Lecture 5 Uses and Applications

CHAOS IN ENGINEERING

Solids (spatial chaos) and Fluids (turbulence)

Chemical Reactions

Telecommunications and Mathematical methods

FRACTALS

Hearts and Lungs

Crystals, Snowflakes and the Fractal Dimension

IMPACTS AND FRICTION

Mooring towers and bouncing balls

Shimmy, chatter and time delays

HEALTH

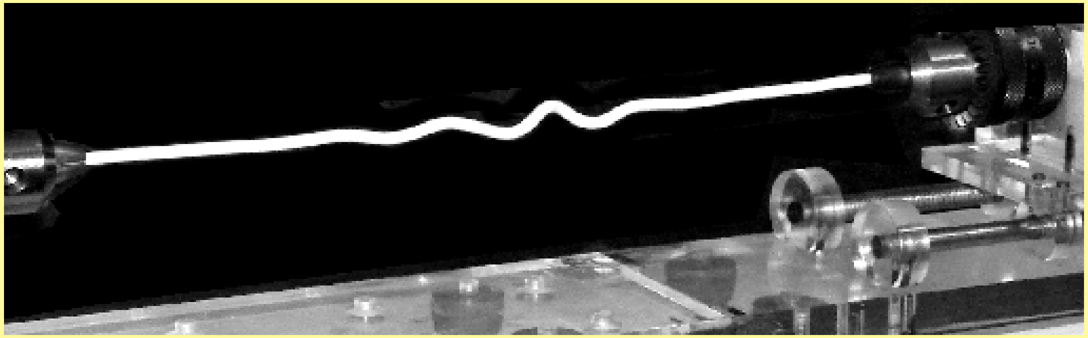
Chaos in your heart and brain may be good for you

Chaos in Space as well as in Time

In a static-dynamic analogy a distance, *x*, replaces time, *t*.

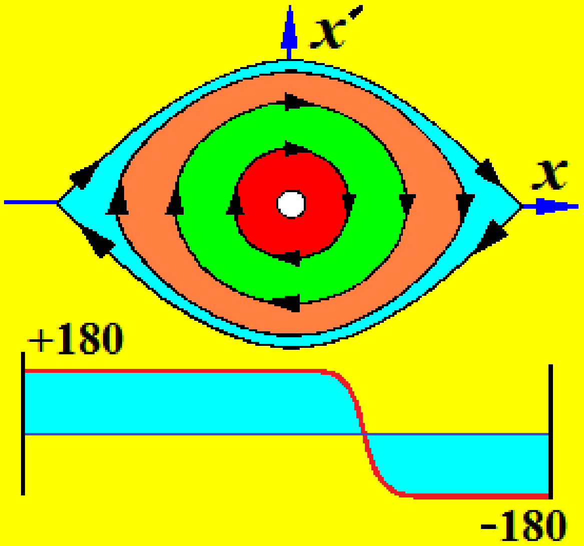
Pendulum = Buckling column. Spinning top = Twisted rod

A homoclinic connection now gives a spatial localization



When a stretched and twisted elastic rod buckles it adopts a localized form.

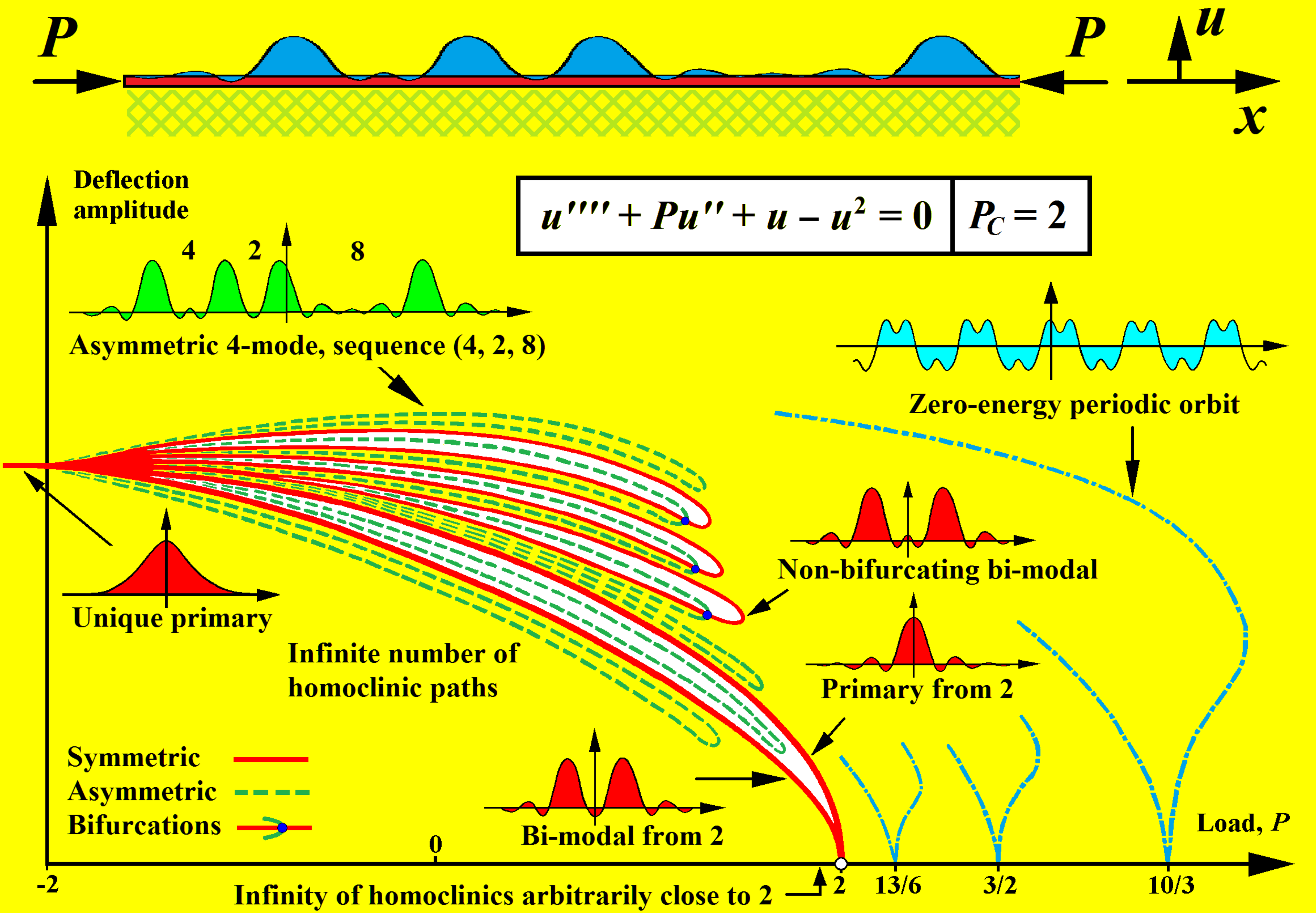
A non-circular rod (= non-symmetric top) also exhibits spatial chaos.



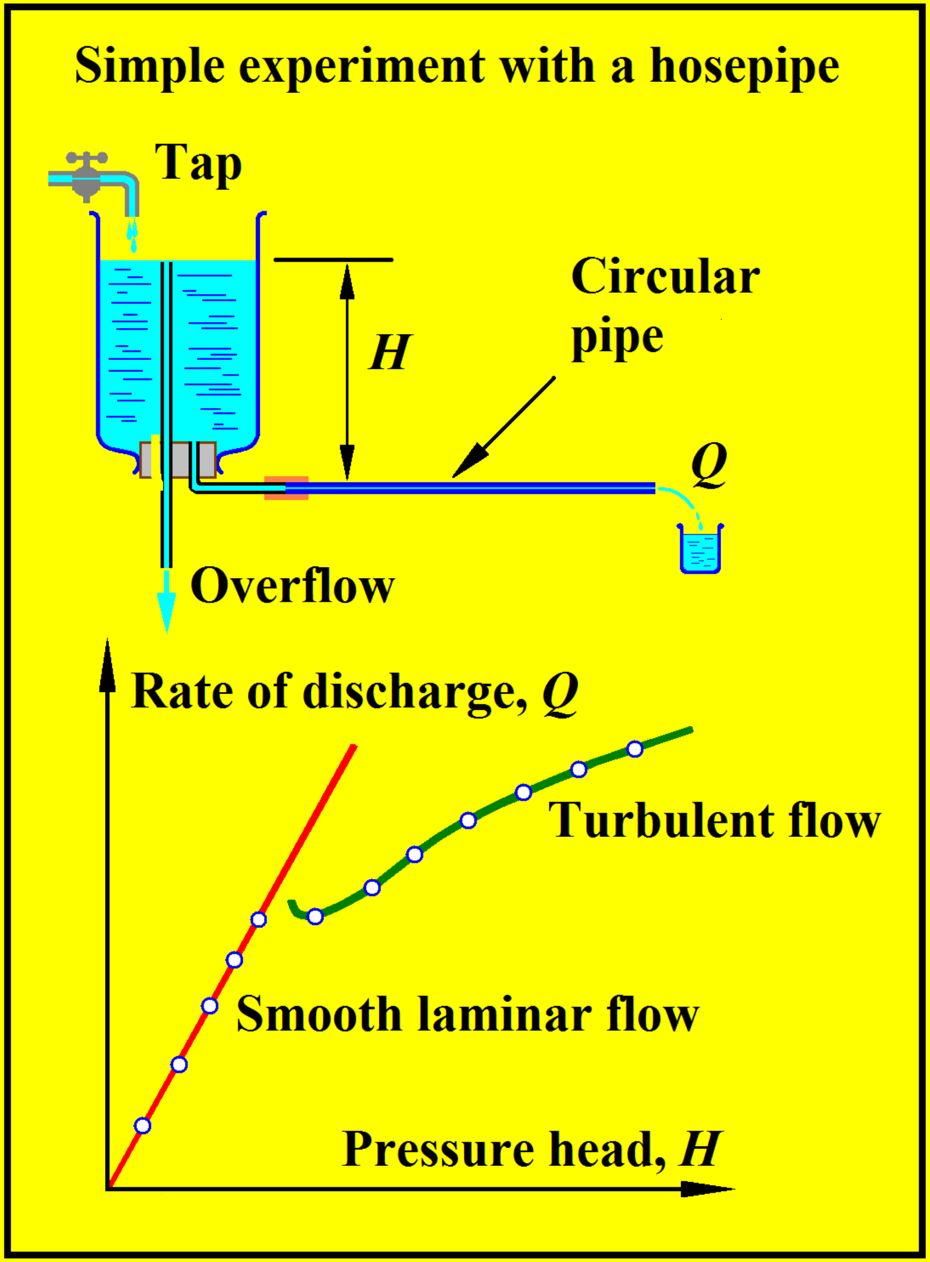
The homoclinic behaviour of the simple pendulum

