Dynamics and Composition of the Mantle: From the Atomic to the Global Scale

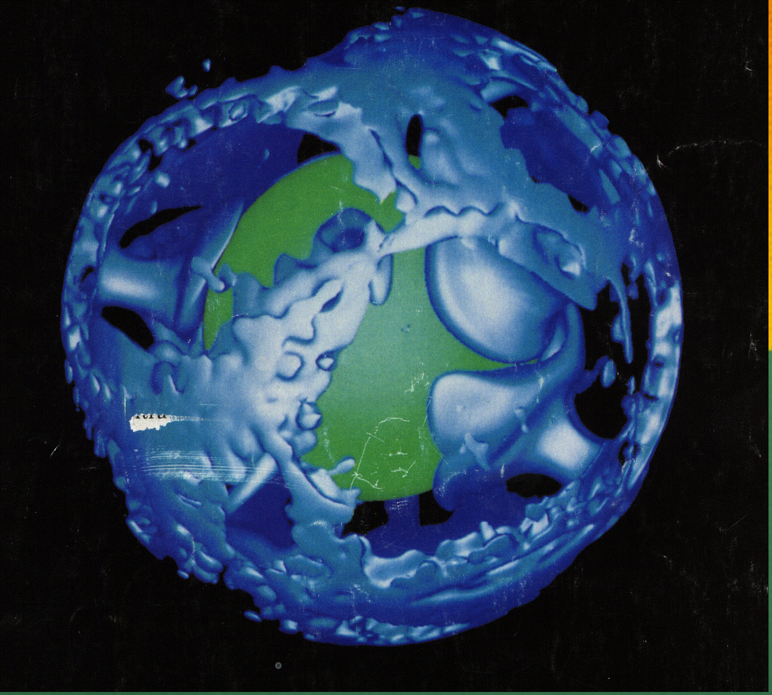
Day	Lecturer	Lectures
2	CLB	Mantle Flow- Deformation; Fluid Mantle
2	CLB	Mantle Flow- Governing Equations; Theory and applications
Tutorial 2: Modelling the geoid and dynamic topography		
3	CLB	Geoid and Dynamic Topography
3	CLB	Dynamics of Plate Motions
Tutorial 2: Modelling the geoid and dynamic topography (continued)		

Short time-scales

Earthquakes

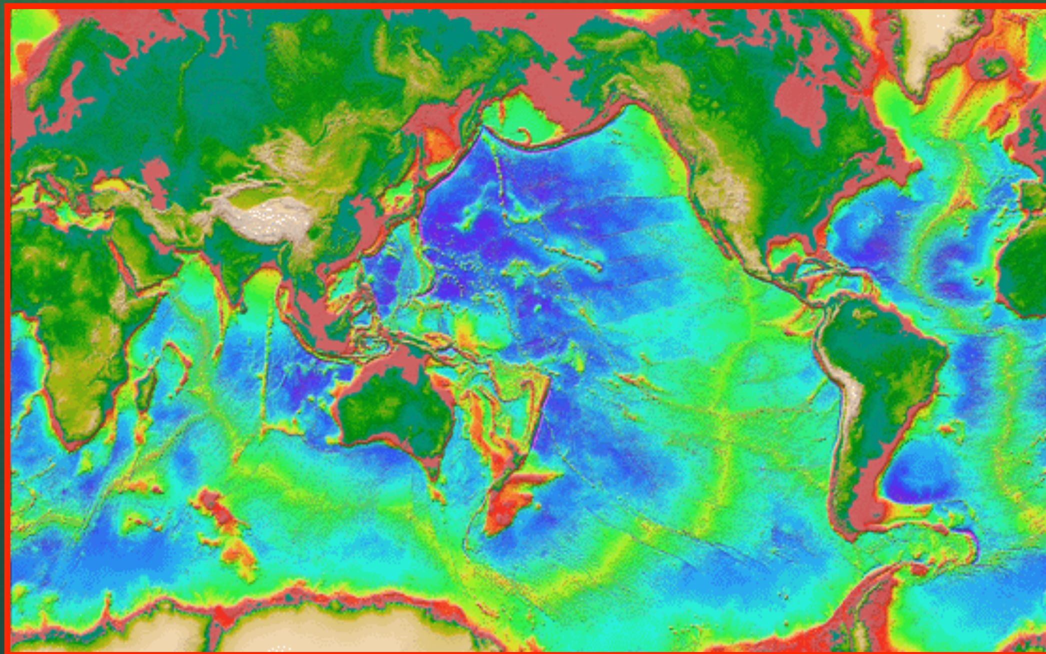
Volcanoes

Long Time-Scales

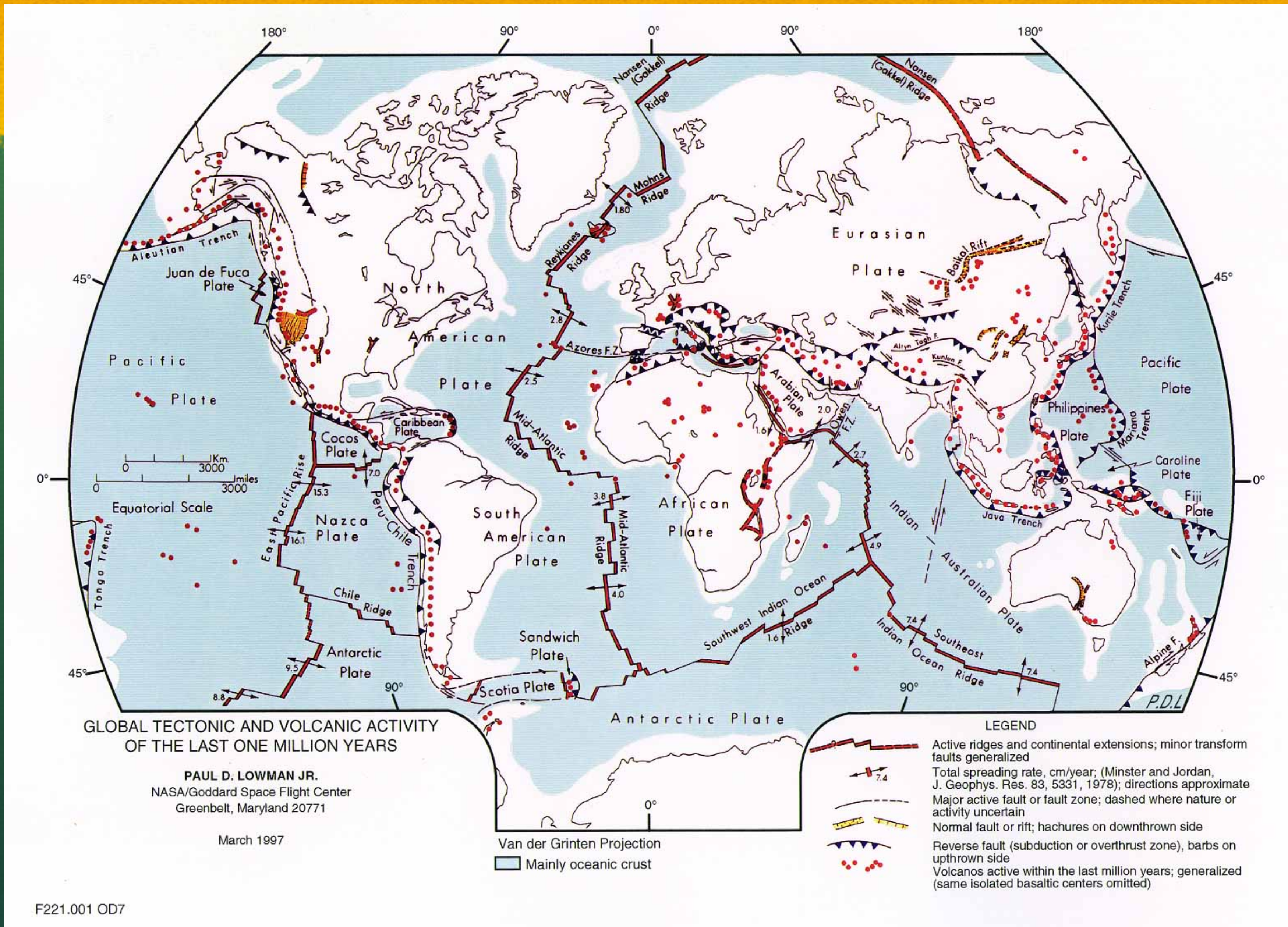


Mantle Convection

Topography



Result of Plate Tectonics



Stress, Strain and Plate Tectonics

What is Tectonic Activity?

For the Earth we mean the large-scale strains or deformation that occur over geological time-scales. There are other geodynamic processes over shorter time-scales, but for now we will deal only with long-term processes.

What are the stresses (or forces) that cause this activity?

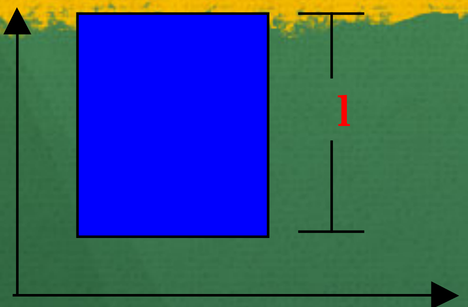
That is the major question we are trying to answer. What are the magnitude, nature, and origin of the stresses (forces) causing tectonic activity?

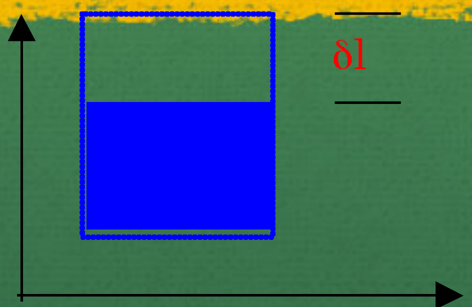
Types of Geologic Strain

Types of Geologic Strain

How do we define strain?

