasticities with Linear Budget Constraints

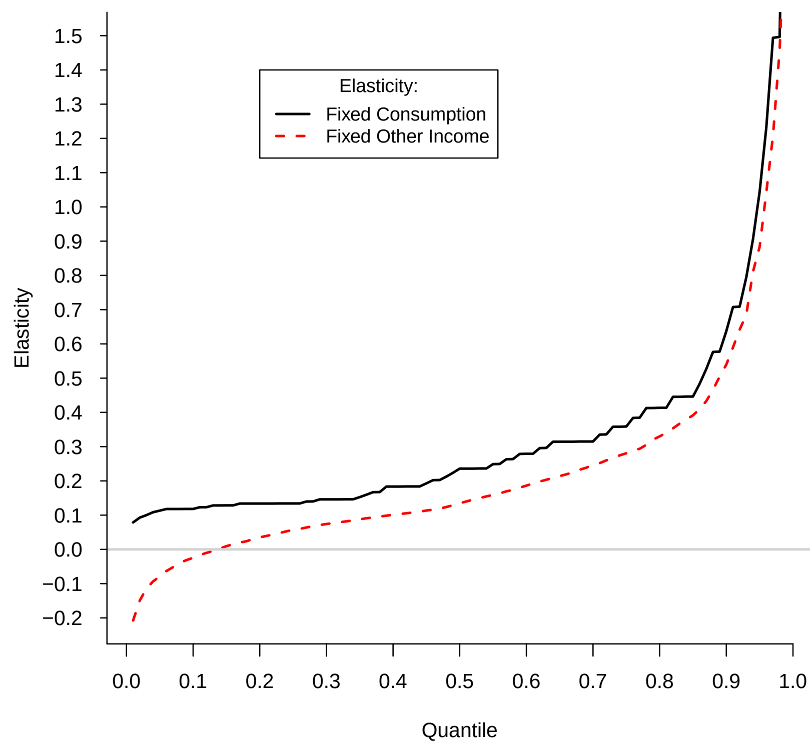



Table 8: Estimation Results for 1997-2002 years: Elasticity Assuming Linear Budget Constraint

| | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Fixed Consumption | | | |
| mean | 0.438 | 0.442 | 0.348 |
| q5 | 0.143 | 0.143 | 0.113 |
| q25 | 0.169 | 0.171 | 0.134 |
| q50 | 0.297 | 0.299 | 0.236 |
| q75 | 0.452 | 0.457 | 0.359 |
| q95 | 1.317 | 1.323 | 1.045 |
| Fixed Other Income | | | |
| mean | 0.436 | 0.376 | 0.244 |
| q5 | 0.14 | 0.017 | -0.079 |
| q25 | 0.168 | 0.129 | 0.057 |
| q50 | 0.29 | 0.227 | 0.135 |
| q75 | 0.451 | 0.405 | 0.281 |
| q95 | 1.312 | 1.216 | 0.884 |

## 5.5 Model Simulations

To describe the importance of hours restrictions and how would women would react to a wage increase we present some model simulations. In these simulations we use observed consumption and covariates from the data. Conditional on these variables, for each woman $i$, we draw wages $w_i^s$, preferences $a_i^s$ and fixed costs $b_i^s$ from the estimated distributions (Model 3). We evaluate expression (3)

$$V(h, w_i^s, b_i^s, c_i, a_i^s) = a_i^s v(h) + c_i^{-\gamma} \left(R(w_i^s, h) - b_i^s\right)$$

whose maximization over the whole possible set of hours yields the chosen unconstrained supply of hours $h_i^u$:

$$h_i^u = \arg \max_{h=0, h\in\{1,\ldots,H\}} V(h, w_i^s, b_i^s, c_i, a_i^s).$$

In the case of the two offers model, we constrain women to choose between two different offers $h_i^{s,1}$ $h_i^{s,2}$ that are drawn from the estimated offer distribution $g$. For each individual, the choice of hours worked is then:

