Joint causal inference on observational and experimental datasets

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What If?, 10th December, 2016

Part I

Introduction

Causal inference: learning causal relations from data

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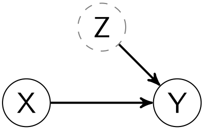

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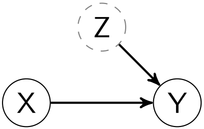

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- Causal inference = structure learning of the causal DAG
- Traditionally, causal relations are inferred from interventions.
- Sometimes, interventions are unethical, unfeasible or too expensive

Causal inference from observational and experimental data

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Learn as much causal structure as possible from observations, integrating background knowledge and experimental data.

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Advantage of constraint-based methods:

- can handle latent confounders naturally

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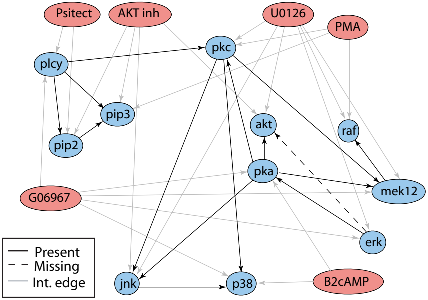
- can formulate joint inference on observational and experimental data and learn the targets of interventions, e.g. [Eaton and Murphy, 2007].



Example from [Eaton and Murphy, 2007]

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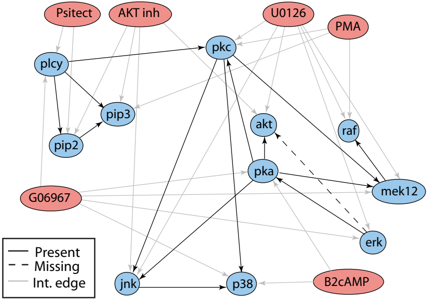

Example from [Eaton and Murphy, 2007]

Goal: Can we perform joint inference using constraint-based methods?

Part II

Joint Causal Inference

Joint Causal Inference: Assumptions

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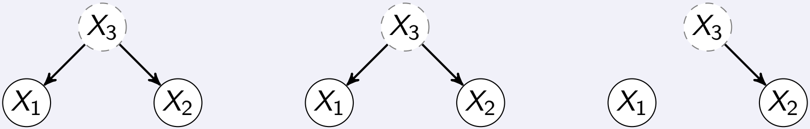

Dataset D_1 unknown ints | Dataset D_2 unknown ints | Dataset D_3 do X_1

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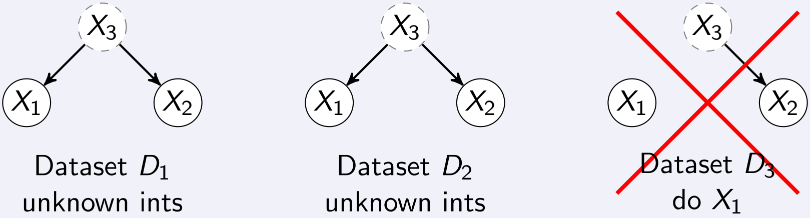

Note: cannot handle certain intervention types, e.g. perfect interventions.

Joint Causal Inference: SCM

We introduce two types of dummy variables in the data:

- a **regime variable** R, indicating which dataset D_r a data point is from
- **intervention variables** $\{I_i\}_{i \in \mathcal{I}}$, functions of R

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We assume that we can represent the whole system as an acyclic SCM:

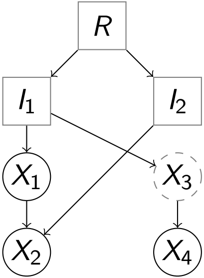
$$\begin{cases} R &= E_R, \\ I_i &= g_i(R), \quad i \in \mathcal{I}, \\ X_j &= f_j(X_{\mathrm{pa}(X_j)\cap\mathcal{X}}, I_{\mathrm{pa}(X_j)\cap\mathcal{I}}, E_j), \quad j \in \mathcal{X}, \end{cases}$$

$$P\big((E_k)_{k\in\mathcal{X}\cup\{R\}}\big) = \prod_{k\in\mathcal{X}\cup\{R\}} P(E_k).$$

We represent the SCM with a causal DAG $\mathcal{C}$ representing all datasets jointly:

R	I_1	I_2	X_1	X_2	X_4
1	20	0	0.1	0.2	0.5
1	20	0	0.13	0.21	0.49
1	20	0	...	...	...
2	20	1	...	...	...
3	30	0	...	...	...
4	30	1	...	...	...

4 datasets with 2 interventions



Causal DAG $\mathcal{C}$

We assume Causal Markov and Minimality hold in $\mathcal{C}$.