Measure of the change of shape and size of a body

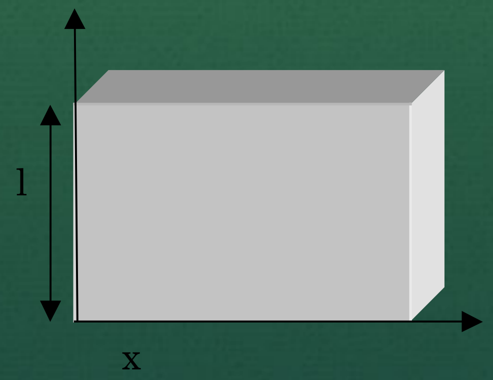


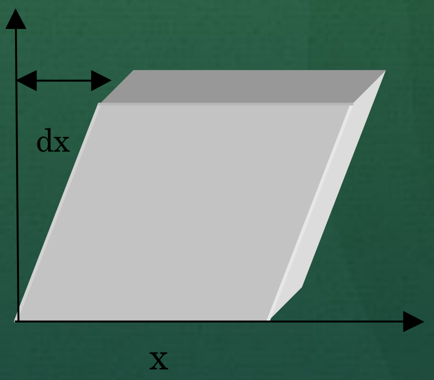


Strain $\varepsilon = \dfrac{\partial \ell}{\ell}$

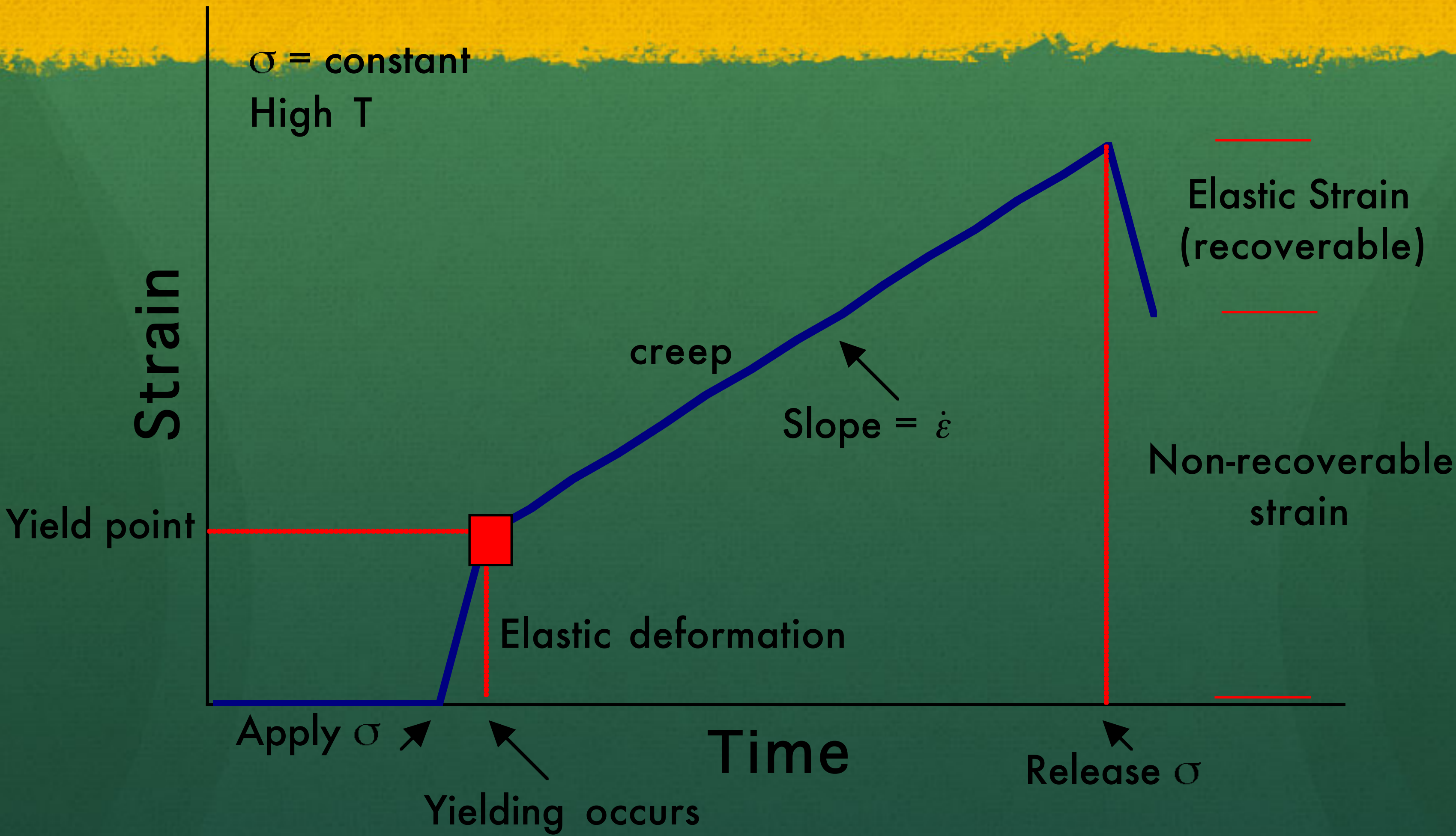
Stress $\sigma = \dfrac{F}{A}$ (Pressure)

Strain $\varepsilon = \dfrac{dx}{\ell}$





Types of strain



Types of strain

What are the different types of strains?

I) Recoverable (elastic)

II) Non-Recoverable (permanent)

i. Brittle Failure

ii. Creep (ductile deformation)

Recoverable Strain- mostly associated with small amounts of deformation

Small stress ← → Small Strain

When Stress (or Force) is released snaps back to original position

Stress is proportional to amount of strain (Hooke's Law)

Non-Recoverable (anelastic) Strain- Associated with large deformations

Large Stresses applied slowly (small strain rate) or fast (large strain rate)

Stress is proportional to strain rate

Strain rate $\dot{\varepsilon} = \frac{d\varepsilon}{dt}$

Brittle	Creep
Low T	High T
Fast $\dot{\varepsilon}$	Slow $\dot{\varepsilon}$

Quantifying types of strain

We need to relate stress to strain for both types of strain. To do so we need the properties of the material (moduli)

Elastic Strain is related to the elastic or rigidity moduli (*G*).

$$G = \frac{\sigma}{\varepsilon}$$

Anelastic Strain to viscosity (η).

$$\eta = \frac{\sigma}{\dot{\varepsilon}}$$

What are some typical values for the solid Earth?
Let's work it out

Strain and Strain Rate

Elastic ~ .2-.5%

Anelastic ~ 100%!

Strain rate ~ $10^{-15} s^{-1}$

In the Earth clearly the permanent, anelastic deformation is much greater than the elastic strain. But over what time scales? We saw that what type of strain develops depends on the P and T conditions and the strain rate. So what is an appropriate characteristic time scale?

Maxwell Relaxation Time

i.e.

How long before anelastic strain becomes the dominant mode of deformation?

Maxwell Relaxation Time

$$\tau_M = \frac{\varepsilon}{\dot{\varepsilon}} = \frac{\sigma/G}{\sigma/\mu} = \frac{\mu}{G}$$

time<< τ_M → Elastic Behavior

time>> τ_M → Fluid (creep) Behavior

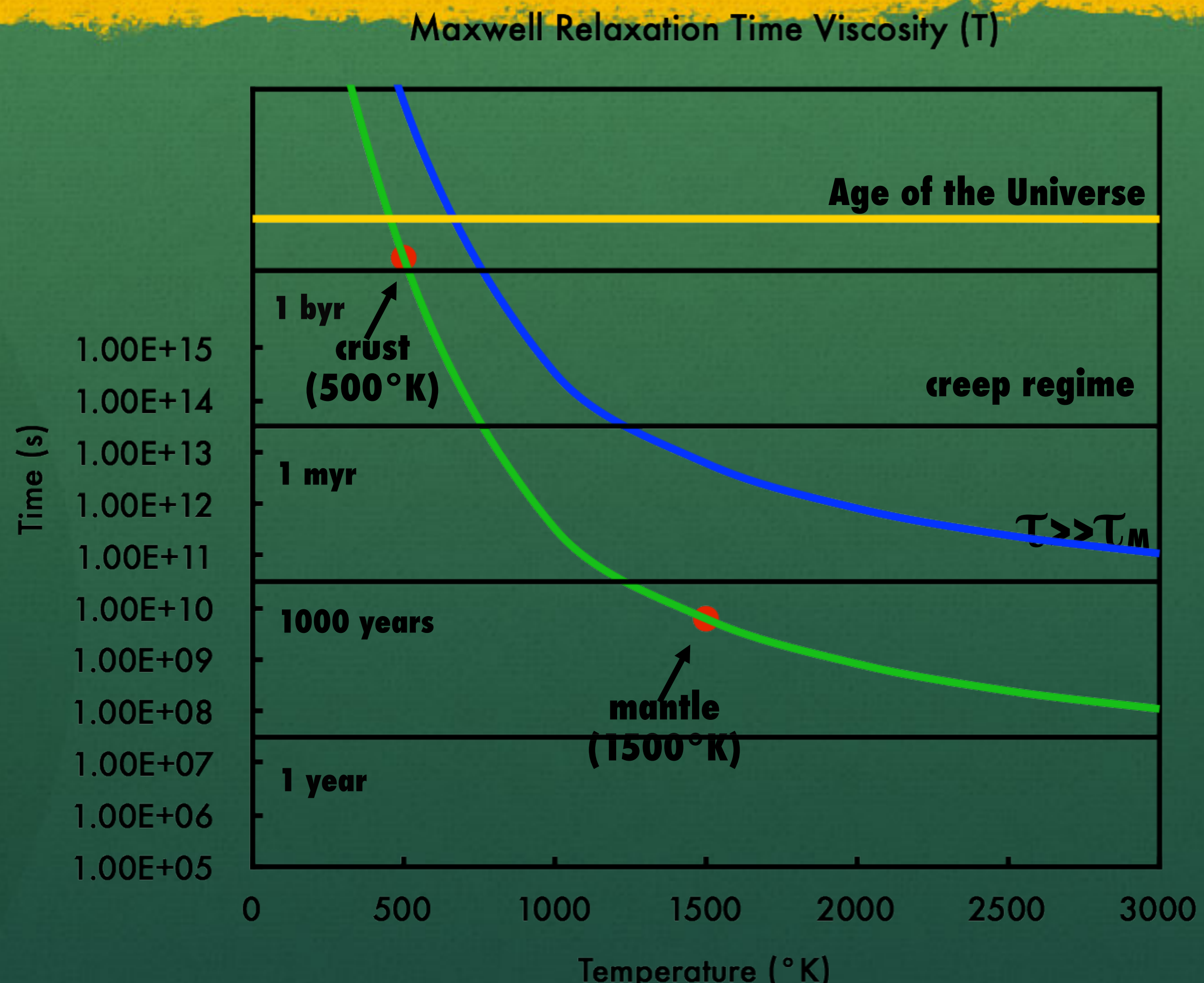
Let's get some characteristic values for the Earth:

Material	μ(Pa s)	G (GPa)	τ_M (s)	
Na-Ca glass (250°C)	4×10^{11}	25	20	
Salt (200°C)	3×10^{16}	20	2×10^6	17 days
Ice (0°C)	10^{13}	4	2.5×10^3	90 minutes
Upper Mantle (1300 °C)	10^{20}	50	2×10^9	300 yr

Mantle creep is clearly the dominant mechanism of deformation for geological time scales $t > 10^{10}$ to 10^{11} s ~ 10^3 to 10^9 years >> τ_M

Maxwell Relaxation Time

Material properties strong function of temperature!