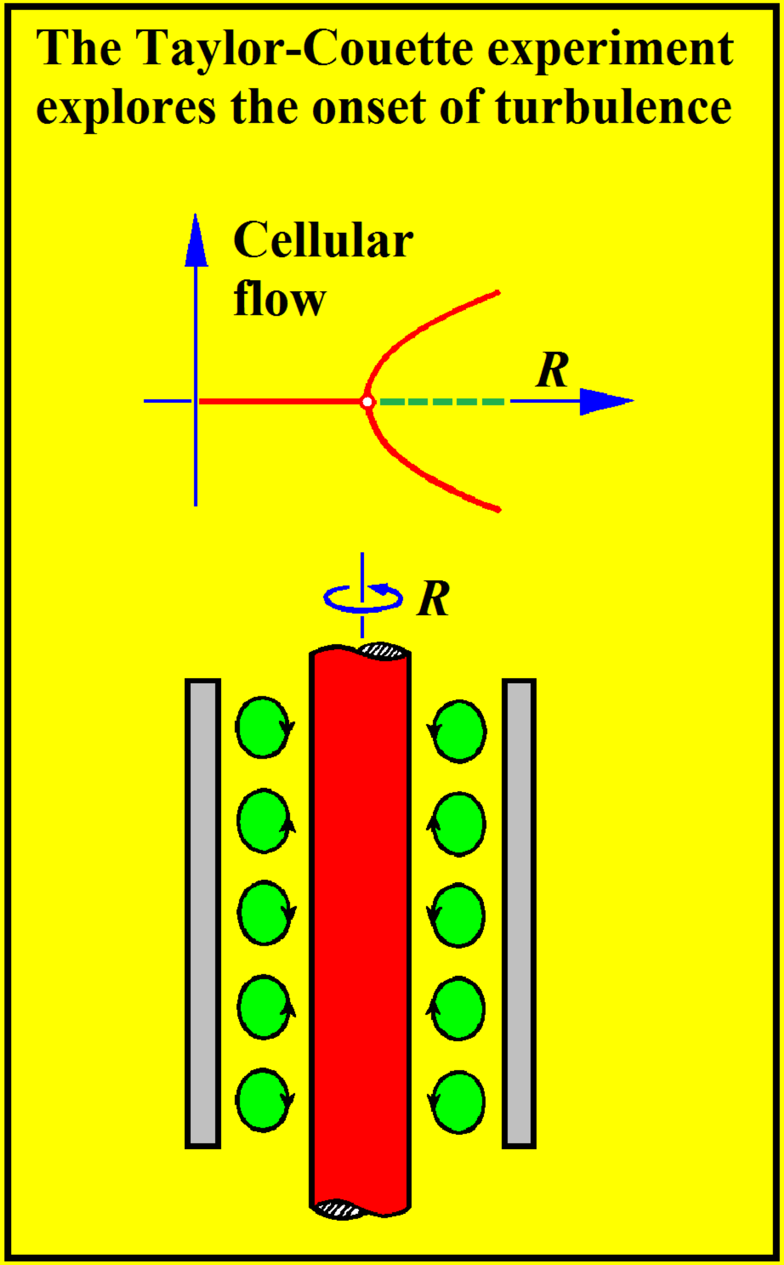
Rig and Rod

Long compressed beam on a nonlinear foundation

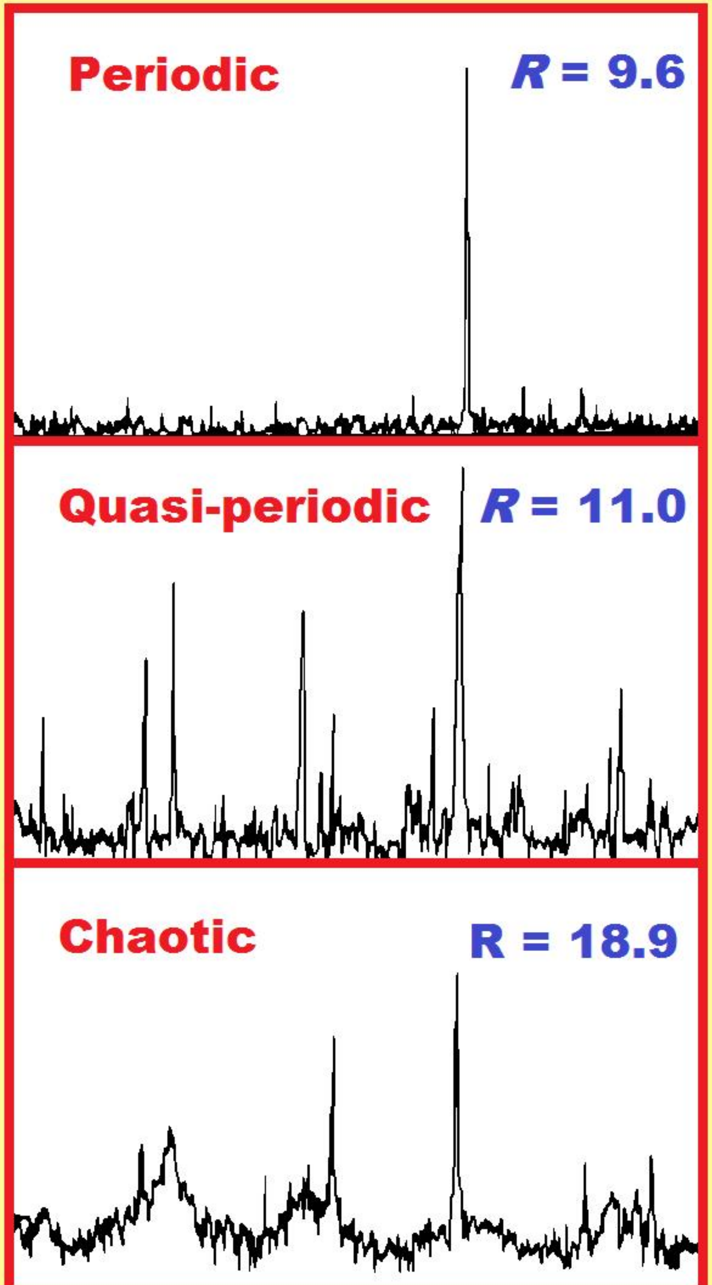


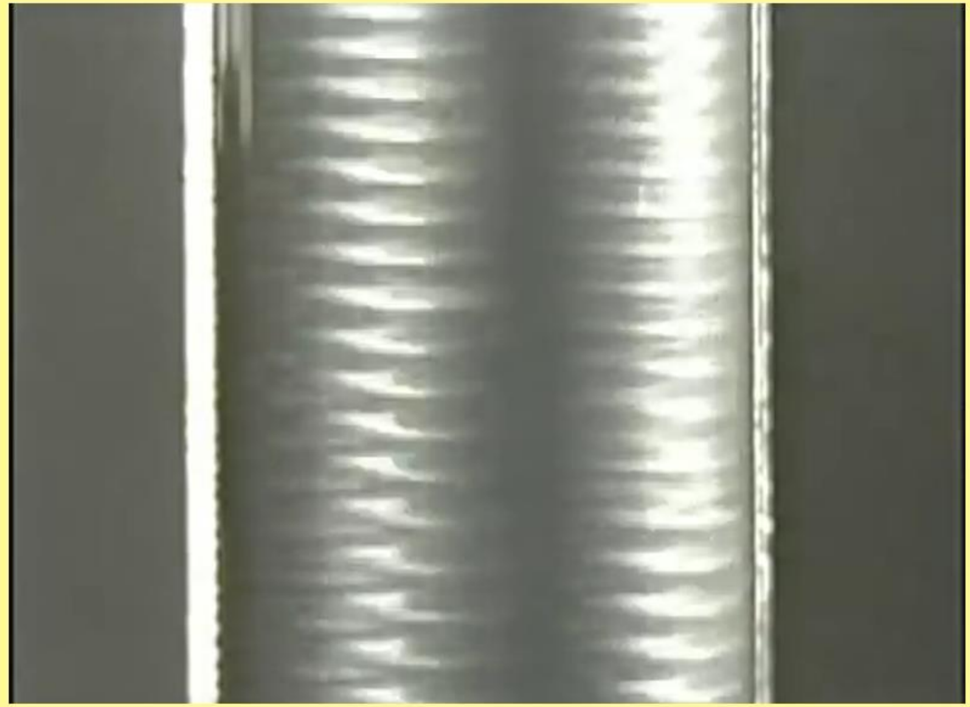
Turbulence is a Major Topic in Fluid Mechanics