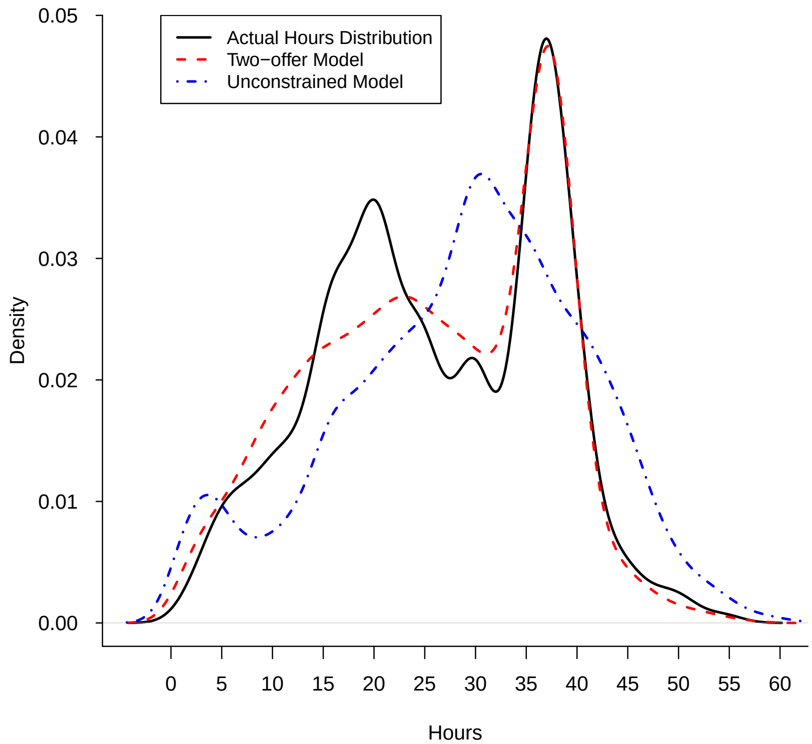
$$h_i^c = \arg \max_{0, h_i^{s,1}, h_i^{s,2}} V(h, w_i^s, b_i^s, c_i, a_i^s).$$

In both cases women can always choose not to work.

### 5.5.1 Importance of Hours Restrictions

Comparing the two simulated distributions of hours, we find an employment rate of 76.01 percent in the unconstrained case, larger than the 62.31 percent obtained in the constrained case. It appears that the restrictions in the two-offer model significantly reduce the number in employment relative to those who would choose to work if they were not constrained. Figure 6 shows the prediction of the hours distribution using the estimated preference parameters assuming that women are not constrained. The resulting distribution of hours is of course very different, reflecting the importance of the specification of the distribution of offers. In addition to the large difference in employment, the modes of the hours distribution move downwards when one goes from the unconstrained to the two-offer case, as well as the average (28.7 vs 26.3 hours).

Figure 6: Predicted hours with and without restrictions




### 5.5.2 Elasticities with Non-Linear Budget Constraints

We now focus on the impact of an increase of before-tax-income on labour supply decisions. We start from the baseline case and, from baseline simulated wage, we consider an increase of 10%. To do so, we compute the corresponding budget constraint functions $R(w_i^s(1+x), h)$, where $x = 10\%$.

Using these new budget constraints, we derive new labour supply decisions. In the case of the unrestricted model, we have:

$$h_i^{(u,x)} = \arg \max_{h=0, h \in \{1,\ldots,H\}} V(h, w_i^s(1+x), b_i^s, c_i, a_i^s)$$

In the case of the restricted model, we keep the offer fixed. The new chosen hours are obtained from:

$$h_i^{(c,x)} = \arg \max_{0, h_i^{s,1}, h_i^{s,2}} V(h, w_i^s, b_i^s, c_i, a_i^s).$$

In both cases, the intensive margin elasticity is obtained as:

$$\varepsilon_{\text{Intensive}} = \frac{1}{\sharp(h_i^k > 0)} \sum_{i/h_i^k > 0} \frac{(h_i^{(k,x)} - h_i^k)}{h_i^k} . \frac{1}{x}.$$

and the extensive margin elasticity as:

$$\varepsilon_{\text{Extensive}} = \frac{(\overline{E_i^{(k,x)}} - \overline{E_i^k})}{\overline{E_i^k}} . \frac{1}{x}.$$

where $\overline{E_i^{(k,x)}}$ and $\overline{E_i^k}$ are equal to one if $h_i^{(k,x)}$ and $h_i^k$ are respectively positive.