Maxwell Relaxation Time

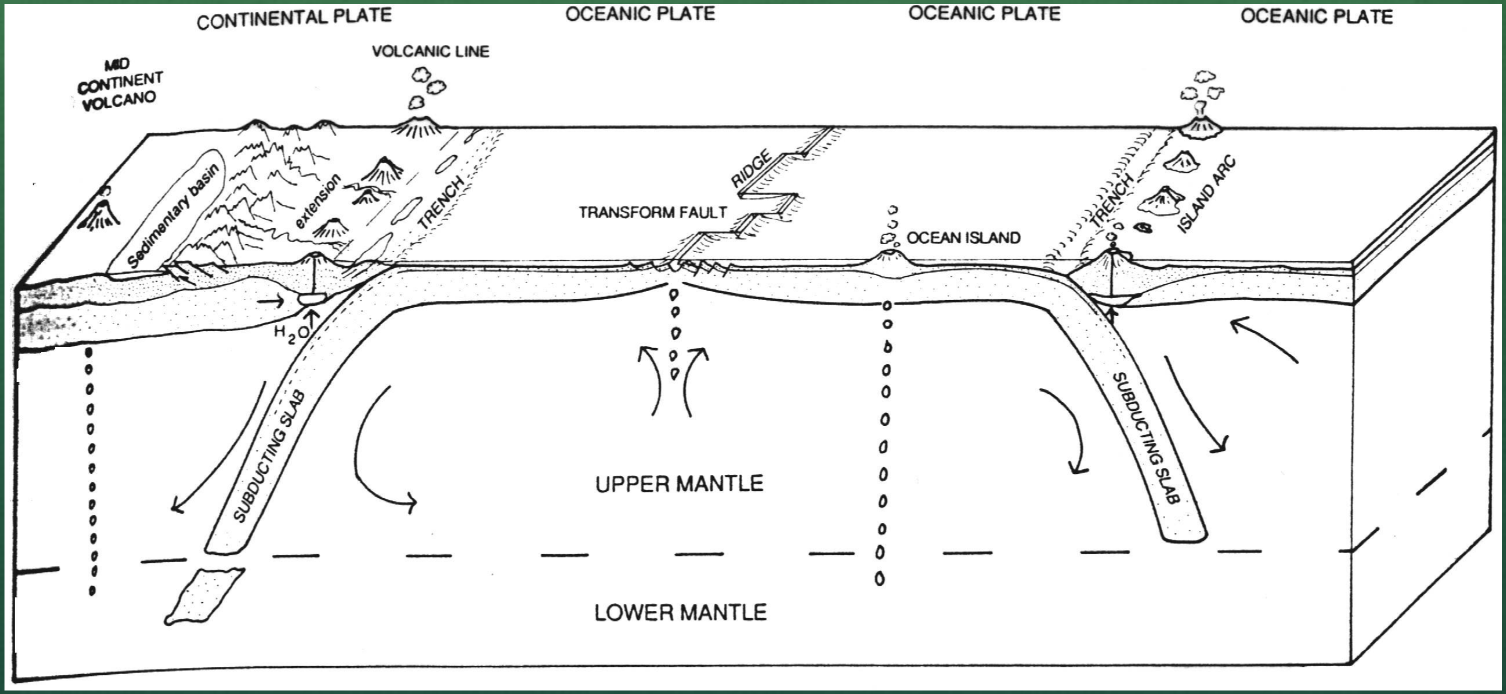
This pictures shows to first order that

1)Only hot planets ($T_{interior}$ > 1500 K) can be "active"

2)Crust and near-surface rocks cannot deform by creep on geological time scales

In the Earth we have – to first order - the simple picture – of plate tectonics

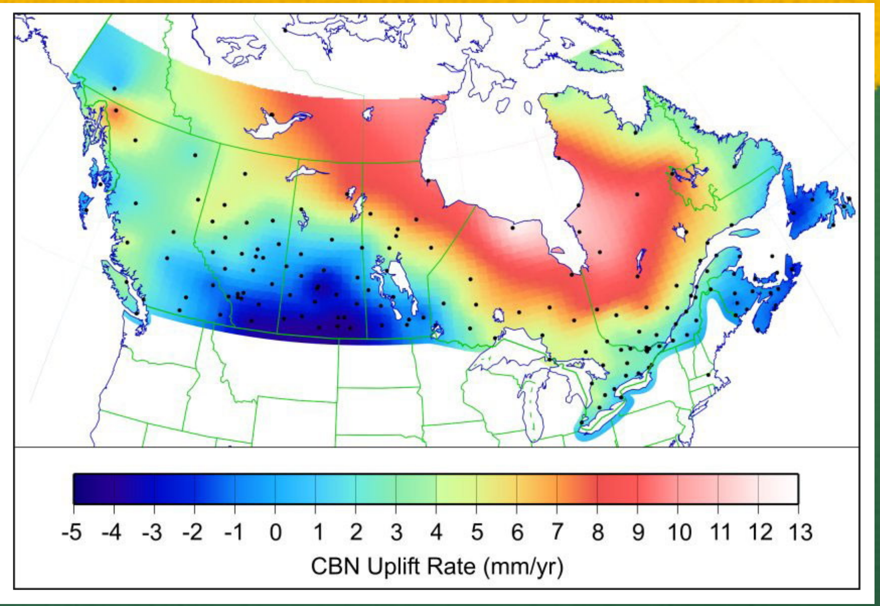
Cold brittle plates over hot creeping mantle



How do we know?

