

Towards a theory of sustainable systems

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Received 20 May 2001 ; accepted 20 August 2001

Abstract

While there is tremendous interest in sustainability, a fundamental theory of sustainability does not exist. We present our efforts at constructing a theory from Information Theory and Ecological Models. We discuss the state of complex systems that incorporate ecological and other components in terms of dynamic behavior in a phase space defined by the system state variables. From sampling the system trajectory, a distribution function for the probability of observing the system in a given state is constructed, and an expression for the Fisher information is derived. Fisher information is the maximum amount of information available from a set of observations, in this case, states of the system. Fisher information is a function of the variability of the observations such that low variability leads to high Fisher information and high variability leads to low Fisher information. Systems in stable dynamic states have constant Fisher information. Systems losing organization migrate toward higher variability and lose Fisher information. Self-organizing systems decrease their variability and acquire Fisher information. These considerations lead us to propose a sustainability hypothesis: “sustainable systems do not lose or gain Fisher information over time.” We illustrate these concepts using simulated ecological systems in stable and unstable states, and we discuss the underlying dynamics. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Fisher information; Information theory; Predator–prey models; Sustainability; Ecological models

1. Introduction

The topic of sustainability is, perhaps, operationally and conceptually one of the most complex that modern science has faced. Possibly, the two most widely known statements on sustainability are those of the World Commission on Environment and Development [1] and the National Research Council [2]. The World Commission on Environment and Development states “. . . development that meets the needs of the present without compromising the ability of future generations to meet their own need”. The

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National Research Council states “the reconciliation of society’s development goals with the planet’s environmental limits over the long term”.

As one can imagine, the topic of sustainability must at least embody in some form elements of physics, engineering, ecology, law, economics, sociology, and politics. This gives rise to severe operational difficulties in cross-disciplinary communication, and multidisciplinary research teams have by consequence a difficult history. One does not know how to easily connect economic demand curves to ecosystem function and to legal issues. Yet, sustainability is an area that cannot be successfully investigated within the confines of any single discipline. The conceptual difficulties arise in part due to the lack of a general underlying theory in the area of sustainability. This gives rise to many problems. For instance, it is unclear what are the appropriate measures of sustainability, and various metrics have been proposed over the years that include species diversity, profitability, human income levels, etc. in various forms. The criteria that identify a sustainable from an unsustainable system are also unknown. In fact, even the principles on which any sustainability criteria could be based are themselves only dimly understood.

To begin to address as many of these issues as possible, we have invoked the concept of information in its mathematical form as the centerpiece of our research work. The reason is that essentially any type of data or model can be converted to information regardless of disciplinary origin. Information can, therefore, serve as common interdisciplinary bridge. We also hypothesize that information theory can serve as an appropriate basis for the construction of a basic theory of sustainability. Information is a very fundamental quantity from which many other known laws of nature can be derived as shown by Frieden [3]. The objective of this article is then to demonstrate our beginning steps in the development of a theory of sustainable systems using ecological models as a starting point. This includes a sustainability hypothesis and illustrations using predator–prey models. In a way, the long term goal of this work is fundamentally to translate into mathematical form the aforementioned statements of sustainability from World Commission on Environment and Development and the National Research Council, and this article documents our initial steps in this effort.

2. Information theory in ecology

Information theory has had four primary applications in biological and ecological research: as an index of physical or structural diversity, as a measure of evolutionary processes, as a measure of distance from thermodynamic equilibrium, and as a measure of transaction propensity in networks [4]. A fifth, more qualitative approach, treating ecosystems as semiotic systems has been recently advanced by Hoffmeyer [5] and Emmeche [6].

The most common application has been the use of Shannon information as a diversity index. Shannon and Weaver [7] described in “The Mathematical Theory of Communication”, the information transfer from a signal to a receiver with the following relationship:

$$H = -k \int p(x) \ln p(x) dx \quad (1)$$

where H is the Shannon information, p the probability density function, k a positive constant, and x a variable. In this formulation, H actually measures the degrees of freedom of the system. Communication systems with few degrees of freedom have a clear signal transmission and high information content. Contrary to communication systems, biological systems with many degrees of freedom (i.e. biodiversity)

were deemed favorable for their apparent stability (see McCain [8] on the diversity-stability debate). Therefore, the application in ecology has been the use of Shannon information as a measure of system capacity particularly as an index of biodiversity.