Taylor-Couette rotating flow experiment: Power Spectra

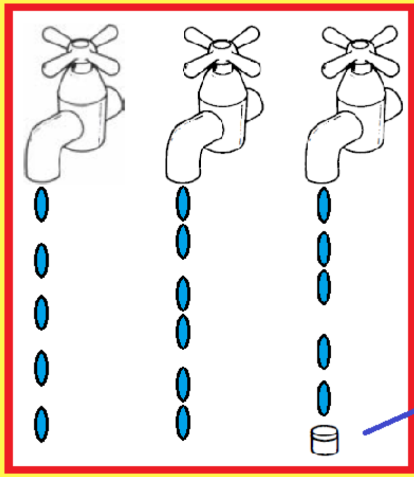


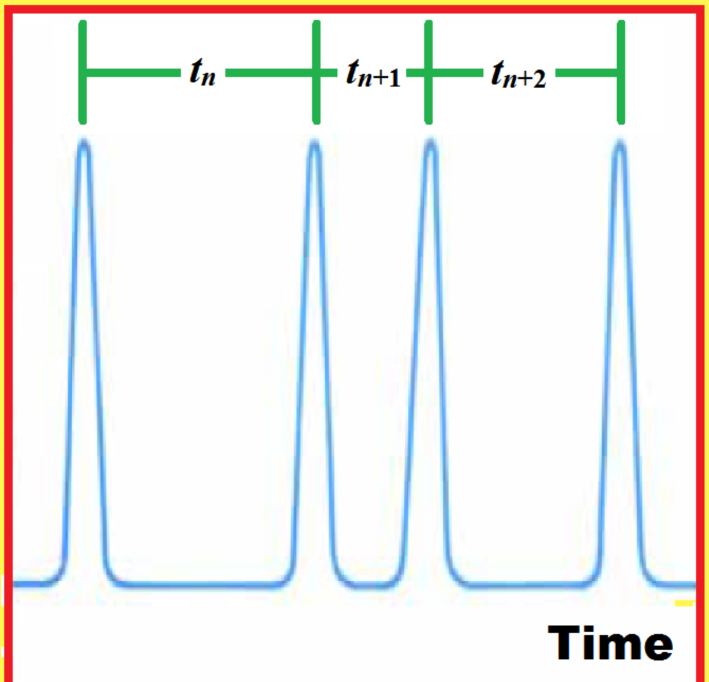


Showing plots of power, *P*, against frequency *w* for different values of the rotation speed, *R*

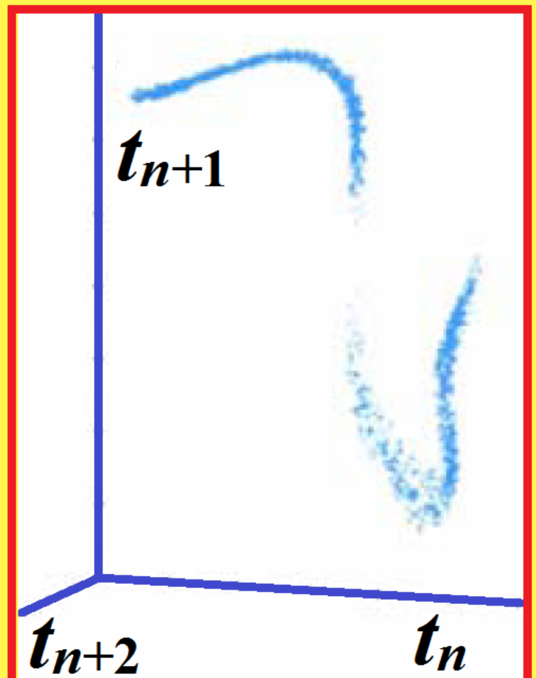
m22 taylor

Chaotic Dripping of a Tap

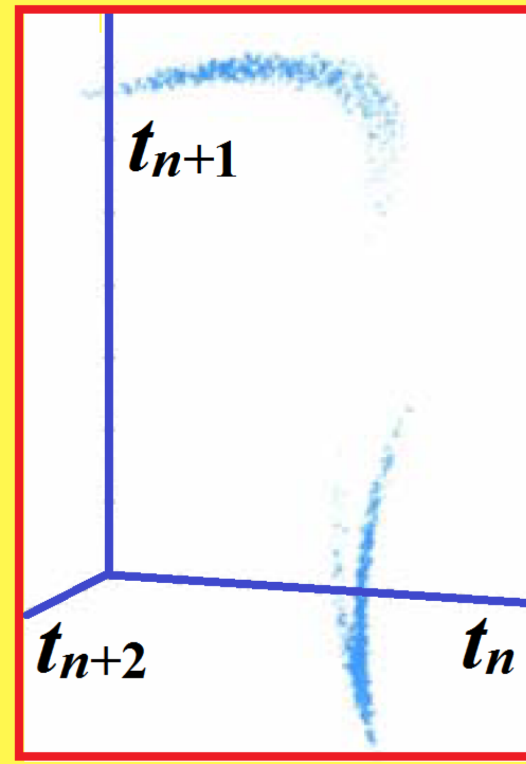




Experiment

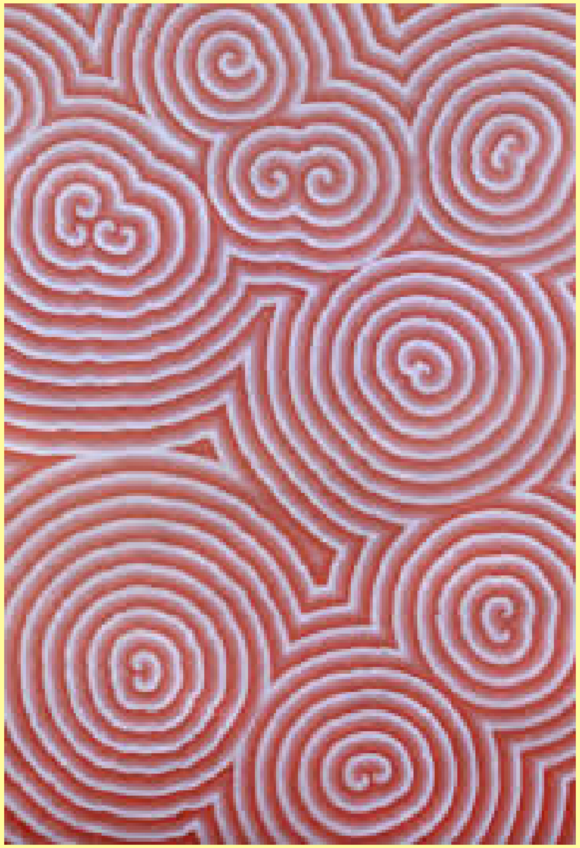


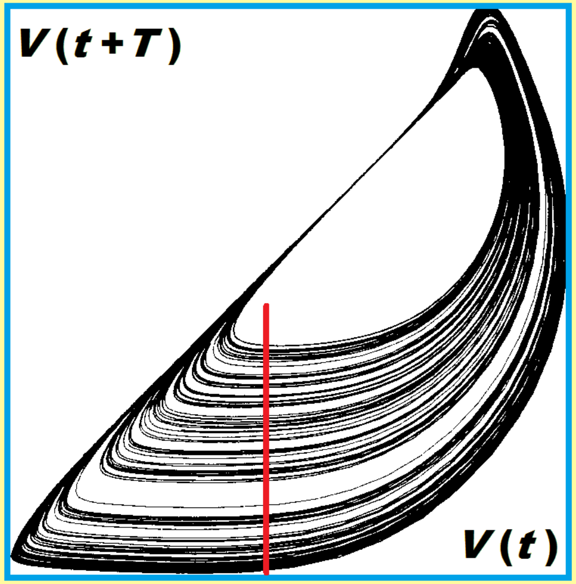
Math Model



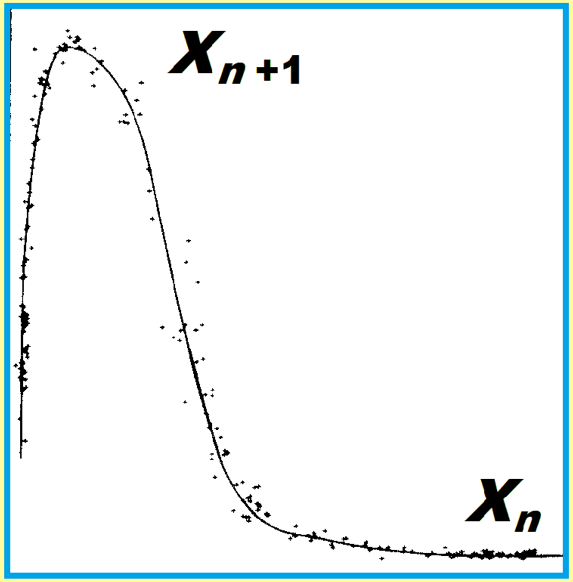
Chaos in oscillating chemical reaction (Belousov-Zhabotinsky)