Isostasy; Post-Glacial Rebound

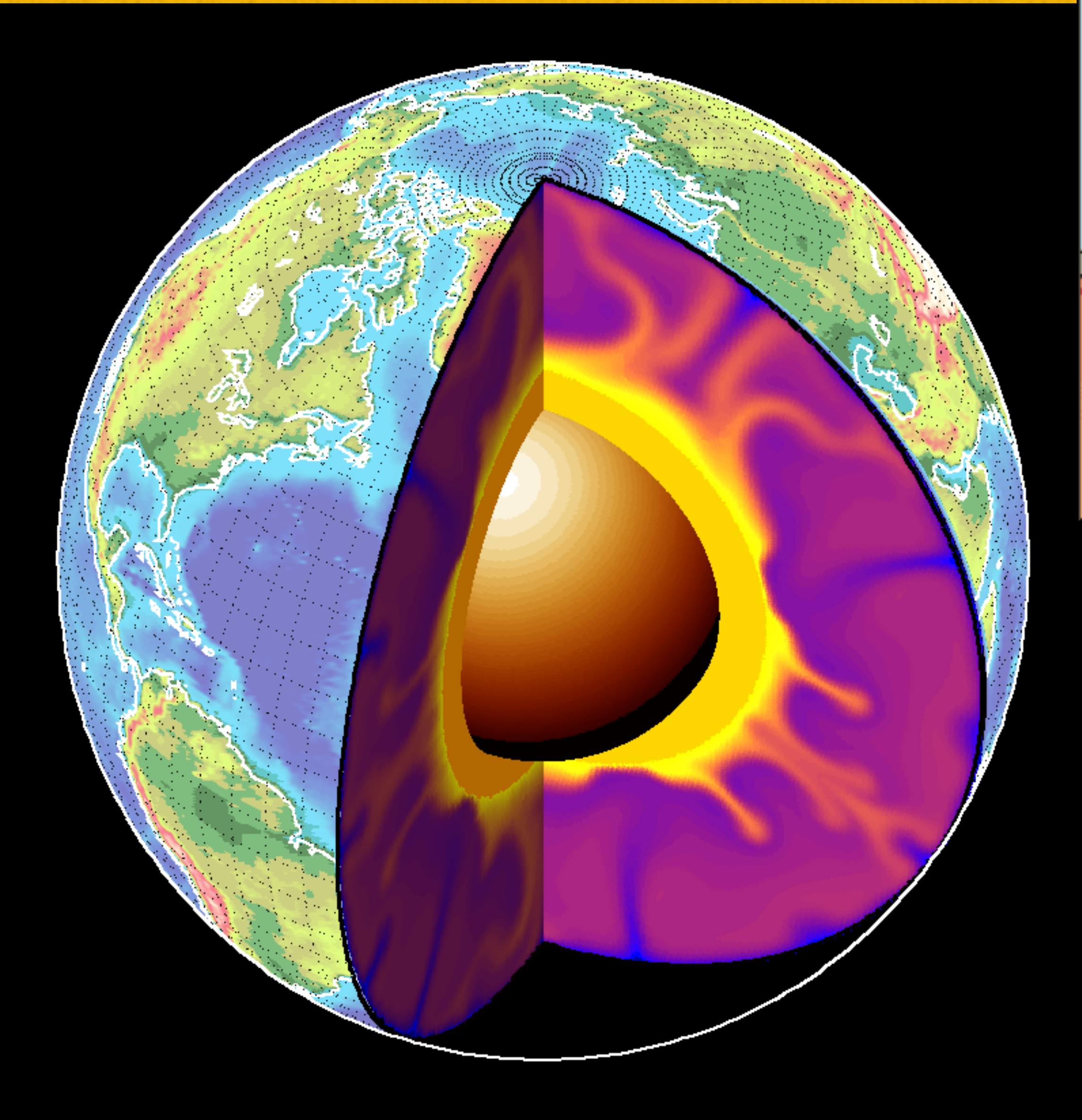
Richmond Gulf, Hudson's Bay, Canada

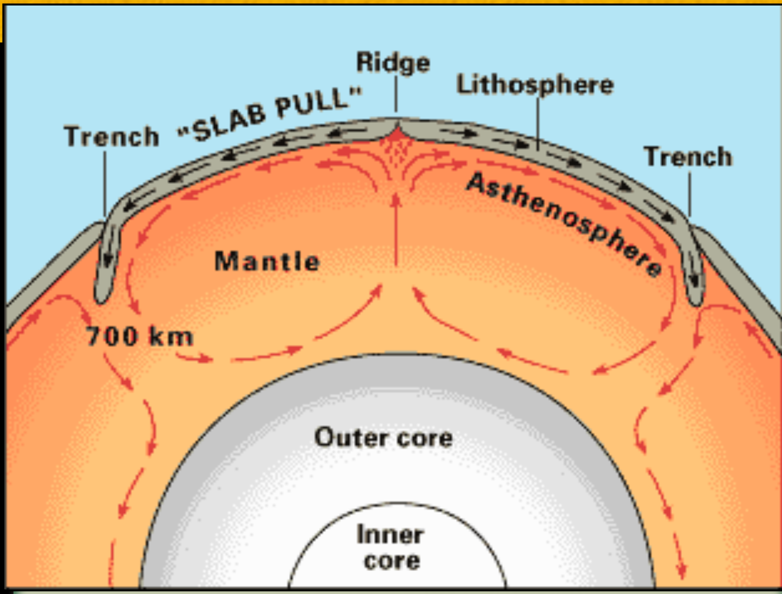


$$\tau = \frac{2\eta k}{\rho g} = \frac{4\pi\nu}{g\lambda}$$

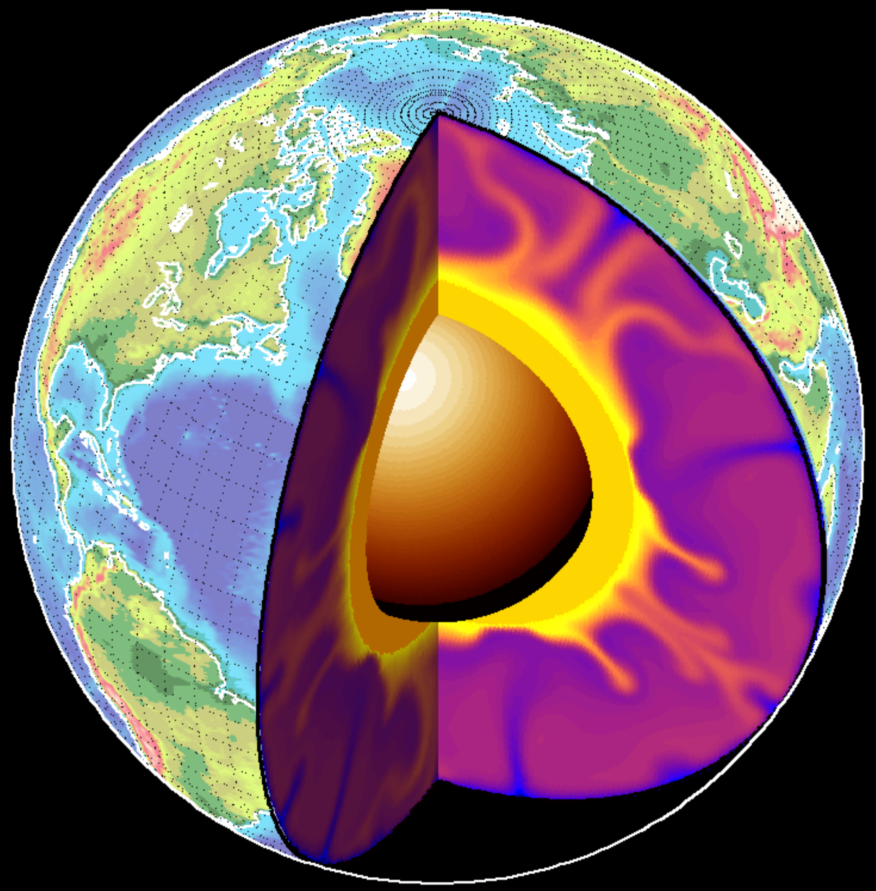
$$\eta = \frac{\rho g \tau}{2k}$$

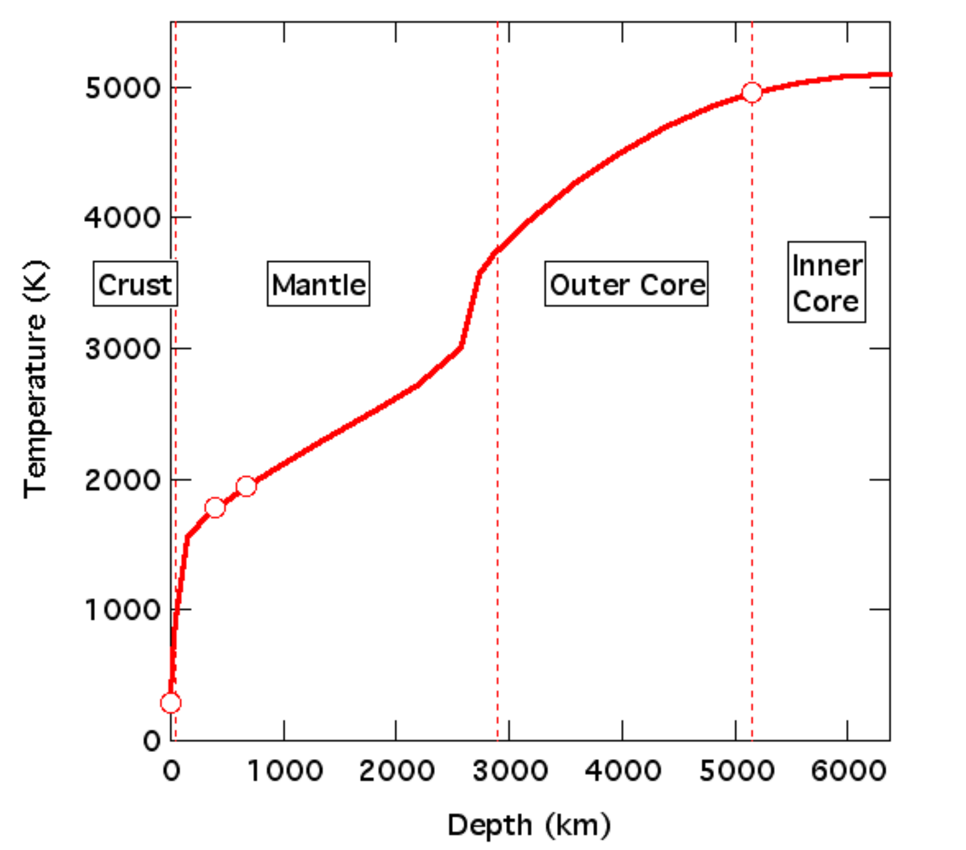
Mantle Convection



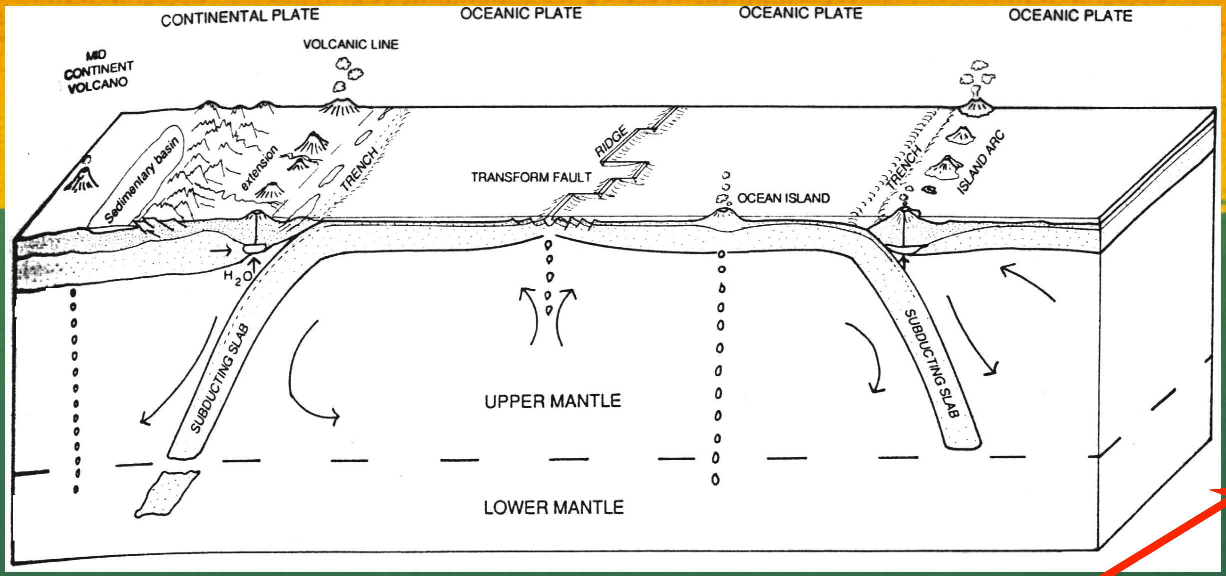


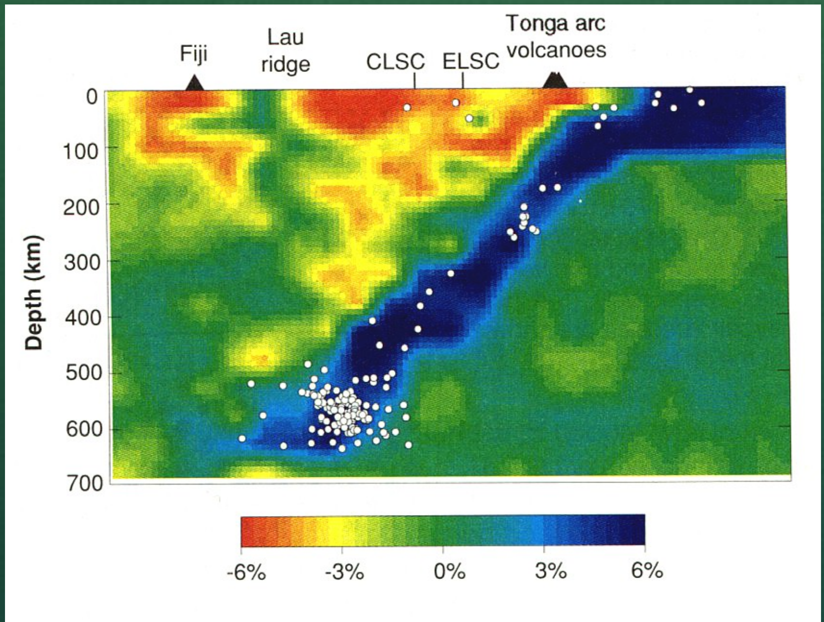
Mantle Convection Boundary Layers





Plates ⟷ Mantle Convection





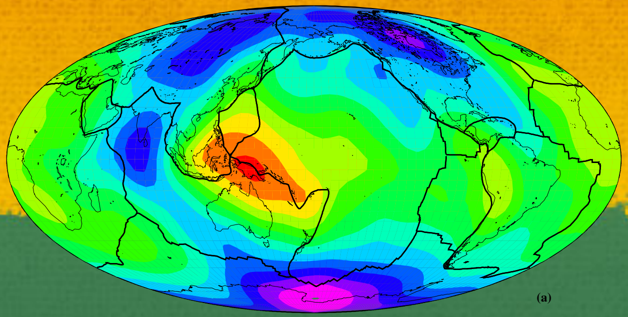
Continuous generation of dynamical (thermal) + geochemical (compositional) = seismic heterogeneity

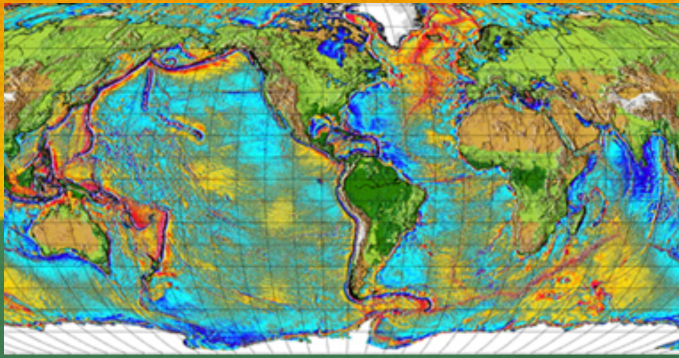
[including phase transitions (TZ!!)]