Biologists have used information theory to address evolutionary processes as irreversible non-equilibrium phenomenon (Brooks and Wiley [9], Weber et al. [10]) and to quantify the organization of genetic and macromolecular structures (Youckey [11]). A measure for the entropy of information was derived using a concept similar to Shannon information. The premise is that information gained during evolution results in a decrease in entropy, and therefore, entropy and information can be used to investigate evolution.

The third approach uses Information Theory to measure the distance a system has moved from thermodynamic equilibrium. This is based mostly on the concept of exergy, a formulation of thermodynamic efficiency. Mejer and Jørgensen [12] and Jørgensen and Mejer [13] introduced the concept of exergy to ecosystems by assuming that ecosystem exergy could be based on the chemical potential of the most essential elements of the system. More recently, an exergy index has been introduced that moves the reference state to that of dead organic matter and also includes a weighting factor for the genetic complexity of the organism.

Ulanowicz [14,15] developed the concept of Ascendency, which has a functional form similar to Shannon information, to describe the organization in reticulated systems. This approach uses conditional probability to ascertain the “average mutual information” in the system based on the likelihood of flow through a network. In this way, it is possible to estimate the total information capacity in a system and differentiate between “organized” complexity (Ascendency) and “encumbered” complexity (Overhead).

While recognizing the contributions and limitations of the previous attempts to understand ecological systems using Information Theory, we introduce another approach, based on Fisher information.

3. Fisher information: general

The work of Ronald Fisher [16] introduced a statistical measure of indeterminacy now called Fisher information. Fisher information can be interpreted as a measure of the ability to estimate a parameter, as the amount of information that can be extracted from a set of measurements, and also as a measure of the state of order or organization of a system or phenomenon [3]. It is the later interpretation that has the most relevance to issues of sustainability. Fisher information, I , for one variable is calculated as follows:

$$I = \int \frac{1}{p(x)} \left(\frac{dp(x)}{dx} \right)^2 dx \quad (2)$$

where the symbols have the previously assigned interpretation. Frieden [3] has proposed Fisher information as a basis for a unifying theory for physics by showing that a Lagrangian consists of two types of Fisher information, intrinsic data information and external phenomenological information. He has used it to derive many fundamental equations of physics such as those describing relativistic quantum physics, classical electrodynamics, general relativity, and classical statistical physics to name a few, and has also applied it to problems of population genetics (Frieden et al. [17]). Unlike Shannon information, which is a global measure of smoothness, Fisher information is a local property since it is based on the derivative of the probability distribution function. This makes it more sensitive to perturbations that affect the probability distribution function. A highly disordered system has a uniform or “unbiased” probability distribution function, which is broad and smooth (Fig. 1a). This lack of predictability results in a system

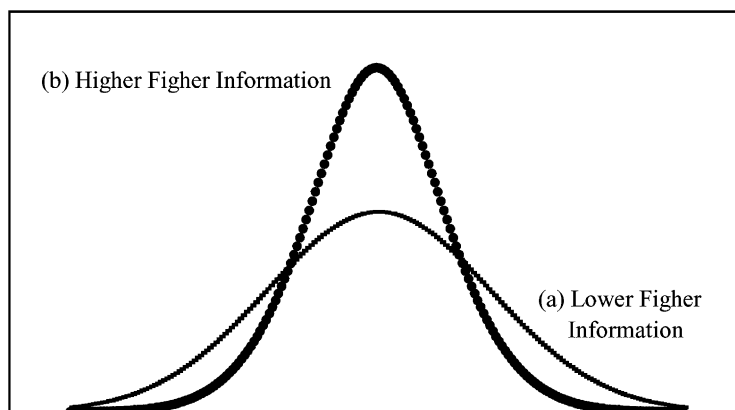


Fig. 1. Comparison of probability distribution functions: a flatter more uniform distribution (a) will have a lower Fisher information than the steeper PDF (b).

with low Fisher information. A highly structured system with low disorder shows bias to particular states and the probability distribution function is steeply sloped about these values (Fig. 1b), in which case Fisher information is high. For physical systems, entropy follows the second law of thermodynamics, increasing monotonically with time. Conversely, Fisher information decreases with time as entropy (system disorder) increase. Ecological (and perhaps sociological) systems, however, are able to create local order by utilizing energy flows through their self-organizing properties. This leads us to our hypothesis, formalized below, that the Fisher information of ecological systems in a sustainable state does not change with time.