Spontaneous pattern formation as conceived by Alan Turing





Chaos in a 2D projection of a 3D phase space using $V(t)$, $V(t+T)$, $V(t+2T)$

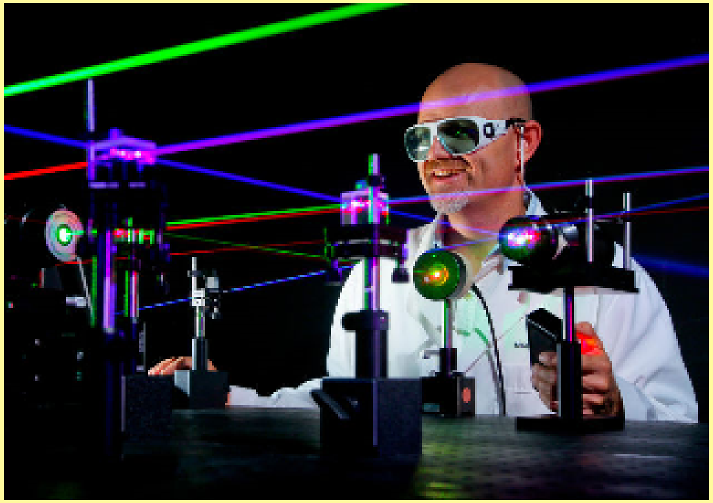


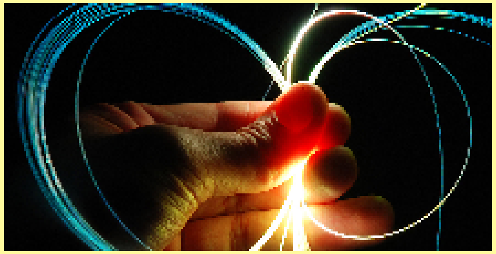
A 1D map from crossings of the red line in the left-hand figure

Chaos for a Secure High-Speed Internet

High-speed web communications use digitally-coded lasers in fibre-optic cables.
Chaos in the lasers caused by reflective feedback needs to be controlled.
Sensitive messages need safe encoding and decoding procedures.

Many books have been written to see if chaos can help.





Early trial of speech encoded chaotically
Top: before encoding   Bottom: decoded

Techniques for Controlling Chaos

Delicately tweak the parameters
Exploit invariant manifolds (OGY)

Using Chaos for Encryption

Two chaotic oscillators, A and B, can synchronize with one another.
This persists if a low-amplitude message is added to the transmission from A.
Subtracting B's stimulated chaos from the received signal reveals the message.

Chaos generated by Euler's Method

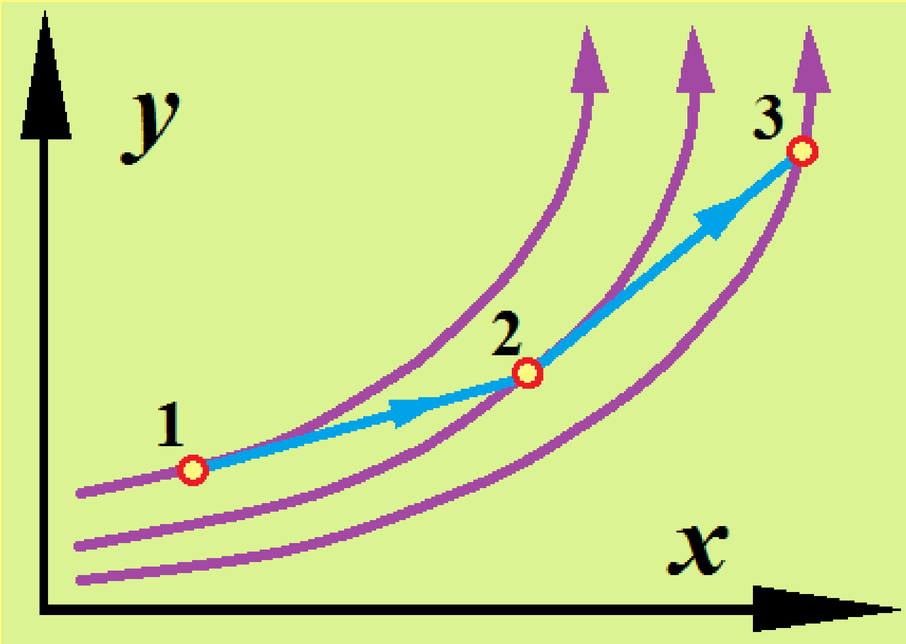
Euler's Method

$\mathrm{d}x/\mathrm{d}t = F(x, y)$

$\mathrm{d}y/\mathrm{d}t = G(x, y)$ giving

$\Delta x = x_{n+1} - x_n = \Delta t\, F(x_n, y_n)$

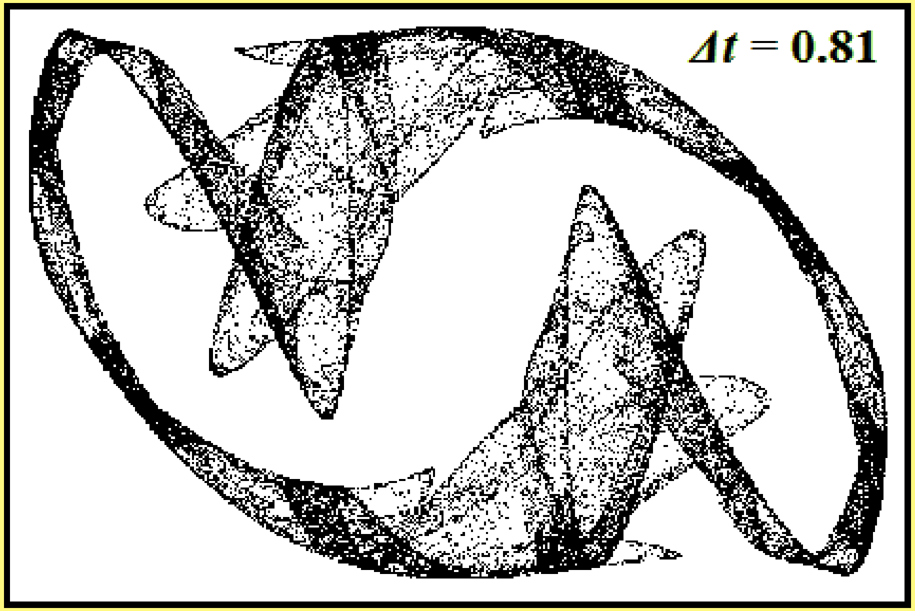
$\Delta y = y_{n+1} - y_n = \Delta t\, F(x_n, y_n)$



Example with a circular periodic attractor (chaos not possible in 2D)