[*Zhao et al.*, 1997]

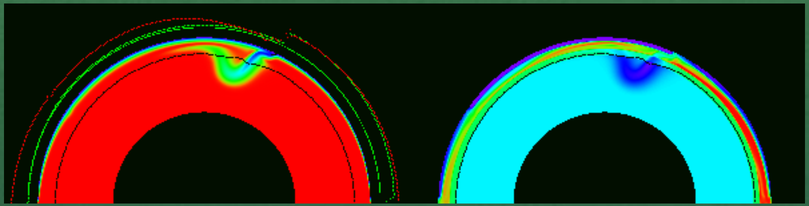
Research Methods

Observational - Modeling

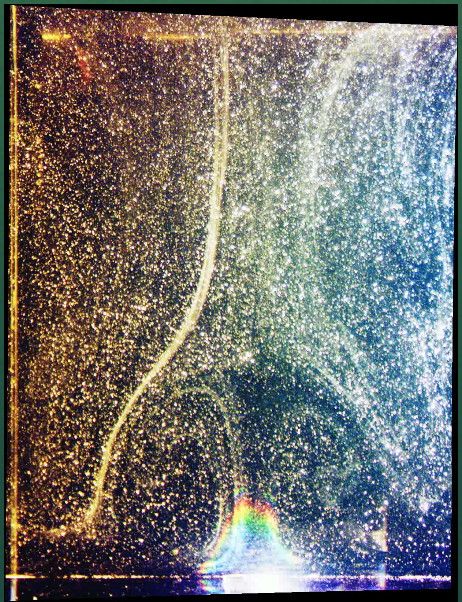




Theoretical - Numerical Simulations



Experimental - Laboratory



Present
Past

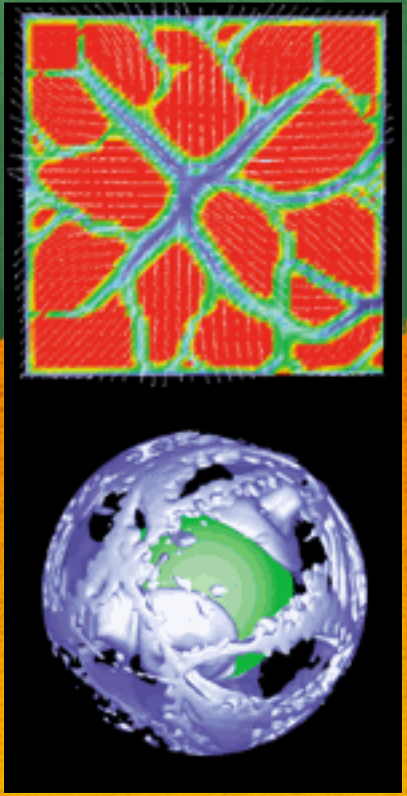
Static Processes
Dynamic Processes

Governing Equations

Momentum- $$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}) = \nabla \cdot \sigma + \mathbf{f}$$

Energy - $$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + H$$

Mass - $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$



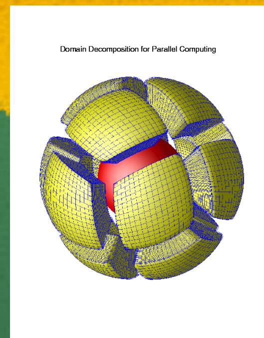
[*Tackley*, 1999]

Non-linear

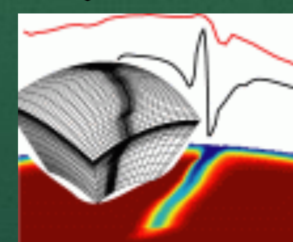
What is right Constitutive Relation?

$$\sigma = -p\mathbf{I} + 2\eta\dot{\mathbf{e}}$$

How to solve?



- Numerical methods for PDE’s
  - Finite Difference, Spectral, Finite element, Finite Volume, etc.
  - Flexibility
    - Grids (geometry, adaptability)
    - Resolution
    - Material property contrasts
  - Speed!
- Regional vs. Global
  - Boundary conditions
  - Resolution, Speed
  - Nature of problem



- Inputs
- Material properties (from mineral physics)
  - α, κ, ρ
  - as a function of (P, T, X)
- Rheology (viscosity, but not only)
  - As a function $(P, T, X, \sigma, \dot{\varepsilon})$
  - P dependence requires compressibility
- Energy sources (from geochemistry, and …)
  - Rate of internal heating
  - Basal heating (heat flow coming out of the core)
- Chemical Composition (from geochemistry in a broad sense)

Mantle convection Movies

http://www.gps.caltech.edu/~gurnis/Movies/movies-more.html

http://www.ipgp.jussieu.fr/~labrosse/movies.html

Approximations

-Infinite Prandtl # fluid: i.e. Inertial forces are not important
-Fluid is Incompressible, Newtonian
-Properties Homogeneous

$$\nabla p - \eta \nabla^2 \mathbf{v} = \mathbf{f}$$

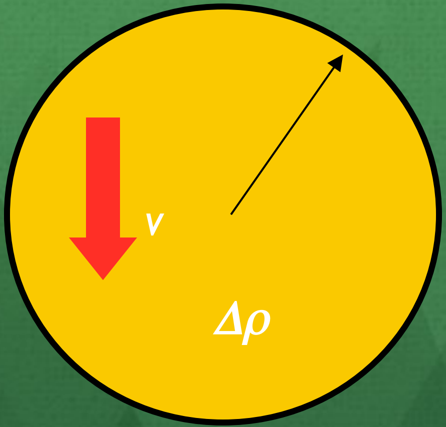
$$\mathbf{f} = \delta\rho\mathbf{g}$$

$$\delta\rho = \alpha\rho_o \Delta T$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \kappa\nabla^2 T + H$$

But suppose you know $\delta\rho$?

Falling Sphere



Buoyancy

$$F_B = g\Delta m = \frac{4\pi r^3 \Delta\rho g}{3}$$

Resistance

$$F_R = A\sigma_R = 4\pi r^2 \eta\dot{\varepsilon} \approx -4\pi r^2 \eta \frac{v}{r} = -4\pi r \eta v$$

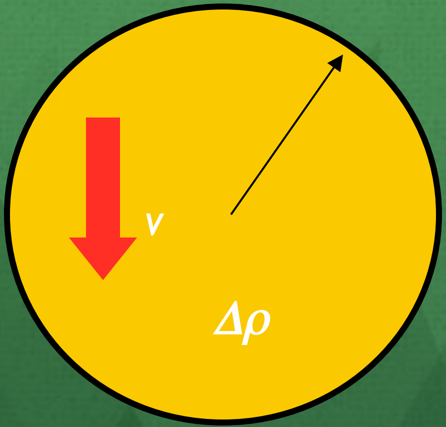
Force Balance: $F_B + F_R = 0$

$$v \approx \frac{1}{3}\frac{r^2 \Delta\rho g}{\eta}$$

Actual coefficient varies between 1/3 (inviscid) and 2/9 (solid)

Falling Sphere

$$v \approx \frac{1}{3}\frac{r^2 \Delta\rho g}{\eta}$$



Object	r	Δρ (kg/m³)	η (Pa s)	v
Plume head	500 km	30	10^{22}	80 km/Ma
Xenolith	10 cm	200	100	90 km /week

How to Solve Stokes..

$$\nabla p - \eta \nabla^2 \mathbf{v} = \delta\rho\mathbf{g}$$

$$\nabla^2 \Phi = 4\pi G \delta\rho$$

Take each variable: $[v_{r,} v_\theta, v_\varphi, \tau_{rr}, \tau_{r\theta}, \tau_{r\varphi}, \delta p, \delta\rho]$

Expand using spherical harmonics, scalar:

$$\delta\rho = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \delta\rho_l^m(r) Y_l^m(\theta, \phi)$$

and vector fields:

$$\mathbf{v}(r,\theta,\phi) = \hat{\mathbf{r}} U(r,\theta,\phi) + \nabla_1 V(r,\theta,\phi) - \hat{\mathbf{r}} \times (\nabla_1 W(r,\theta,\phi))$$

[Alterman et al., 1959; Takeuchi and Hasegawa, 1965; Kaula, 1975; Hager and O'Connell, 1979; 1981]

Aside: Spherical Harmonics

The spherical harmonics are the angular portion of the solution to Laplace's equation in spherical coordinates

$$\nabla^2 \Phi = 0$$

$$Y_l^m(\theta, \phi) = (-1)^m \left[\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}\right]^{\frac{1}{2}} P_l^m(\cos\theta) e^{im\phi}$$

$$\nabla_1^2 Y_l^m = -l(l+1) Y_l^m$$

$$\frac{\partial}{\partial \phi} Y_l^m = im Y_l^m$$

Spherical Harmonics

Continuing…..

$$v_r = a_\ell^m(r)Y_\ell^m$$
$$v_\theta = b_\ell^m(r)Y_{\ell,\theta}^m + c_\ell^m(r)Y_{\ell,\varphi}^m$$
$$v_\varphi = b_\ell^m(r)Y_{\ell,\varphi}^m - c_\ell^m(r)Y_{\ell,\theta}^m$$

$$Y_{\ell,\varphi}^m = \frac{1}{\sin\theta}\frac{\partial Y_\ell^m}{\partial\varphi}$$
$$Y_{\ell,\theta}^m = \frac{\partial Y_\ell^m}{\partial\theta}$$

[So for example the continuity (mass conservation) equation]

$$\dot{a}_\ell^m = \frac{-2a_\ell^m}{r} + \frac{\ell(\ell+1)b_\ell^m}{r}$$

…. Substitute the expansions for velocity, stress, with those for pressure and density… We end with 6 coupled ODEs

…..We could solve NUMERICALLY… but if we perform a simple variable substitution for each spherical harmonic coefficient

…..We get two nicely defined differential equations

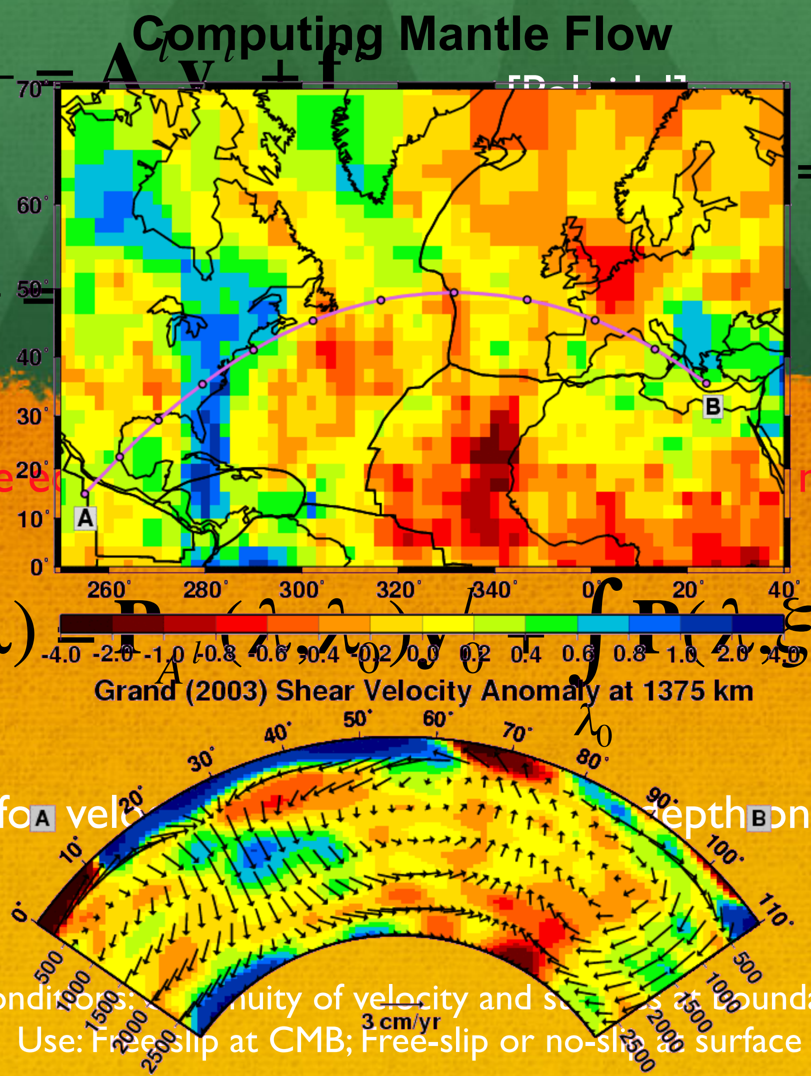
[See Hager and O'Connell, 1979 and 1981]

Propagator Matrices

$$\frac{d\mathbf{y}^l}{d\lambda} = \mathbf{A}_l\mathbf{y}^l + \mathbf{f}^l$$

$$\frac{d\mathbf{t}^l}{d\lambda} =$$

$= \ln(r/R_E)$



These e... matrices

$$\mathbf{y}(\lambda) = \mathbf{P}_A(\lambda,\lambda_0)\mathbf{y}_0^l + \int_{\lambda_0} \mathbf{P}(\lambda,\xi)\mathbf{f}^l(\xi)d\xi$$

We can now solve fo... velo... depth once P is known!