These elasticities are obtained by keeping $c_i$ constant. As in the case of linear budget constraints, a second type of elasticity can be obtained by fixing other income $\mu_i$ in the current period budget constraint:

$$c_i = R(w_i^s, h) - b_i^s + \mu_i$$

Fixing other income is equivalent to make the assumption that any increase in earnings will be directly consumed by the individual, who does not increase or decrease savings by definition. So the future value of labour market decisions is constant, and changes in the value today can be summarized by changes in the direct utility function. As a consequence, labour supply decisions will maximize the following function:

$$U(h, w_i^s, b_i^s, \mu_i^s, a_i^s) = \frac{[R(w_i^s, h) + \mu_i - b_i^s]^{1-\gamma}}{1-\gamma} + a_i^s \frac{(L-h)^{1-\phi}}{1-\phi}$$

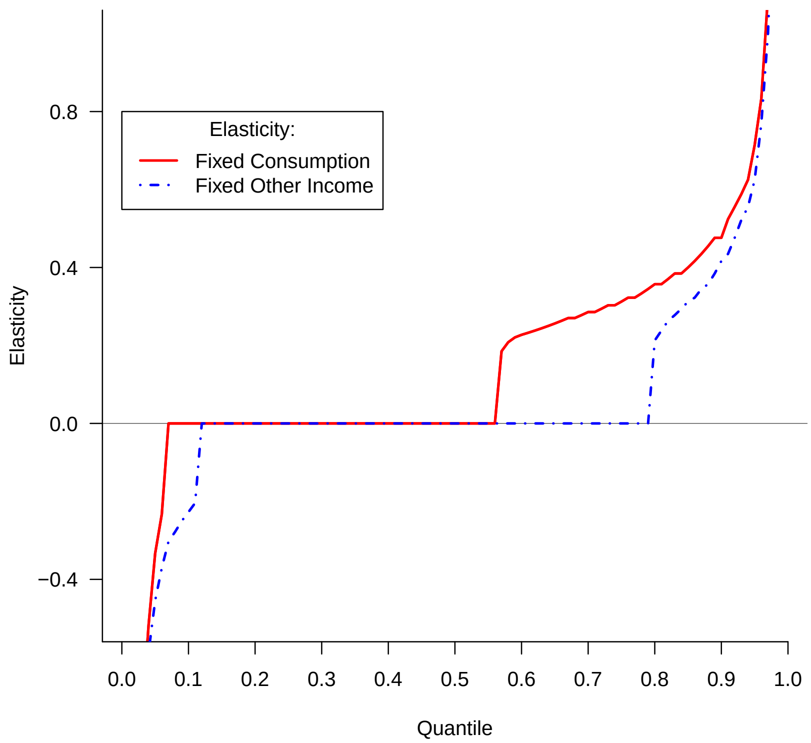
and the choice set will depend on the hours constraints that women face.

Figure 7 presents the distribution of elasticities corresponding to a 10% change in wages. The first two columns of Table 9 presents the mean of these elasticities that were obtained without hours constraints by fixing either consumption or other income.

As in the linear case presented in Figure 5 fixed-income-elasticities are higher than elasticities obtained by fixing other income. Accounting for non-linearities in the budget constraint leads to higher elasticities (.15 vs .348 in the case of the Frisch Elasticity). We can also note that in the non-linear case many women do not react to a change in wages leading to a large range of zero elasticities.

Table 9 also shows the differences in elasticity if we account for hours restrictions. In

Figure 7: Elasticities with Non-linear Budget Constraints




that case, intensive margin elasticities are close to zero (.01 when we fix consumption and .02 when we fix other income). But extensive margin elasticities are higher in the constrained than in the unconstrained case: 0.29 vs 0.21 if consumption is kept constant and .18 vs .17 if we fix other income.

Table 9: Elasticities with non-linear budget constraints

| | Unconstrained | | 2 offers | |
|---|---|---|---|---|
| | Fixed Consumption | Fixed Other income | Fixed Consumption | Fixed Other income |
| Extensive Margin | 0.21 | 0.17 | 0.29 | 0.18 |
| Average Intensive Margin | 0.15 | 0.05 | 0.01 | 0.02 |

## 6 Summary and Conclusions

In this paper we have developed a new model of employment and hours in which individuals face restrictions over possible hours choices. Hours choices are made on a random subset of possible hours, and observed hours reflect both the distribution of preferences and the limited choice set. Consequently observed choices do not necessarily satisfy the revealed preference conditions of the standard labour supply model with unrestricted hours choices.