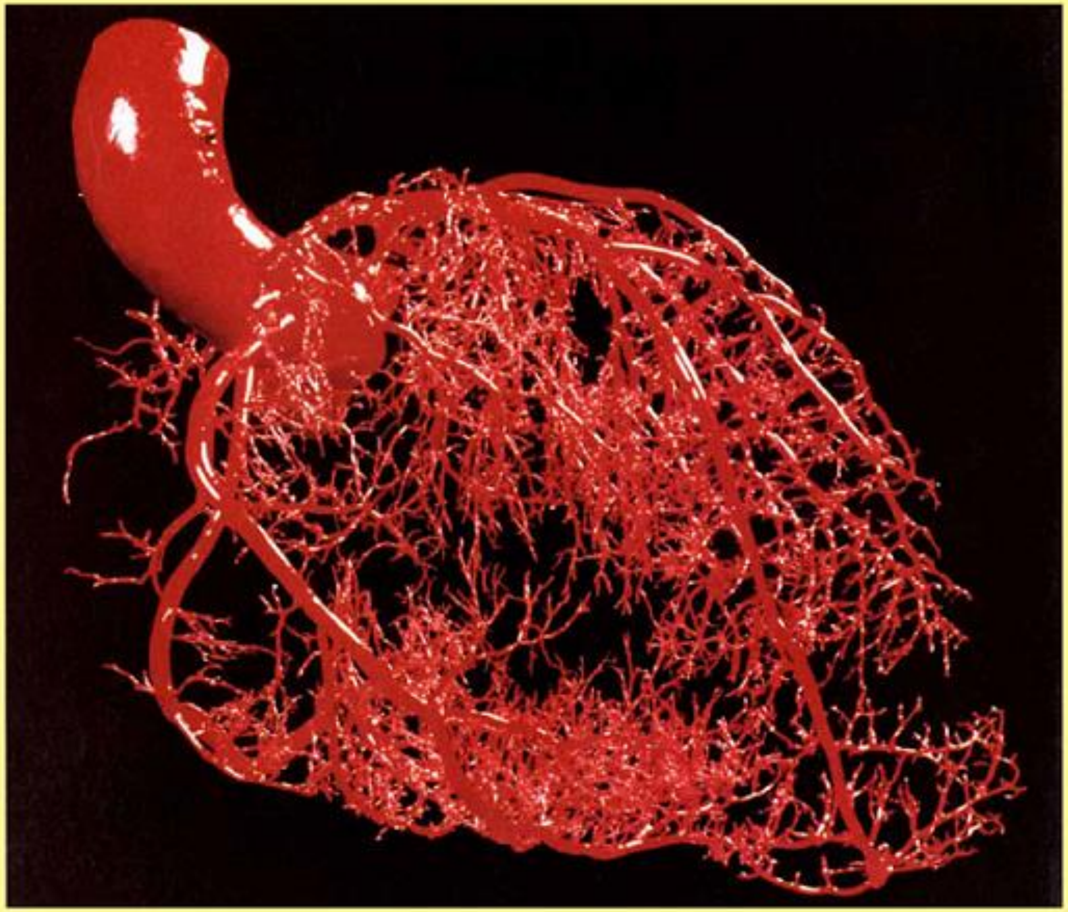
$\mathrm{d}x/\mathrm{d}t = y$

$\mathrm{d}y/\mathrm{d}t = (1 - x^2 - y^2)y - x$



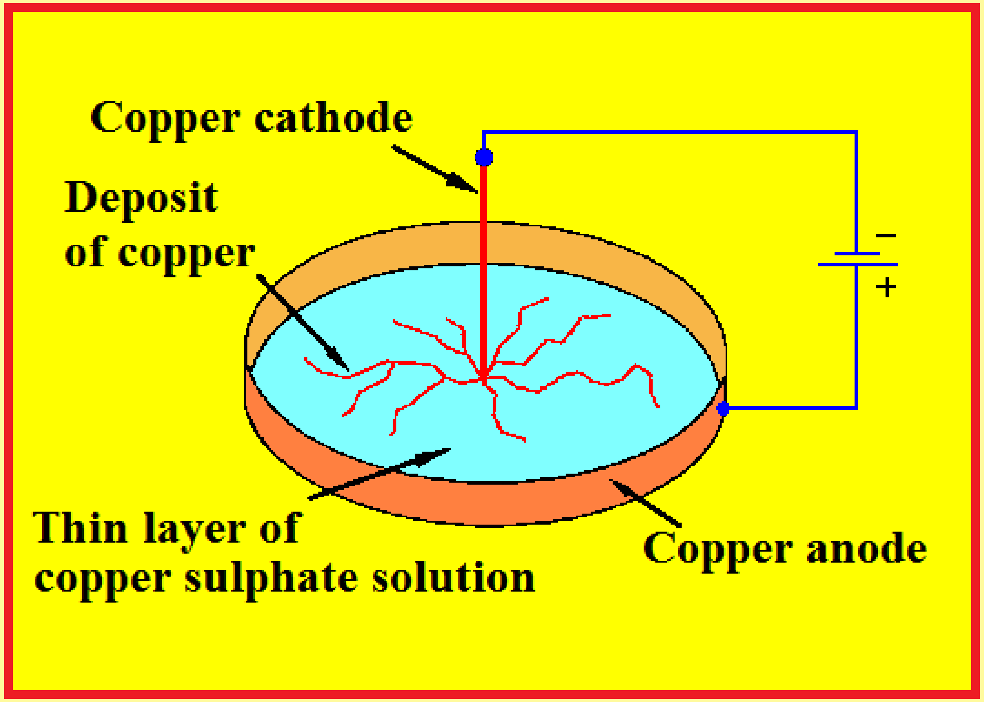
Fractal-like structures emerge naturally in plants and animals

AIRWAYS of the LUNG
Fractal geometry in our bronchial tubes.

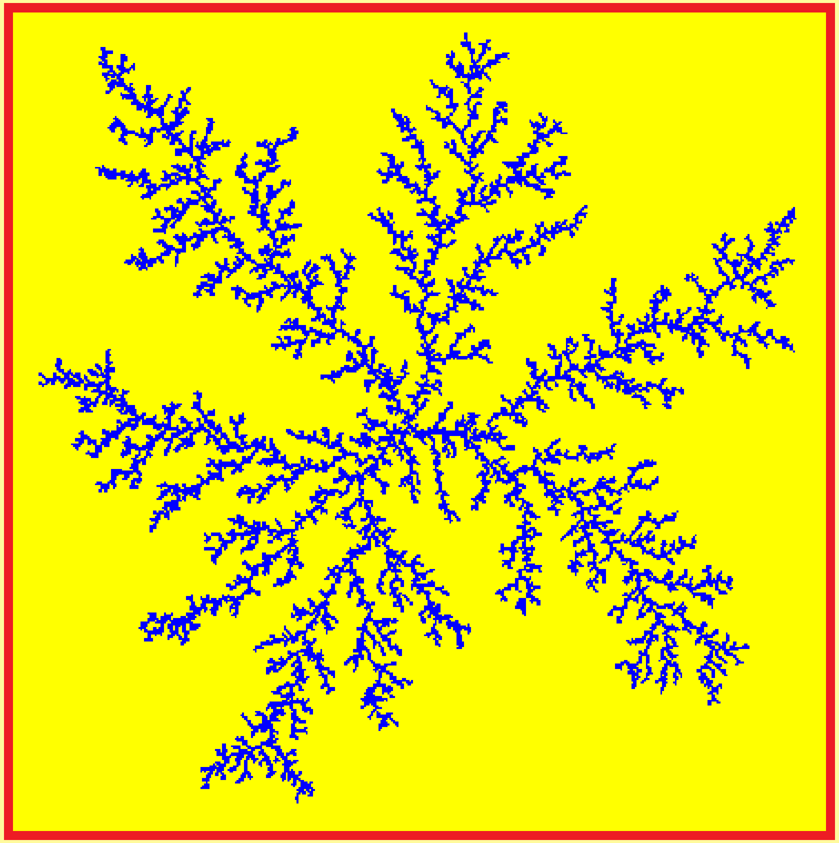


BLOOD VESSELS of the human HEART
These exhibit fractal branching into smaller and smaller blood vessels.

Fractal growth of a copper deposit during electrolysis



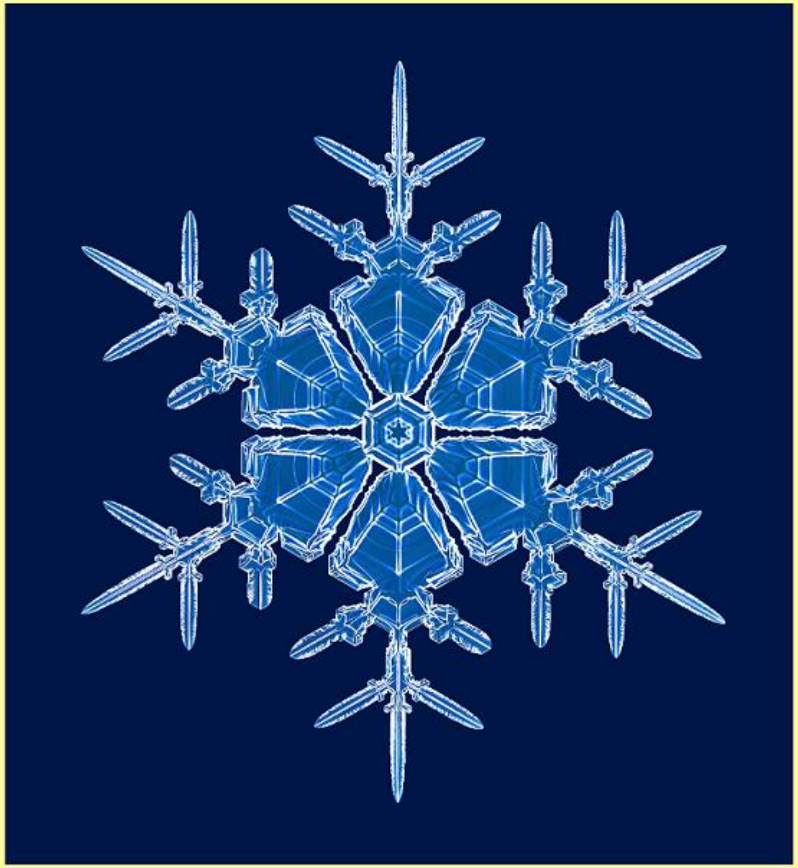
The experimental set-up in the laboratory

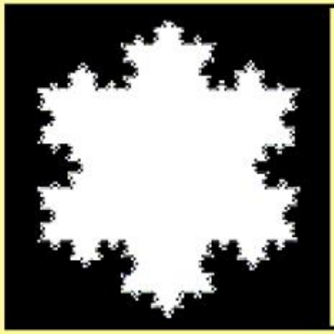


A theoretical prediction using 'diffusion-limited aggregation' modelling

Photographs of two 'fractal-like' real snowflakes

Hexagonal symmetry reflects shape of a water molecule





Mathematical Koch flake,
$d = \log 4 / \log 3$
$\simeq 1.26186.$

Fractal dimensions (d) of real and simulated snowflakes are are usually similar (in the range 1.4 to 2.0).