Boundary Conditions: ...uity of velocity and s... at boundary interfaces

Use: Free-slip at CMB; Free-slip or no-slip at surface

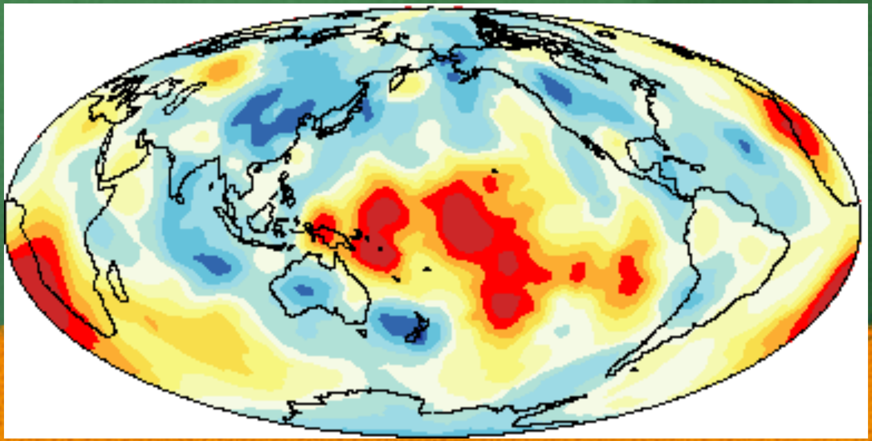
Advantages & Disadvantages

-Spectral Solutions VERY FAST! (You'll see)
-Can change radial viscosity profile (explore effects of viscosity structure)
-Spherical Shell
-Predict observables (Geoid, Topography, Plate Motions, Flow (Anisotropy))
-Can explore compressibility (less than 10% effect at long wavelengths)

-LACK OF RHEOLOGICAL COMPLEXITY
Lateral viscosity variations
Plate boundary rheology
-MUST ASSUME A DENSITY HETEROGENEITY
-No TIME DEPENDENCE

So now what?

Seismic Tomography- Convert velocity to density---- BUT HOW?

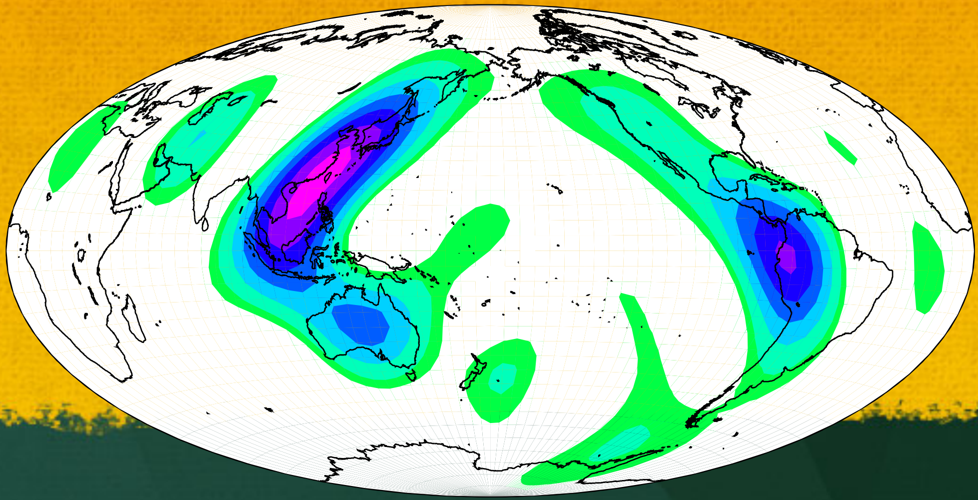


Mantle Density Heterogeneity Model

[*Masters and Bolton*]

Based on Geologic Information-Plate Motion History

Depth = 1000 Km [*Lithgow-Bertelloni and Richards,* 1998]

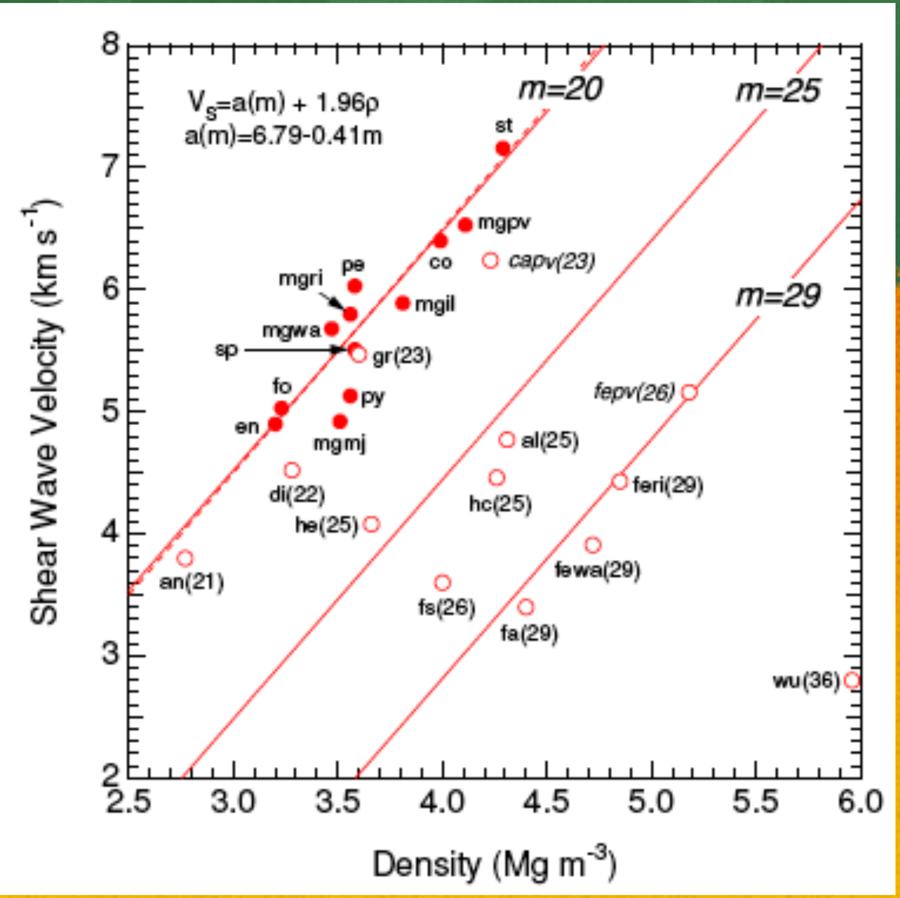


Velocity-Density Scaling

Birch's law

$\delta v = a \delta \rho$

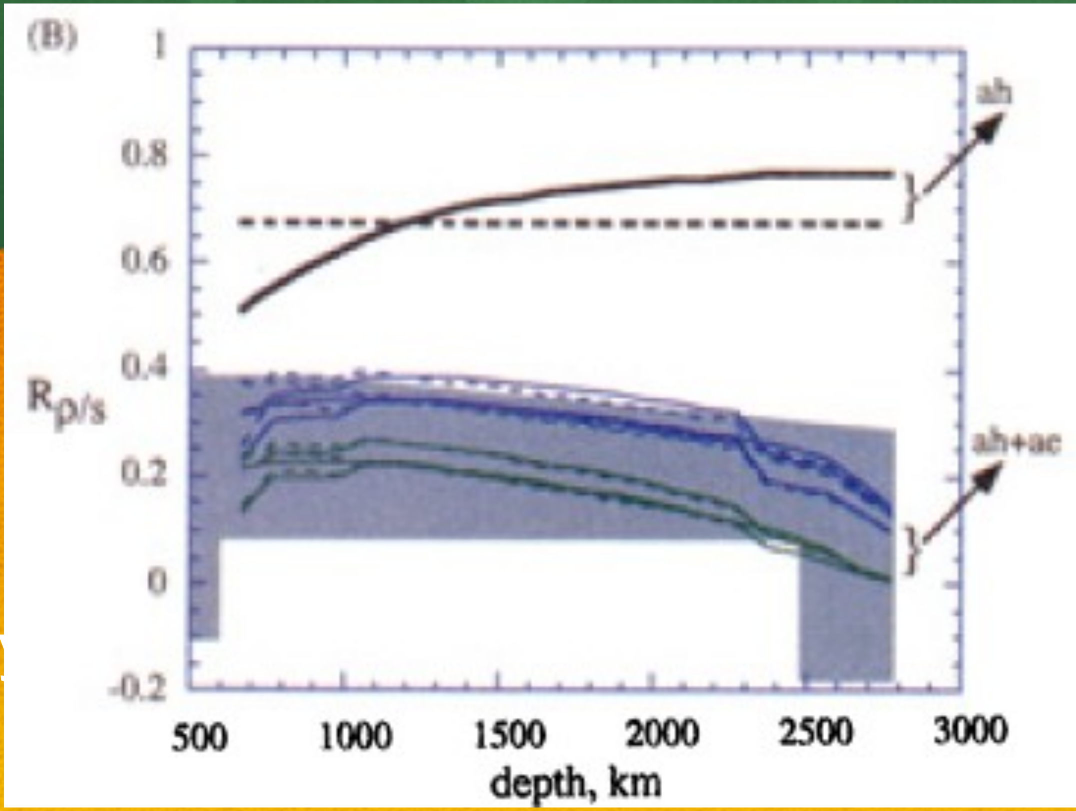
Factors=0.1-0.5 g s/km cm^3



Velocity-Density Scaling

$$R_{\rho/S} = \left(\frac{\delta \ln \rho}{\delta \ln V_S}\right)_{Depth}$$

Karato and Karki (2001)