The leading example we explore in detail in this paper is of individuals selecting from two offers. We show first that, when the offer distribution is known, preferences can be identified. Second, we show that, where preferences are known, the offer distribution can be fully recovered. We then develop conditions for identification of both the parameters of preferences and of the offer distribution.

The new framework is used to study the labour supply choices of a large sample of women in the UK, accounting for nonlinear budget constraints and fixed costs of work. With nonlinear budget sets observed labor supply may not be reconciled with standard optimisation theory. The results point to a small but important group of workers who fail the standard choice model with unrestricted choices. This motivates the estimation of a two-offer model, which provides a satisfactory fit of the data. We specify a mixture of normals for the offer distribution in which the mixture probability is allowed to depend on education, region and calendar time. The estimated offer distribution features the observed twin peaks centered around full-time and part-time hours.

Accounting for restrictions on the choice set changes the estimated pattern of preference parameters. Individuals appear more responsive once restrictions are accounted for and the model simulations predict a higher level of employment were restrictions to be removed.

The two-offer specification we adopt in the application in this paper is nevertheless restrictive. In future work we intend to develop the n-offer case, allowing a much more flexible specification of the effective choice set. In particular, we could allow the number of alternative choices to vary by location, age, education and point in the business cycle.

# A Wage and Consumption Equations

Table 10: First Stage Reduced Form (1997-2002 combined)

| | $\log(w)$ | $\log(c)$ | $E$ |
|---|---|---|---|
| Const. | $-6.168$ | 0.134 | $-13.413$ |
| | (0.9495) | (0.6896) | (3.2841) |
| y1998 | $-0.025$ | $-0.04$ | $-0.042$ |
| | (0.0204) | (0.0158) | (0.0752) |
| y1999 | 0.02 | 0.012 | $-0.005$ |
| | (0.0201) | (0.0156) | (0.0746) |
| y2000 | 0.079 | 0.035 | 0.023 |
| | (0.0203) | (0.016) | (0.0764) |
| y2001 | 0.144 | 0.028 | 0.101 |
| | (0.0196) | (0.0154) | (0.0742) |
| y2002 | 0.119 | 0.035 | 0.134 |
| | (0.02) | (0.0158) | (0.0763) |
| age | 2.336 | 0.804 | 2.34 |
| | (0.6744) | (0.5099) | (2.4233) |
| ages | $-0.538$ | $-0.182$ | $-0.55$ |
| | (0.1781) | (0.1361) | (0.6488) |
| agec | 0.039 | 0.011 | 0.015 |
| | (0.0154) | (0.0118) | (0.0565) |
| edu | 0.384 | 0.276 | 0.984 |
| | (0.0426) | (0.0302) | (0.146) |
| edu2 | $-0.782$ | $-0.752$ | $-3.096$ |
| | (0.1067) | (0.0728) | (0.3495) |
| edu*age | 0.061 | 0.176 | 0.748 |
| | (0.0406) | (0.0312) | (0.1552) |
| london | 0.161 | 0.033 | $-0.462$ |
| | (0.0226) | (0.0156) | (0.0731) |
| 2 kids | $-0.041$ | 0.053 | $-0.376$ |
| | (0.0135) | (0.0106) | (0.0517) |
| 3 kids | $-0.126$ | 0.051 | $-0.825$ |
| | (0.0219) | (0.0144) | (0.068) |
| 4 kids or more | $-0.205$ | 0.036 | $-1.551$ |
| | (0.043) | (0.0215) | (0.1042) |
| Cohab | . | 0.679 | 0.937 |
| | (.) | (0.0112) | (0.0529) |
| Kid btw 0 4 | . | $-0.023$ | $-0.811$ |
| | (.) | (0.0156) | (0.0766) |
| Kid btw 5 9 | . | $-0.051$ | $-0.19$ |
| | (.) | (0.0137) | (0.0686) |
| $R(w,0)$ | . | . | $-5.312$ |
| | (.) | (.) | (3.2897) |
| $\sigma$ | 0.469 | 0.462 | 1 |
| | (.) | (.) | (.) |
| $\rho(.,\varepsilon_E)$ | 0.449 | 0.628 | 1 |
| | (.) | (.) | (.) |
| $\rho(.,\varepsilon_w)$ | 1 | 0.351 | 0.449 |
| | (.) | (.) | (.) |

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