4. Stable dynamic states

The state of the system is defined by its state variables. That is, if one knows the values of the state variables, then one “knows” the state of the system. The behavior of these state variables determines the stability of the system. A static steady-state is reached when the state variables all maintain a constant value. A dynamic steady-state occurs when the state variables oscillate repeatedly over a fixed region in phase space. We call this latter a stable dynamic state. Both cases are said to be stable. Non steady-state systems may be dynamically stable depending on how the behavior responds to perturbations. If a perturbed system returns to its original state then it is said to exhibit Lagrange stability. A relaxed stability condition, Liapounov stability, only requires the system to return to within a particular bounded region (Svirezhev and Logofet [18]). Also key to stability, is that the system is not gaining or losing dimensionality by gaining or losing species. Our hypothesis, in effect, maintains that stability of the state of the system is sufficient for sustainability of the state of the system. Since Fisher information is a measure of the system stability (organization), it is an indicator of sustainability of the state of the system as well. Note, however, that stability is not necessary for the sustainability of the system itself, which is consistent with the evolutionarily adaptive behavior of nature. Sustainable systems change over time from state to state losing and gaining parts. This is addressed by two corollaries, formalized below, to the sustainability hypothesis which state that a system gaining

information is still maintaining its self-organized state while a system losing information is losing its self-organization.

5. Fisher information: ecological systems

The system's state variables determine a probability distribution function for that system based on the probability of finding the system in a particular state, i.e. a given set values for the state variables. In this manner, there is only one PDF for the entire system. For simple systems that reach steady-state or have a stable limit cycle, it is possible to create a histogram corresponding to the possible outcome space. However, when dealing with dynamic systems it may not be possible to create a histogram based on all possible state variable combinations. The state trajectories may never repeat on themselves, in which each state may be a nearly unique event. The probability of finding the system in a particular state, i.e. the state variables with a particular set of values, corresponds directly to the time that the system spends in that state. The longer the system is in a specific state, the more likely one would be to find it in that state when sampling. When normalized over the entire space of possibilities, a probability distribution function results. For a complex dynamic system including the ecological system represented by model Eq. (5), there exist relationships between the state variables and time such that once time is fixed, the values of all the variables are determined. The probability density function is, therefore, strictly speaking only a function of time. Thus, the probability of finding the system in a given state is:

$$p = A \Delta t (y_1(t), \dots, y_n(t)) \quad (3)$$

where A is a normalization constant, Δt is the time that the system spent in the state, y_i is a state variable, and t is time. The Fisher information can then be computed from the probability density function applying the chain rule from:

$$I = \int \frac{1}{p} \left(\frac{dp}{dt} \right)^2 dt = \int \frac{1}{p} \left[\sum_i^n \left(\frac{\partial p}{\partial y_i} \frac{dy_i}{dt} \right) \right]^2 dt \quad (4)$$

where all the symbols have their previously assigned interpretation.

6. Sustainable systems hypothesis

Using Fisher information as a basis, we have constructed a hypothesis for a sustainability criterion. The sustainability hypothesis states that: the time-averaged Fisher information of a system in a sustainable state does not change with time. There are, in addition, two corollaries to the sustainability hypothesis which state: (1) if the Fisher information of a system is increasing with time, then the system is maintaining a state of self-organization and (2) if the Fisher information of a system is decreasing with time, then the system is losing its state of self-organization.

The sustainability hypothesis establishes a means of differentiating systems in sustainable states from systems in unsustainable states. Basically, when the state of the system becomes unsustainable, the system begins to either gain or lose Fisher information, i.e. the system is no longer in a stable dynamic state. Because Fisher information is obtained from observations, it is possible to apply this criteria to measured variables from a system regardless of whether the variables are of ecological, economic, technological

or other origin. Recalling that here, we define a system in a sustainable state as a system in a dynamic stable state, we note that in the case of simple systems such as the predator–prey model below, it is trivial to decide by inspection whether the system is or is not in a dynamic stable state. But, for very complex systems such as those necessary for sustainability questions (a planet or a large portion of a planet including the associated economic, ecological, and other parts), deciding whether or not the system is in a dynamic stable state may not be possible. For such complex cases, the use of a hypothesis or principle such as the one above is necessary.