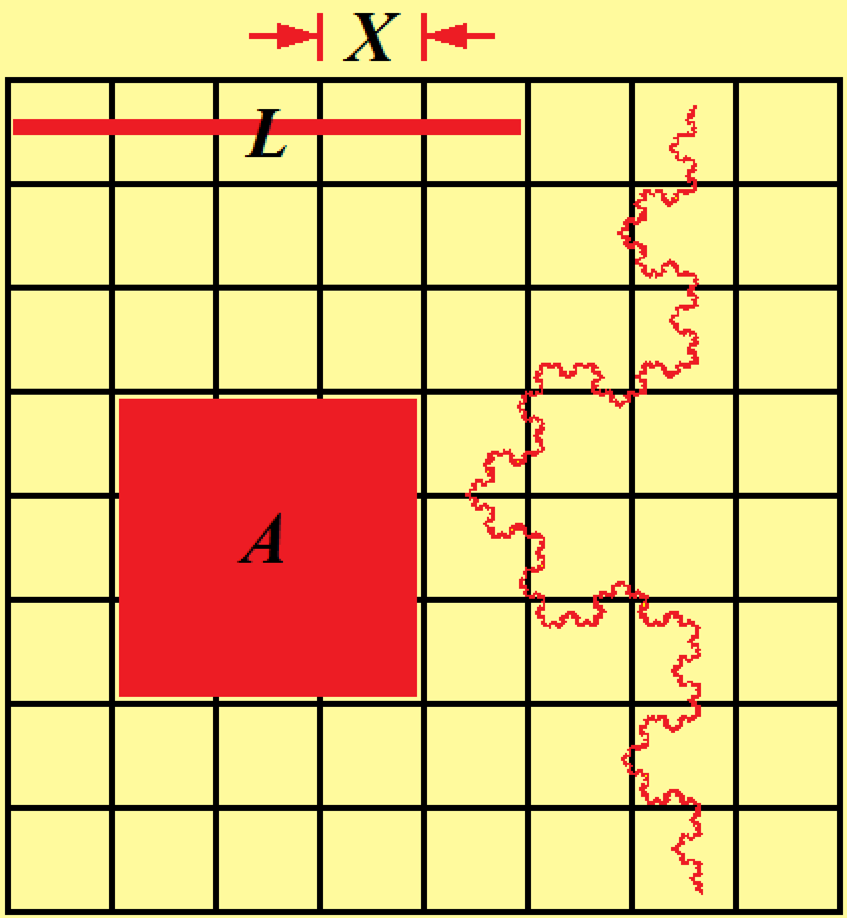
Calculation of the fractal 'box' dimension

N = number of squares, size X, needed to cover an object

For a line of length L, $N = L/X$. So N varies as X^{-1}. $DIM = 1$

For a square, area A, $N = A/X^2$. So N varies as X^{-2}. $DIM = 2$

A fractal is awkward to cover, and if we apply the above method we invariably arrive at a non-integer value for *DIM* (often written as *d*). Strictly the method should be used in the limit as $X \to 0$.

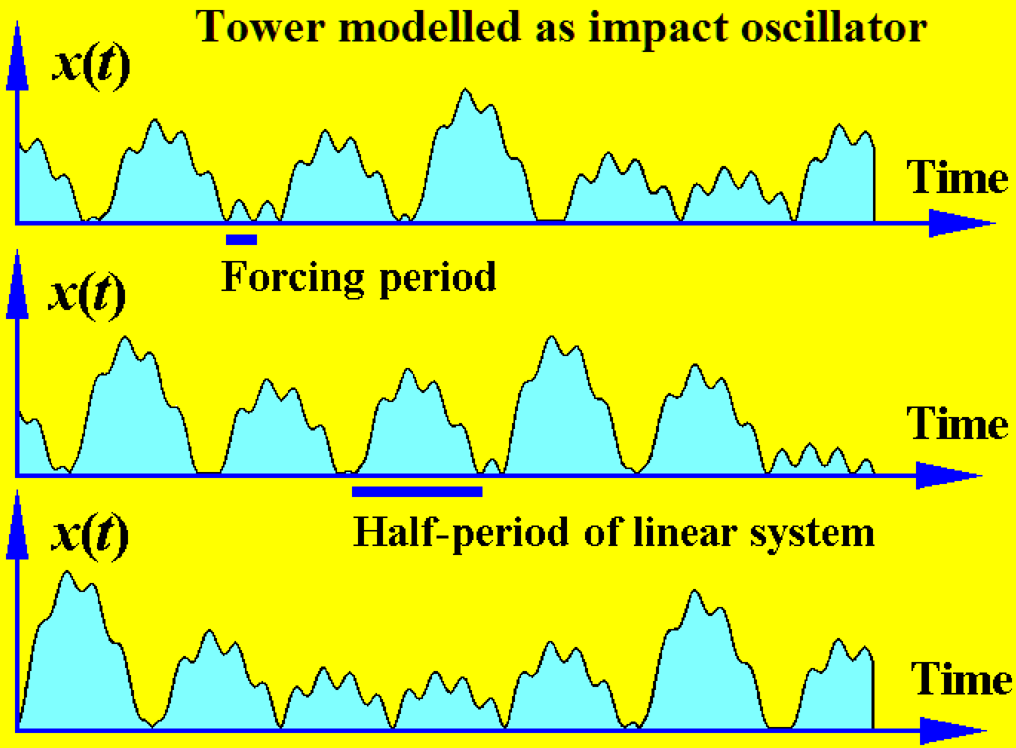


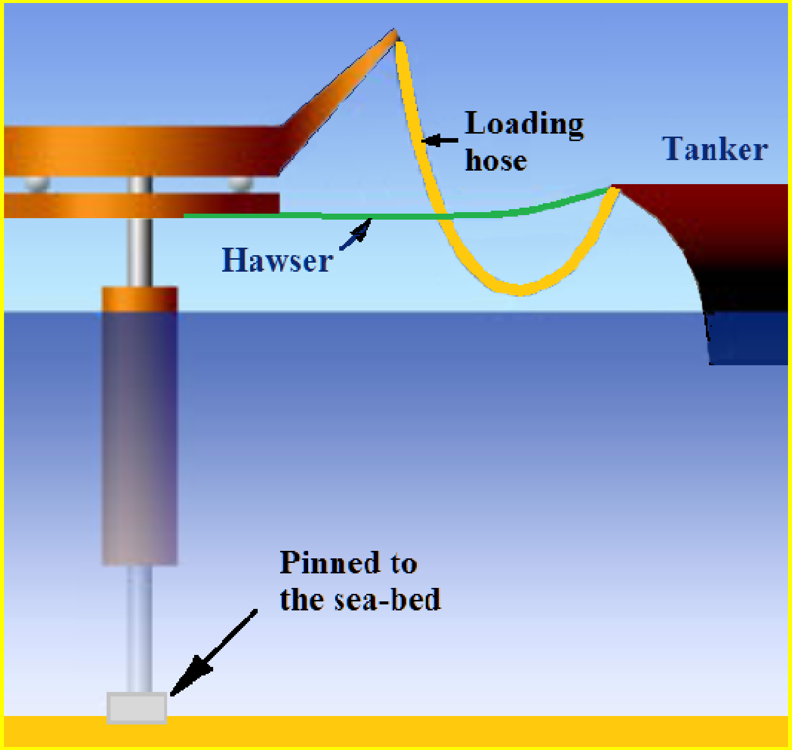
When applied to the Koch boundary curve, as shown, we would find the answer:

$DIM \simeq 1.26186.$

Articulated mooring tower

Oil tankers are moored to towers hinged at the base and held vertical by a buoyancy chamber. Waves drive these towers to oscillate, causing the hawser to the tanker to slacken and snatch in an 'impact'. Subharmonics and chaos are generated.