If due to lateral *T* variations:

$$R_{\rho/S} = \left(\frac{\partial \ln \rho}{\partial T}\right)_P \left(\frac{\partial \ln V_S}{\partial T}\right)_P^{-1}$$

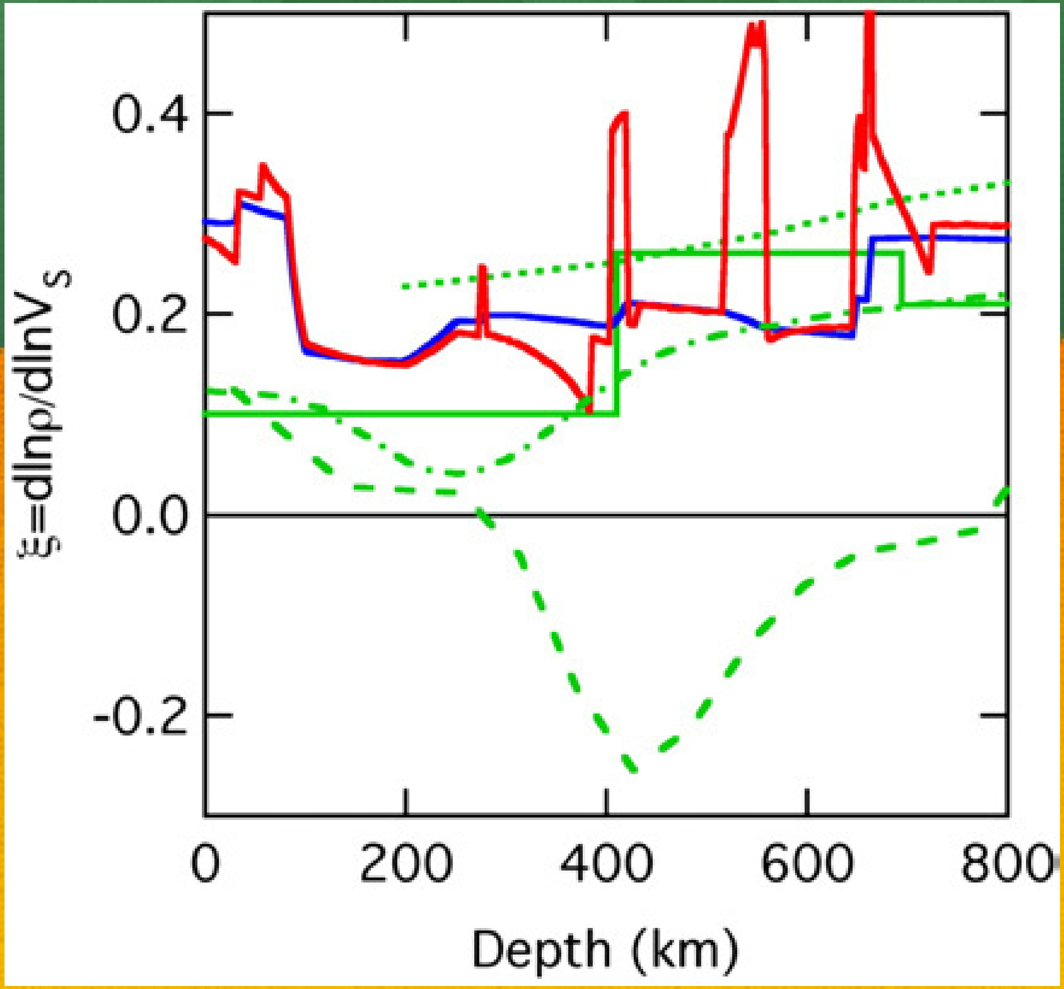
Attenuation (anelasticity) decreases value significantly

Claim: Cannot be negative

Not so! (phase transformations)

$$\left(\frac{\partial \ln \rho}{\partial T}\right)_P = \left(\frac{\partial \ln \rho}{\partial T}\right)_{P,\mathbf{n}} + \left(\frac{\partial \ln \rho}{\partial \mathbf{n}}\right)_{P,T} \left(\frac{\partial \mathbf{n}}{\partial T}\right)_P$$

Velocity-Density Scaling



Stixrude and Lithgow-Bertelloni (2007)

Predict Geoid

Best fitting viscosity structure
Lithosphere-10 * UM
Lower Mantle-50 * UM

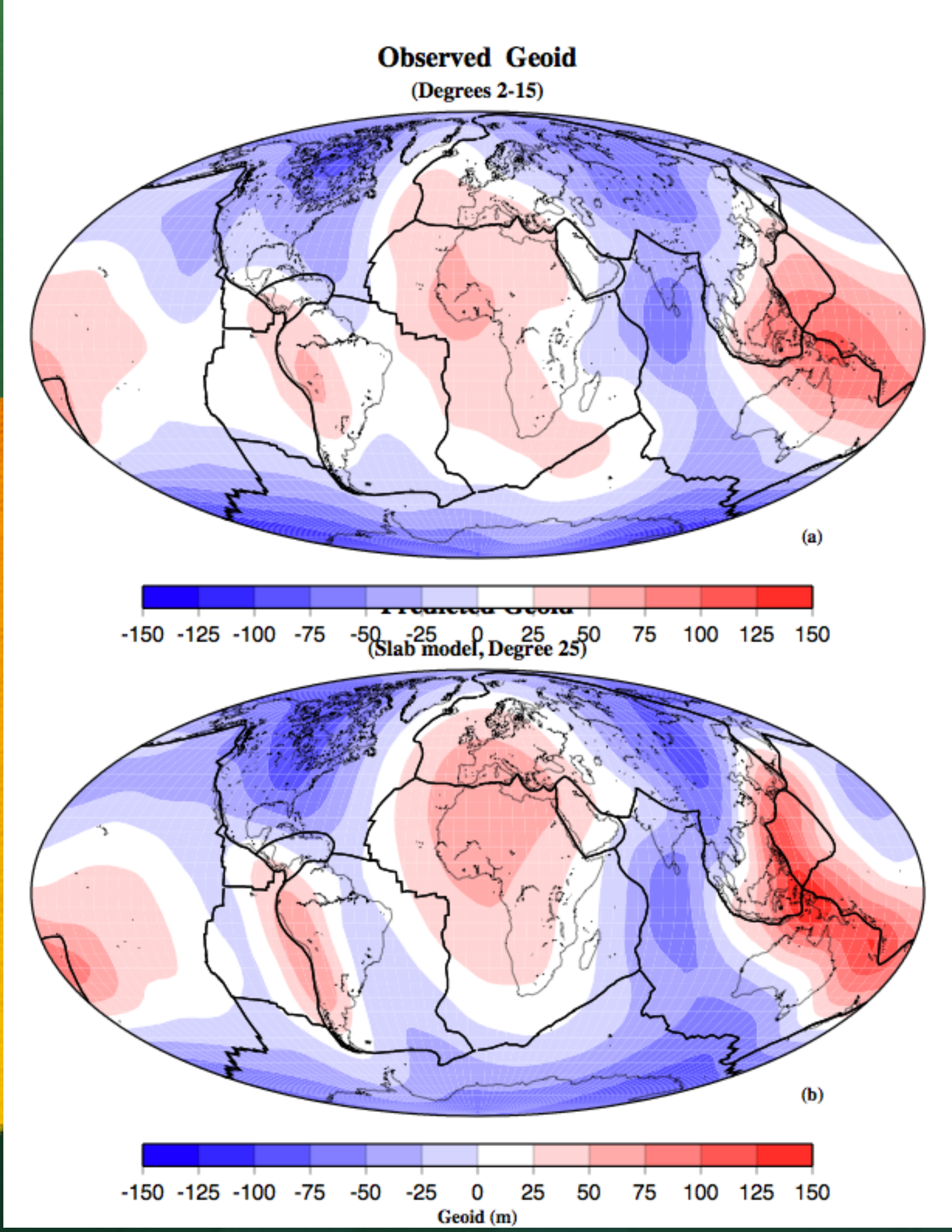
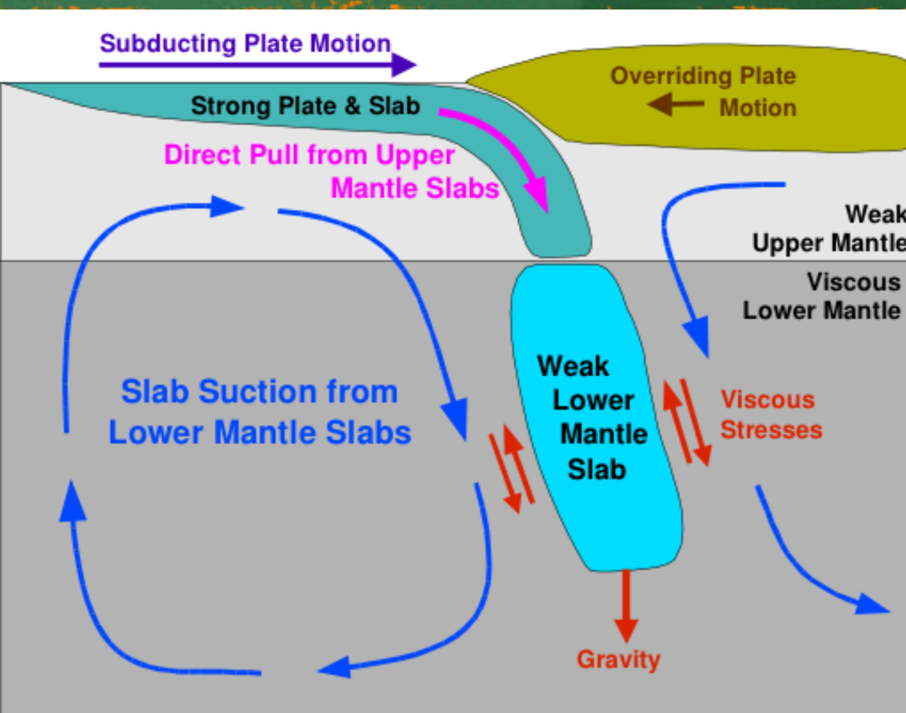
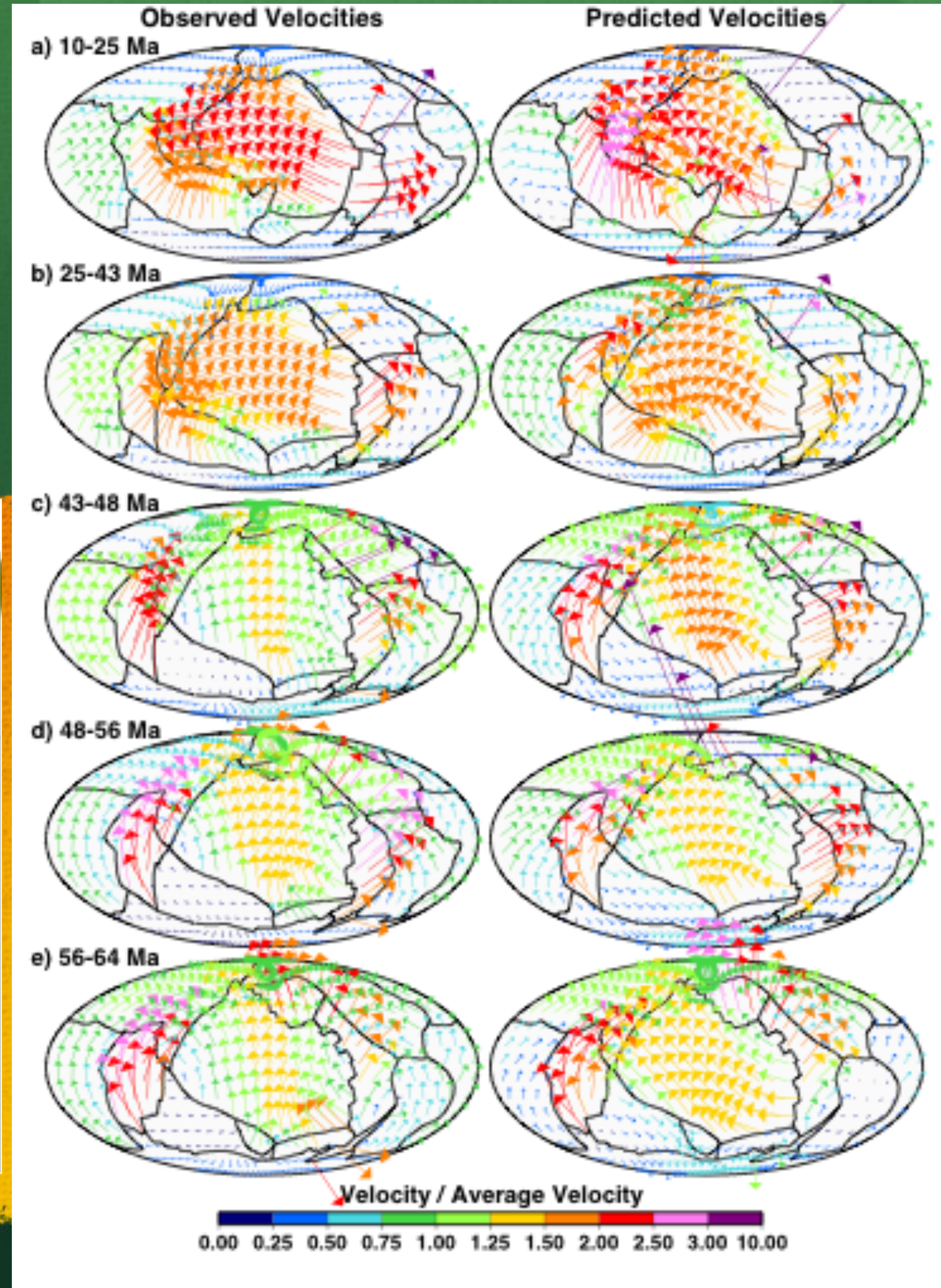