The two corollaries to the sustainability hypothesis give us a way to decide on the quality of change, if change is occurring. The first corollary to the sustainability hypothesis indicates when the direction of change is favorable to the sustainability of the system. If the Fisher information is increasing, then the system, although migrating to a different state, is at least moving in the direction of increasing self-organization. In the case of living systems, a self-organized system is one that is able to remain alive and functioning. So, increasing Fisher information is likely to lead to a system in a sustainable state. The second corollary to the sustainability hypothesis indicates when the direction of change is not favorable to the sustainability of the system. If the Fisher information is decreasing, then the system is simply losing its self-organization. In the case of living systems, the loss of self-organization simply means that the system is ceasing to function, i.e. dying. A dying system is not sustainable.

7. Illustration: predator–prey models

Our objective is to develop a methodology to calculate the Fisher information for a dynamical ecological system. As stated above, Fisher developed a fundamental method to relate observed information with intrinsic information as estimation error. The result can be interpreted as a way to quantify the degree of system organization. We extend the use of Fisher information to measuring the probability of observing a system in a particular state (as defined by its state variables) by substituting natural variation for estimation error in Fisher’s formulation. The probability comes not from the estimation associated with a single measurement, but from the probability of sampling the system in a particular state as it plays out its inherent dynamics. Our working hypothesis is that the homeostatic qualities of sustainable systems will keep natural variation stable over time resulting in systems in which Fisher information will be constant over a given time period. This is our primary line of pursuit applying Fisher information to sustainability theory.

We test and demonstrate the Fisher information-sustainability hypothesis using a two-species predator–prey Lotka–Volterra model. Although, there are valid criticisms of this simplistic model as a realistic descriptor of ecological dynamics (Hall [19], Straskraba [20]), its use here is primarily to demonstrate a new methodology: calculating the Fisher information for a particular system. The Lotka–Volterra model provides a commonly understood and well-studied model for this purpose. The two-species equations describe a simple interaction between a prey species, y_0 , and its predator, y_1 , with four parameters: (a) prey growth rate, (b) prey mortality rate, (c) predator mortality rate, and (e) predator growth rate. The state of the system is defined by the amount of biomass in each state variable, y_0 and y_1 . The population dynamics of the two-species are described with two ordinary first-order differential equations:

$$\frac{dy_0}{dt} = ay_0 - by_0y_1 \tag{5a}$$

$$\frac{dy_1}{dt} = -cy_1 + ey_0y_1 \tag{5b}$$

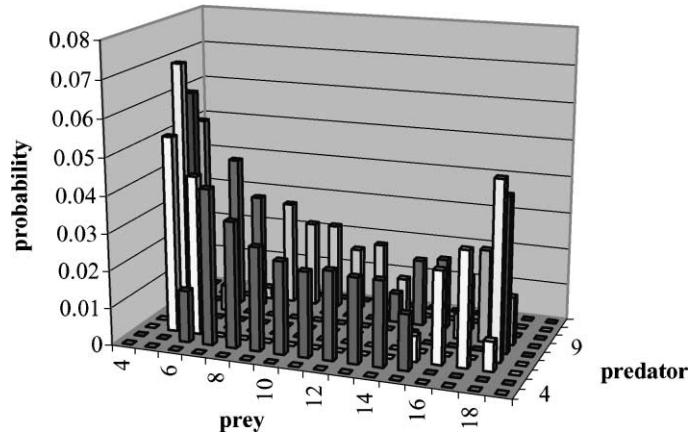


Fig. 2. Histogram corresponding to probability of finding the predator–prey model in specific integer combinations (for given initial conditions and parameter values above with the exception that $b = 2$).

This is a completely deterministic system and the solution to the equations reaches a stable limit cycle whose amplitude and period depend on the initial conditions and parameter values. Every starting point lies on a closed loop and altering the initial conditions produces different trajectories that form a family of nested closed loops. This system of equations is solved numerically using an arbitrary set of initial conditions ($y_0 = 12, y_1 = 5$) and parameter values ($a = 15, b = 3, c = 5, d = 0.5$) to obtain the possible system states. For this system it is possible to construct a histogram of the various states (Fig. 2).