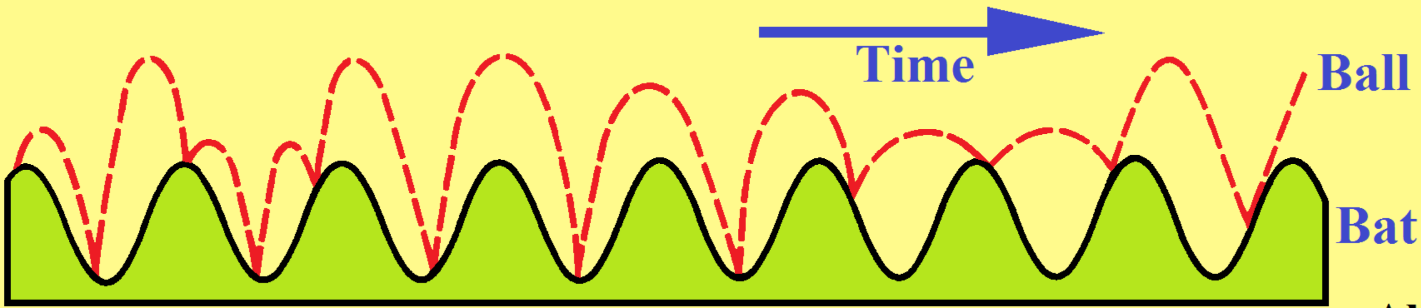


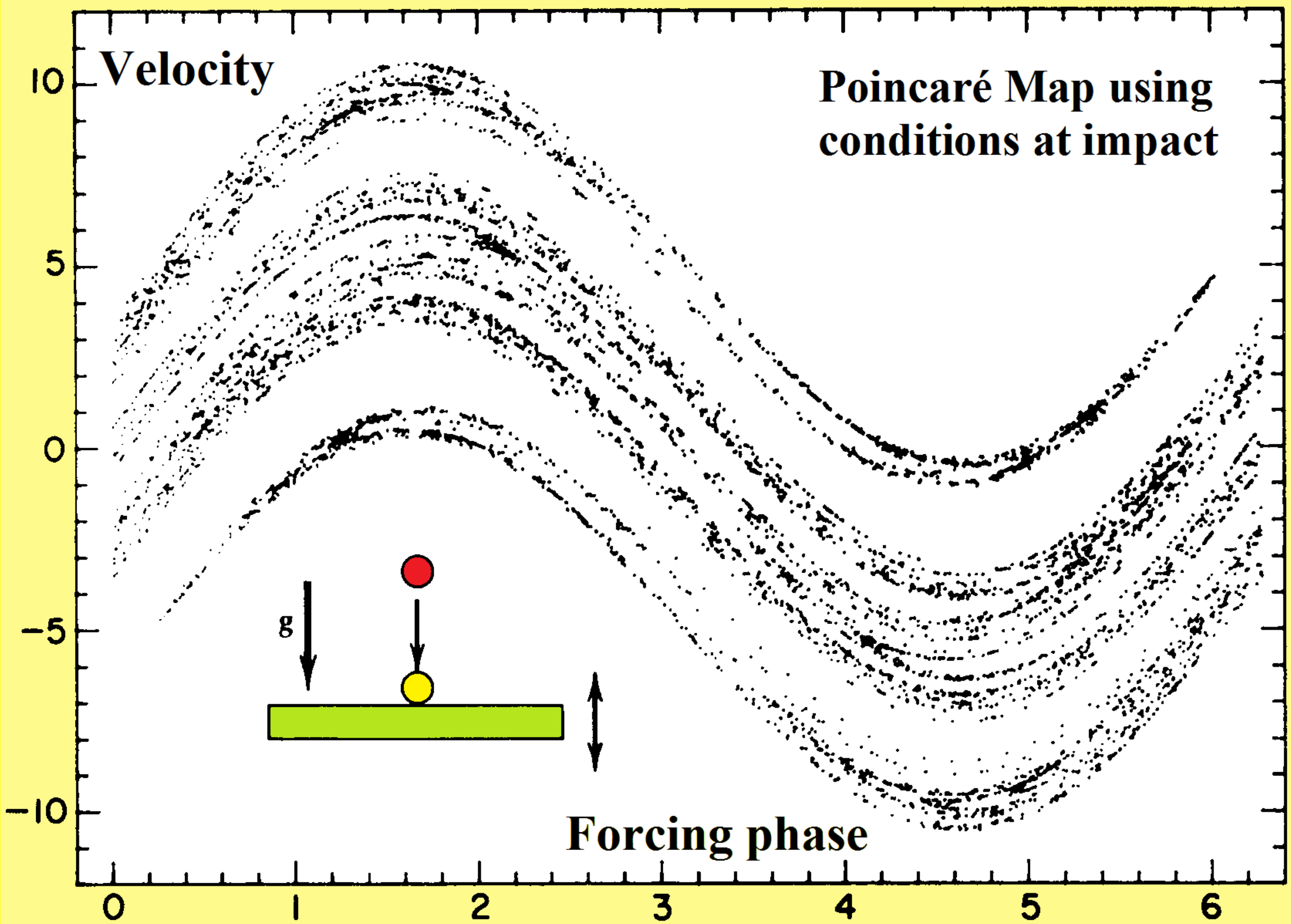


m28 lv impct

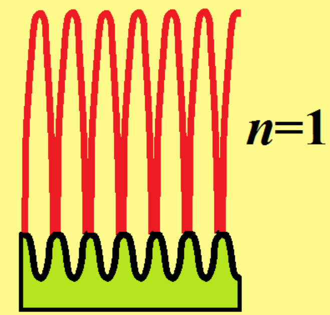
Chaotic bouncing on a periodically vibrating bat

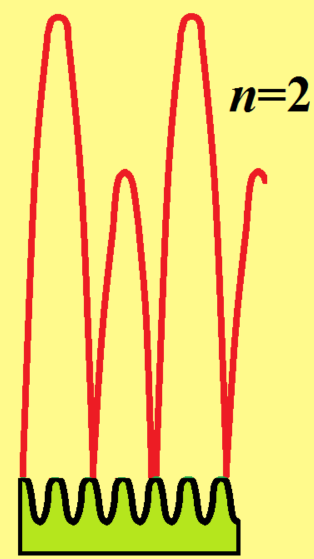
clck





Alternative steady states ...





Shimmying of motorbikes

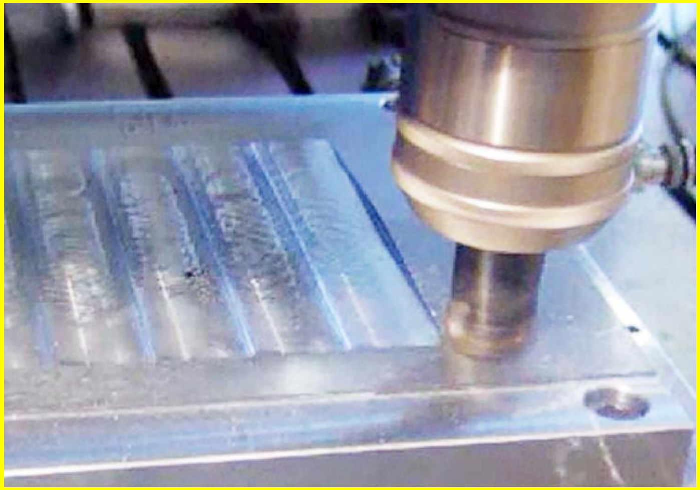
Due to slipping and time delays

m23 m-bik3

Chattering of machine cutting tools

Again associated with sliding friction and time delays

Surface milling

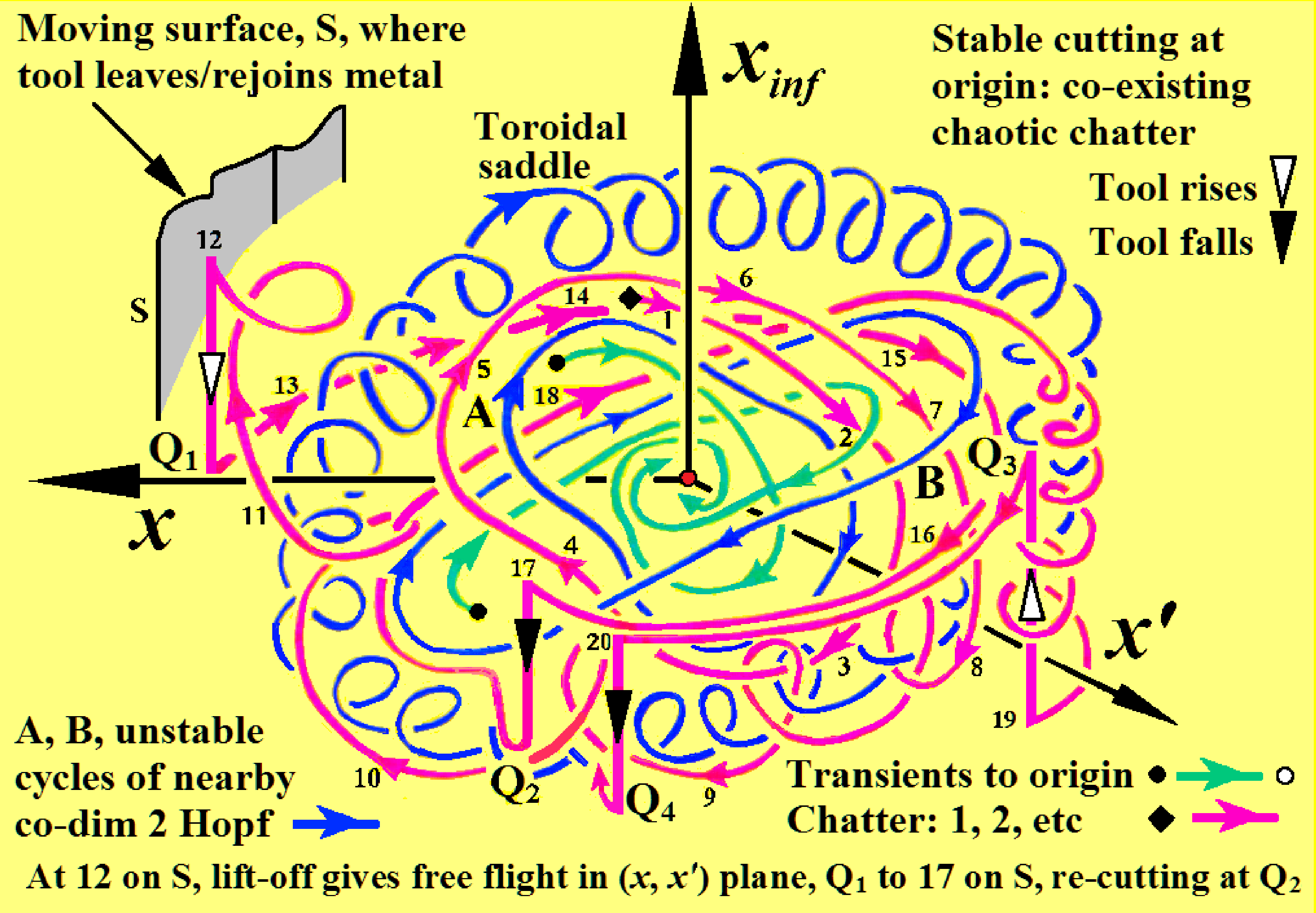


Stick-slip cutting and drill string buckling

(in an under-sea drilling experiment)