Joint Causal Inference: Faithfulness violations

Causal Faithfulness assumption:

$$\boldsymbol{X} \perp\!\!\!\perp \boldsymbol{Y} \mid \boldsymbol{W} \implies \boldsymbol{X} \perp_d \boldsymbol{Y} \mid \boldsymbol{W} \; [\mathcal{C}]$$

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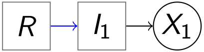

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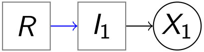

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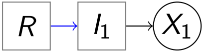

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Solution: restrict deterministic relations to only $\forall i \in \mathcal{I}: \; I_i = g_i(R)$

D-Faithfulness: $\boldsymbol{X} \perp\!\!\!\perp \boldsymbol{Y} \,|\, \boldsymbol{W} \implies \boldsymbol{X} \perp_D \boldsymbol{Y} \,|\, \boldsymbol{W} \; [\mathcal{C}]$.

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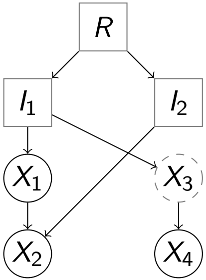

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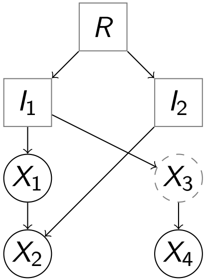

Problem: Current constraint-based methods cannot work with JCI, because of faithfulness violations.

Part III

Extending constraint-based methods for JCI

A simple strategy for dealing with faithfulness violations

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Conjecture: sound also for a larger class of deterministic relations.

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- cannot handle partial inputs
- cannot exploit the rich background knowledge in JCI

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⟹ we implement the strategy in an extension of ACI called:

Rephrasing ACI rules in terms of d-separations

ACI rules:

Example

For X, Y, $\boldsymbol{W}$ disjoint (sets of) variables:

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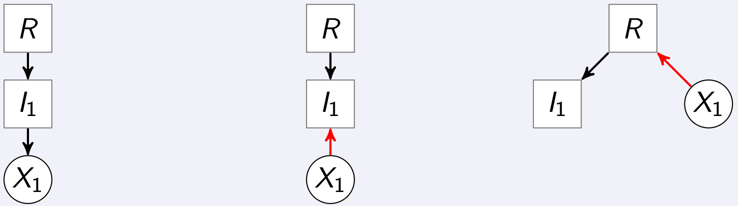

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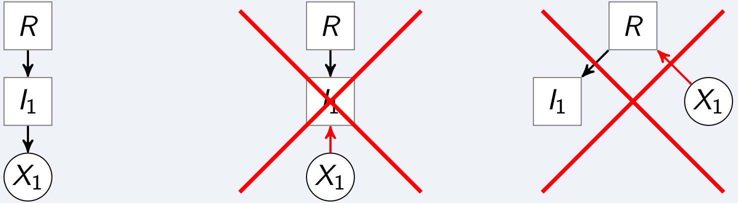
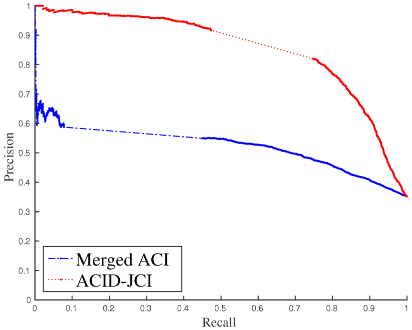


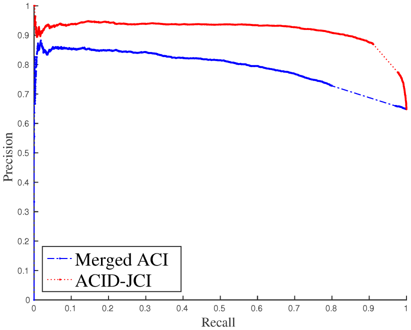
Part IV

Evaluation

Simulated data accuracy: example Precision Recall curve



Ancestral ("causes") relations



Non-ancestral ("not causes")

- Precision Recall curves of 2000 randomly generated causal graphs for 4 system variables and 3 interventions
- ACID-JCI substantially improves accuracy w.r.t. merging learnt structures (merged ACI)

Conclusion

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Future work:

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Working paper: `https://arxiv.org/abs/1611.10351`

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Collaborators:

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Ancestral Causal Inference (ACI)

- Weighted list of inputs: $I = \{(i_j, w_j)\}$:
  - E.g. $I = \{\ (Y \perp\!\!\!\perp Z \,|\, X, 0.2), \ (Y \not\perp\!\!\!\perp X, 0.1)\}\ \}$
  - Any consistent weighting scheme, e.g. frequentist, Bayesian
- For any possible ancestral structure C, we define the loss function:

$$\mathcal{L}oss(C, I) := \sum_{(i_j, w_j) \in I:\ i_j \text{ is not satisfied in } C} w_j$$

- Here: "i_j is not satisfied in C" = defined by ancestral reasoning rules
- For each possible causal relation $X \dashrightarrow Y$ provide score:

$$Conf(X \dashrightarrow Y) = \min_{C \in \mathcal{C}} \mathcal{L}oss(C,\ I + (X \not\dashrightarrow Y, \infty))$$

$$- \min_{C \in \mathcal{C}} \mathcal{L}oss(C,\ I + (X \dashrightarrow Y,\ \infty))$$