As stated above, the likelihood of finding the system in one of these combinations depends on the time that the system spends in that state. The time spent in each state depends on the distance traveled through that space and the speed at which it is traveling. When plotted as a phase plot, it is clear that the system passes through a distance, ds , to cross this region (Fig. 3).

The total distance traveled through phase space for the population trajectory can be approximated linearly using the Pythagorean theorem:

$$ds = \sqrt{dy_0^2 + dy_1^2} \tag{6}$$

As the grid size gets smaller this approximation becomes exact. Substituting from the original ordinary differential equations $dy_0 = dt (ay_0 - by_0y_1)$ and $dy_1 = dt (cy_1 + ey_0y_1)$ and solving for dt gives the time spent in each state (distance divided by the velocities).

$$dt = \sqrt{\frac{ds^2}{(dy_0/dt)^2 + (dy_1/dt)^2}} = \sqrt{\frac{ds^2}{(ay_0 - by_0y_1)^2 + (-cy_1 + ey_0y_1)^2}} \tag{7}$$

Therefore, the probability can be written as

$$p = Adt = A \sqrt{\frac{ds^2}{(ay_0 - by_0y_1)^2 + (-cy_1 + ey_0y_1)^2}} \tag{8}$$

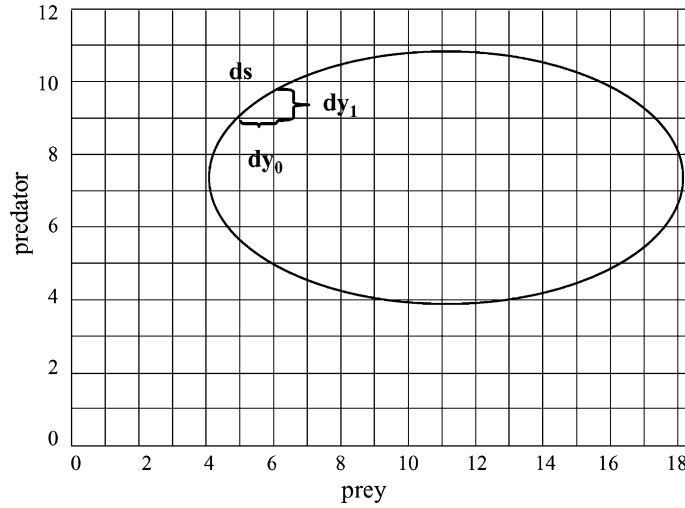


Fig. 3. Two-dimensional phase plot of a predator–prey system. The total time spent in a particular state (as defined by the grid size) depends on the distance, ds , and the velocities, dy_0/dt and dy_1/dt , with which the system moves through the phase space.

Fisher information for the two-species system can be calculated by the following expression:

$$I = \int \frac{1}{p} \left[\sum_{i=1}^2 \left(\frac{\partial p}{\partial y_i} \frac{dy_i}{dt} \right) \right]^2 dt \quad (9)$$

The velocities dy_0/dt and dy_1/dt are known and the derivatives of $p(y_0, y_1)$ are computed analytically to yield:

$$\frac{\partial p}{\partial y_0} = \left(\frac{-A\sqrt{ds^2}}{2} \right) \frac{[2(ay_0 - by_0y_1)(a - by_1) + 2(-cy_1 + ey_0y_1)(ey_1)]}{[(ay_0 - by_0y_1)^2 + (-cy_1 + ey_0y_1)^2]^{3/2}} \quad (10)$$

$$\frac{\partial p}{\partial y_1} = \left(\frac{-A\sqrt{ds^2}}{2} \right) \frac{[2(ay_0 - by_0y_1)(-by_0) + 2(-cy_1 + ey_0y_1)(-c + ey_0)]}{[(ay_0 - by_0y_1)^2 + (-cy_1 + ey_0y_1)^2]^{3/2}} \quad (11)$$

Substituting these back into the equation for Fisher information and estimating the integrals as discrete sums gives:

$$I = \sum (Ads) \Delta t \frac{[((ay_0 - by_0y_1)(a - by_1) + (-cy_1 + ey_0y_1)(ey_1))(ay_0 - by_0y_1) + ((ay_0 - by_0y_1)(-by_0) + (-cy_1 + ey_0y_1)(-c + ey_0))(-cy_1 + ey_0y_1)]^2}{[(ay_0 - by_0y_1)^2 + (-cy_1 + ey_0y_1)^2]^{5/2}} \quad (12)$$

Using this equation with the parameter values and the state variable values from the solution, one can calculate the Fisher information for the two-species predator–prey ecological model. A similar methodology can be applied to more complex systems provided time-series values for the state variables of the system and the system velocity in its phase space can be estimated.