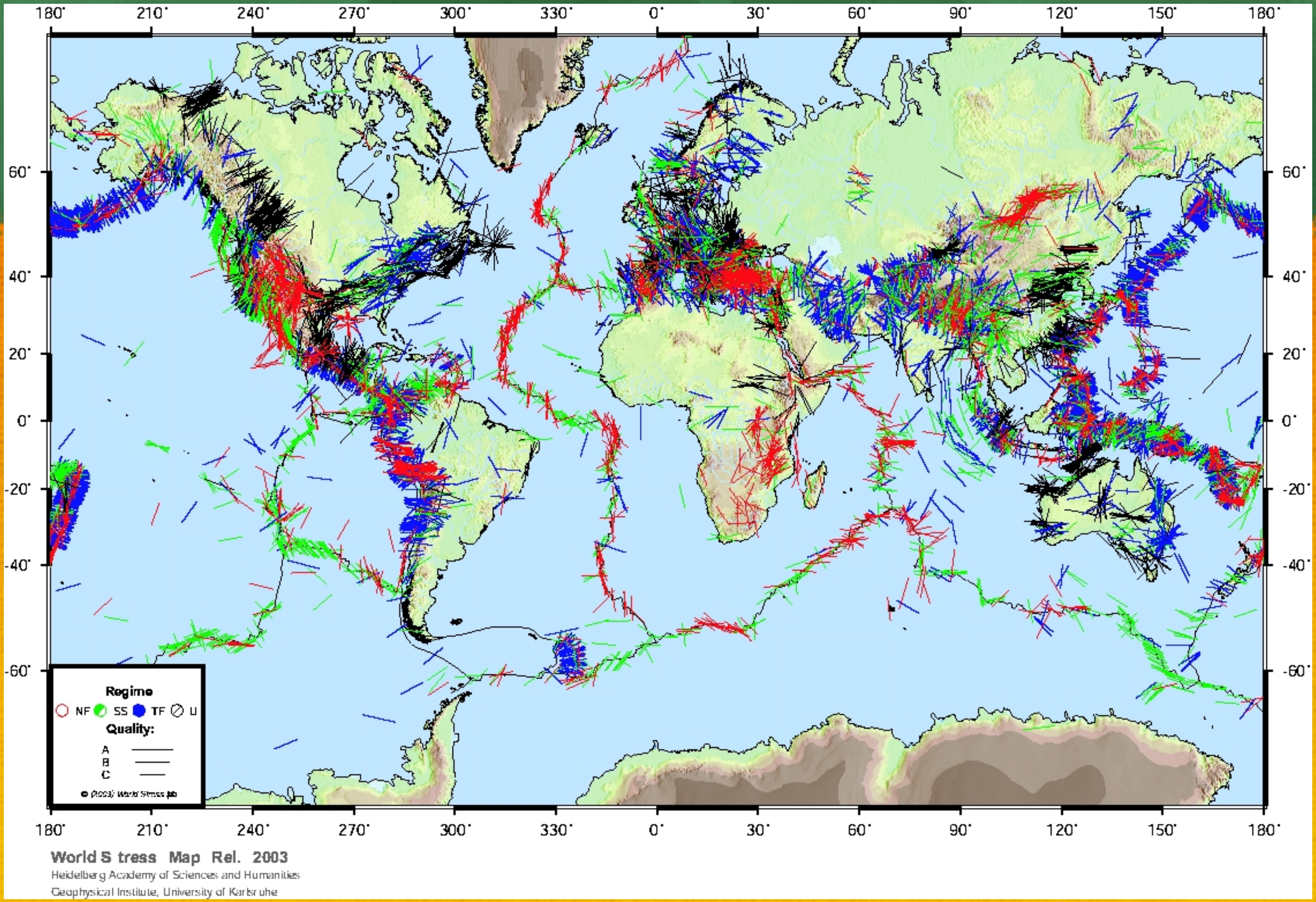
Plate Motions





Earth's Stress Field

Contributions: Mantle Stresses; Crustal Heterogeneity

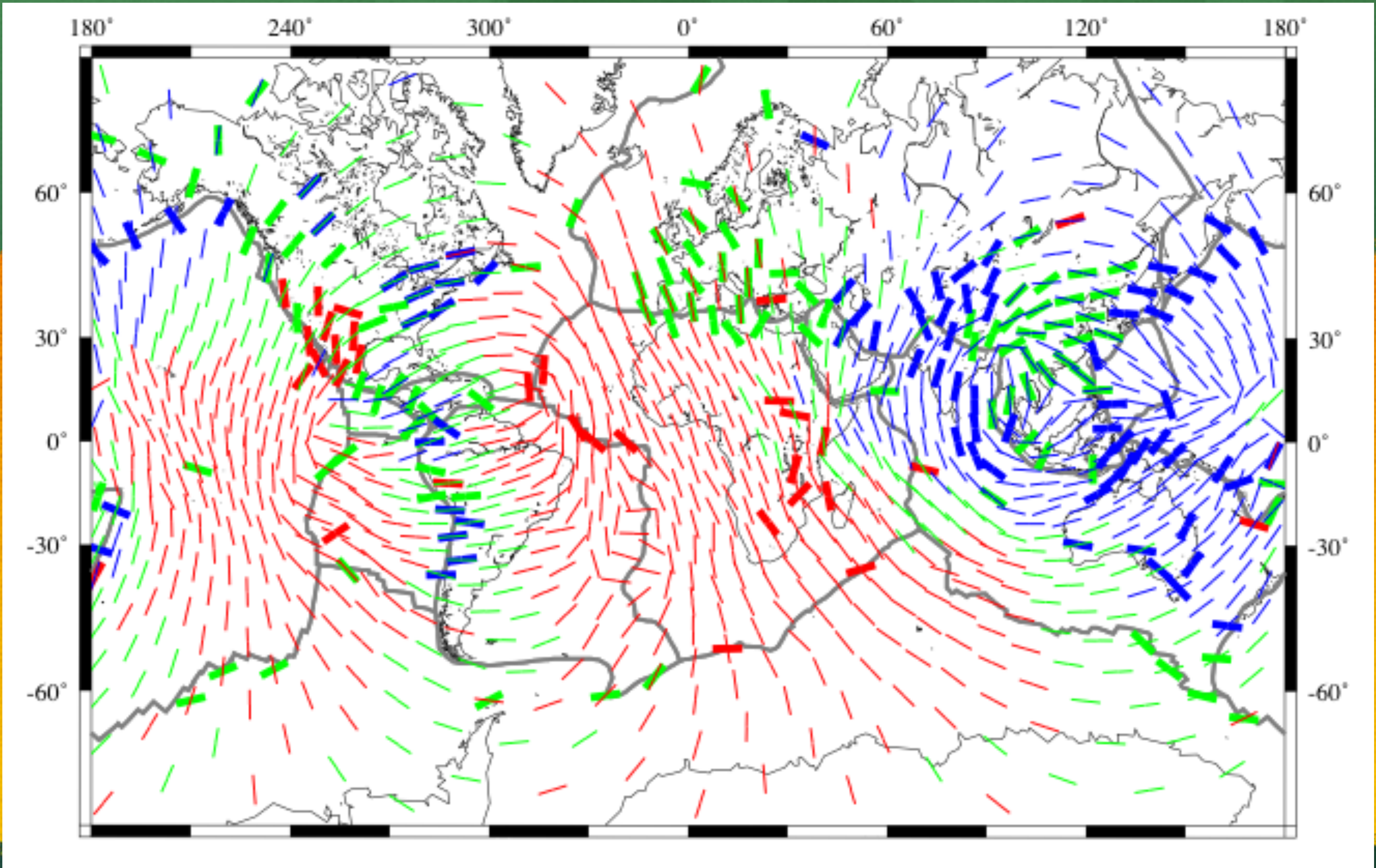


[*Reinecker, J., Heidbach, O. and Mueller, B., 2003*]
(available online at www.world-stress-map.org)

LVC+TD0
Fit to observations (Variance Reduction)
Azimuth-59%
Regime-61%

Stress Field

Combined effect of crustal contribution and mantle flow



[*Lithgow-Bertelloni and Guynn*, 2004]