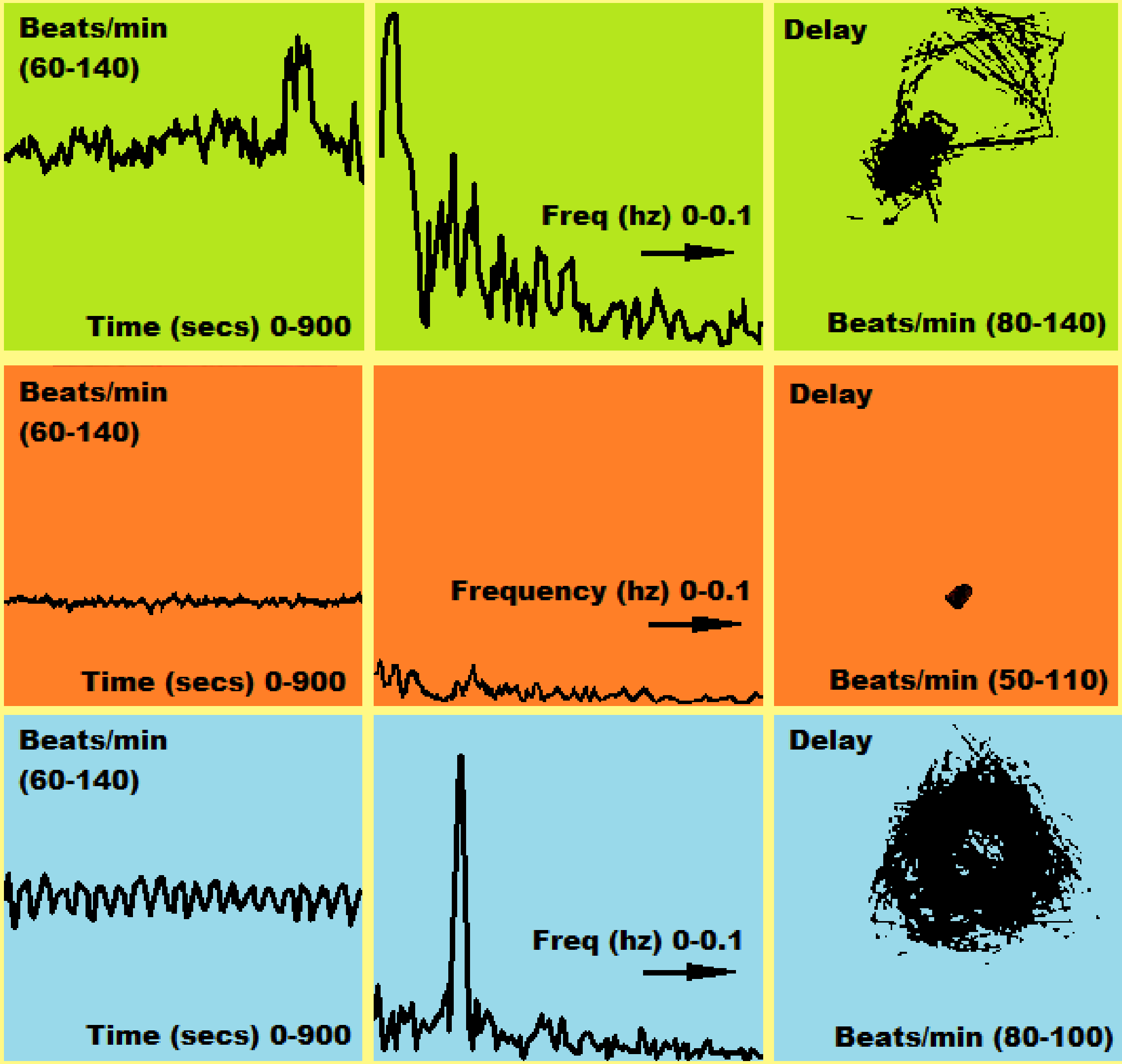
m25 ab drill

Sketch by Gabor Stepan to understand transient chaos in chatter (and shimmying)



Chaotic Heartbeat may be Good for You !!

Time series of beat-rate | **Power spectrum** | **Phase space**



Healthy heart:

Wave form seems erratic

Broad power spectrum

Like a chaotic attractor

Unhealthy (1):

Nearly constant

Flat power spectrum

Like a point attractor

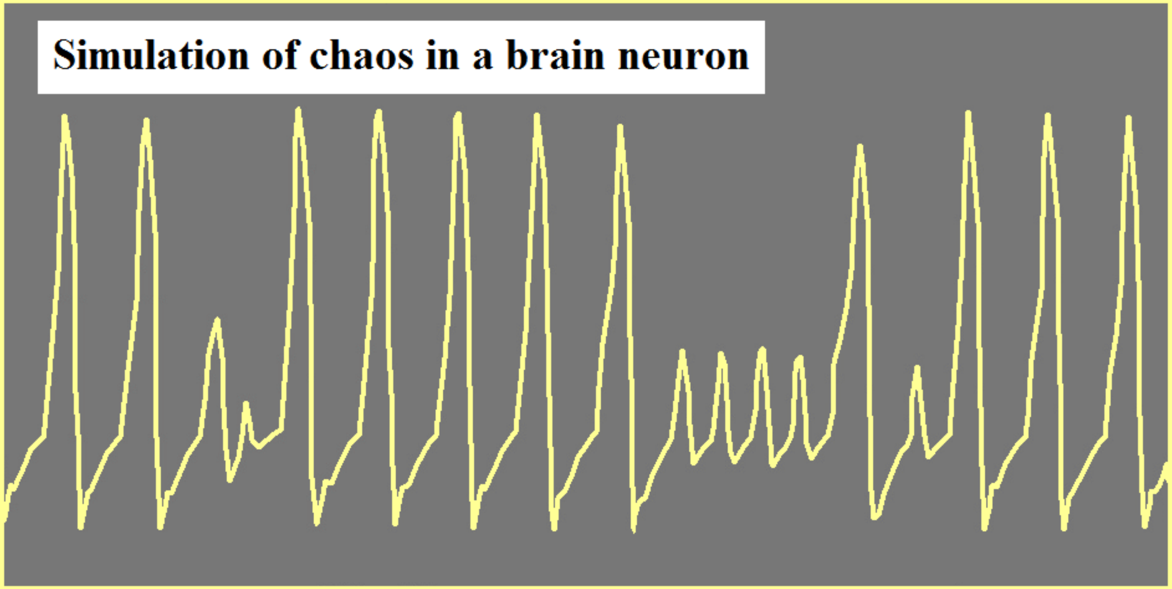
Unhealthy (2):

Quite periodic

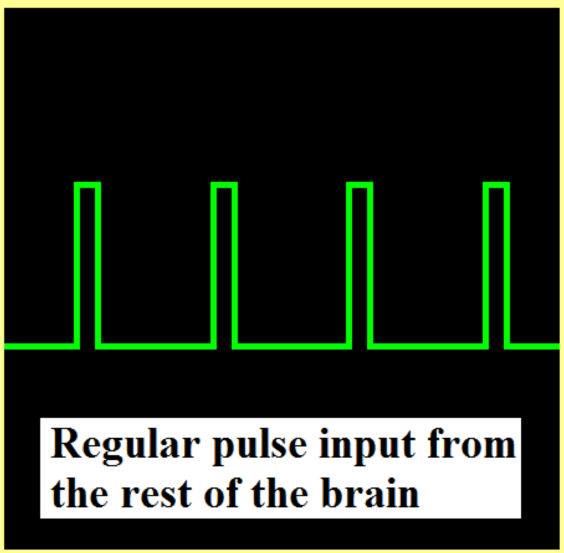
Spike in spectrum

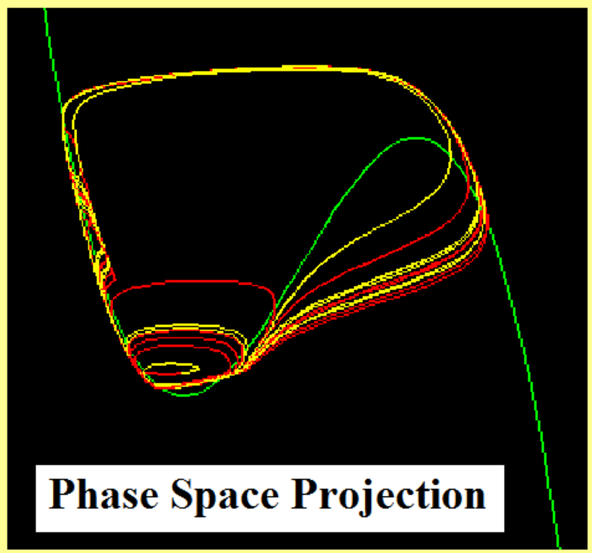
Like noisy cycle

Chaos in the Firing of a Brain Neuron under Regular Excitation



It is said that human brains work best in a chaotic state.





So if your brain
is in chaos, the lecture was a success!

CONCLUDING REMARKS

Chaos = random response of a precise deterministic system

It exhibits extreme sensitivity to initial conditions

Chaotic solutions are mathematically unsolvable

Numerical computations are prone to enormous errors

Detailed prediction is impossible

However there is order within chaos

FURTHER LECTURES AND INFORMATION

My homepage: http://www.ucl.ac.uk/~ucess21/

Norah Boyce Lecture, 10 May 2016, The butterfly effect

Same course of 5 lectures next year. Tell your friends!

Comments welcome on jmtt@ucl.ac.uk