8. Fisher information and stability

Once we have a general method for calculating the Fisher information of the state of the system, we perform a sensitivity analysis to test the relationship between the system stability and Fisher information. The stability of the system is best viewed in the phase space diagram because it shows the spread of the distribution of the variables (Fig. 4). We chose to alter the dynamics of the system by changing the mortality rate of the prey species (b). For a baseline case, $b = 3$, the phase plot is very tightly distributed. This system is stable because the oscillations of both state variables are relatively small. There is little chance for an under-shoot or over-shoot into an unstable state (one or both species being eliminated). The Fisher information is highest around this value as seen in Fig. 5. When the prey mortality is lowered, the range in the phase space increases. In this case the lower prey mortality allows the prey to increase, which also spurs on greater growth of the predator, until they outpace the prey resource and the system returns again. The state variables go through large oscillations and the system has lower Fisher information. When prey mortality increases, a large predator population cannot be supported limiting their oscillations to the lower range of the phase space. This low level of predation allows the prey to expand greatly resulting in wide elliptical phase diagrams. The probability of finding the system in a particular state is low since each variable travels through a wide range of states, and therefore the Fisher information is low. This situation is considered less sustainable because a small perturbation might destabilize the system.

The point that we illustrate with Figs. 4 and 5 is that depending in which stable dynamic state the system happens to be in, a change in a model parameter may cause the system to gravitate to a different state which may or may not be equally sustainable. A less sustainable state will have larger fluctuations that can lead to species extinction than a more sustainable state. Therefore, the sustainability hypothesis

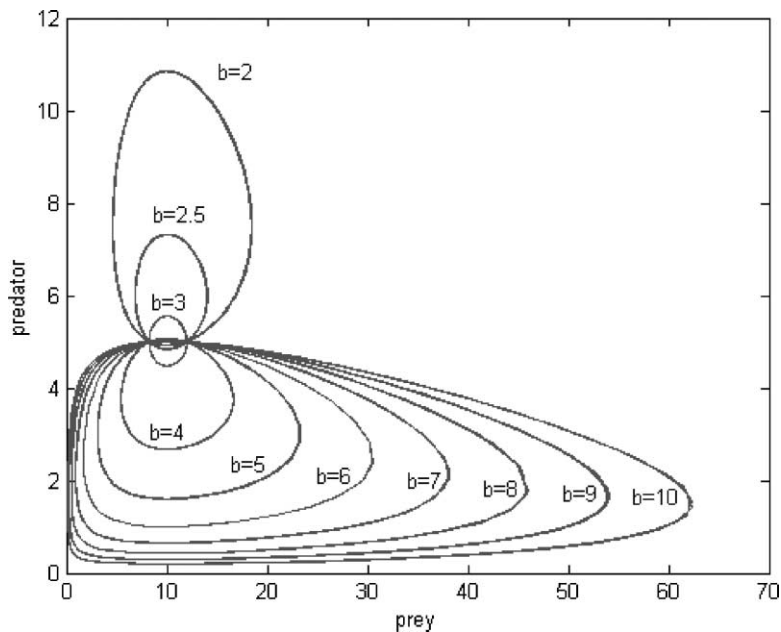


Fig. 4. Phase plots of the predator–prey system for various values of the prey mortality coefficient (b).

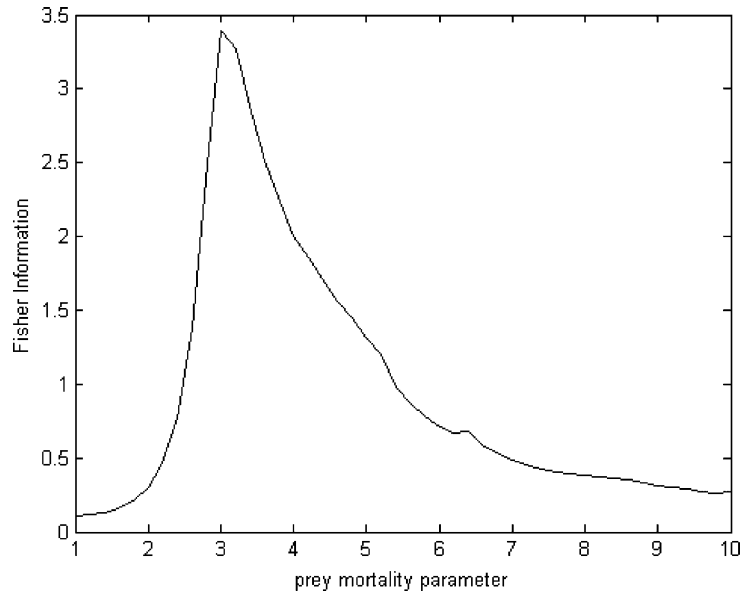


Fig. 5. Average Fisher information in two-species predator–prey model for varying prey mortality coefficient.

and its two corollaries tell us that if the prey mortality is changing, the dynamic state of the system is changing and that state is not sustainable. If the Fisher information is increasing, then the system is at least moving to another dynamically stable state that may be equally sustainable. But if the Fisher information is decreasing, then the system is moving to a dynamic state that is less sustainable and this is certainly moving in the “wrong” direction. The Fisher information is a very sensitive measure of these effects as shown in Fig. 5. This is consistent with current thinking in population biology to the extent that as stated by Noss et al. [21] “populations that fluctuate widely are more vulnerable than populations that are more stable.” Our analysis is relevant to the sustainability or unsustainability of a system that already exists in a given state. This is, however, the most important practical question in sustainability. Is the present state sustainable or not? We contend that the sustainability hypothesis and its two corollaries appear to be at least necessary conditions for the sustainability of a dynamic state of the system.

9. Summary

The principal contributions in this paper are the sustainability hypothesis and its two corollaries. The sustainability hypothesis states that Fisher information of dynamic systems in a sustainable state does not change with time. This is important because it establishes a possible criterion for sustainability. The two corollaries state that increasing Fisher information leads the system on a path through self-organized states while decreasing Fisher information leads the system to a loss of self-organization. This is important because self-organization is a critical and necessary property of living, functioning systems. We illustrate these principles with an example of simple predator–prey ecological model using the Fisher information. The example illustrates the fact that the Fisher information very sensitively tracks the stability of the dynamic state of the system, and hence the sustainability of the dynamic state. It should be noted that

these results address not the sustainability of the system, but the sustainability of the dynamic state of the system. We suspect that the sustainability of the system itself may well be an ill-posed concept because the system, e.g. the Earth, will always exist in some state.

An important consideration in applying Fisher information is determining the state variables of the system in question. For most real systems this is not trivial. An advantage of this approach is that it is sufficiently flexible to accept variables from disparate disciplines such as ecology, thermodynamics, hydrology, or economics. Currently, we are building a series of models and associated computer simulations for systems of increasing complexity starting with simple food webs and then systematically adding further complexity including an economic and legal system. In addition to testing model with simulated ecological systems, we are planning to test the methodology on empirical ecological field data. Work is being planned in this direction also.

List of symbols

A	normalization constant, dimensionless
a	prey growth rate parameter (s^{-1})
b	prey mortality rate parameter ($kg^{-1} s^{-1}$)
c	predator mortality rate parameter (s^{-1})
ds	characteristic length of a state of the system (m)
e	predator growth rate ($kg^{-1} s^{-1}$)
H	Shannon information, dimensionless
I	Fisher information, dimensionless
k	positive constant in definition of Shannon information, dimensionless
p	probability density function, dimensionless
t	time (s)
x	variable, dimensionless
y_i	state variable, kilograms of biomass
y_0	biomass of prey (kg)
y_1	biomass of predator (kg)
Δt	integration time step (s)

Acknowledgements

The authors wish to acknowledge the support of the National Risk Management Research Laboratory and Sustainable Technology Division. In particular we acknowledge the support and encouragement of Dr. Subhas K. Sikdar, the director of the Sustainable Technology Division. BDF is a research associate with the US Environmental Protection Agency Postdoctoral Research Program.

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