Lecture 4

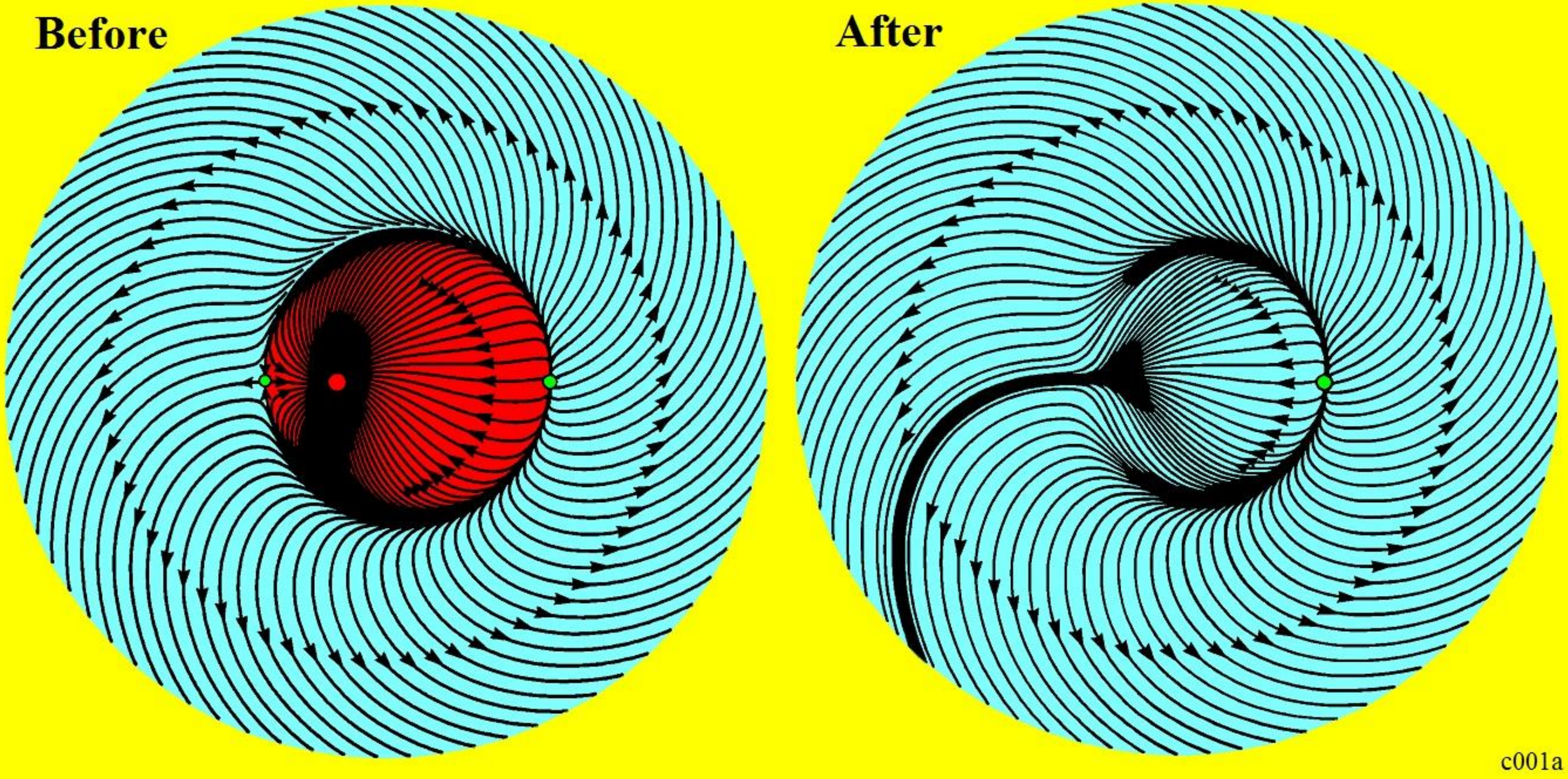
ABC of dynamics, populations and weather, ship capsize

- Attractors, basins and bifurcations
- Stabilizing a pendulum on a jigsaw
- Population dynamics and logistic map
- Weather forecasting
- Escape from a potential well
- Ship capsize in waves

ABC of Nonlinear Dynamics

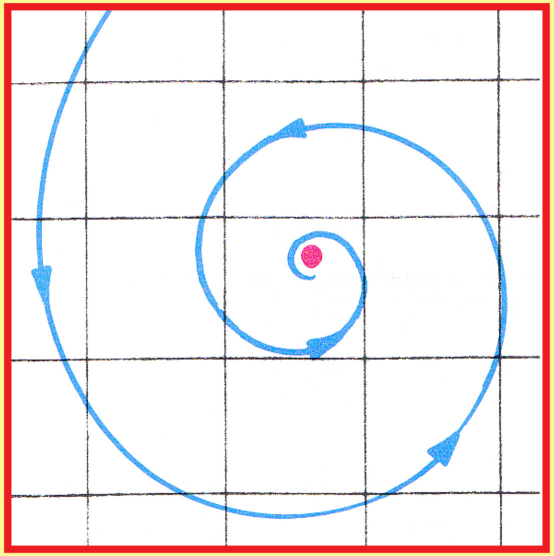
Key concepts of dissipative dynamics are:

Attractor, Basin, Catastrophe (bifurcation)

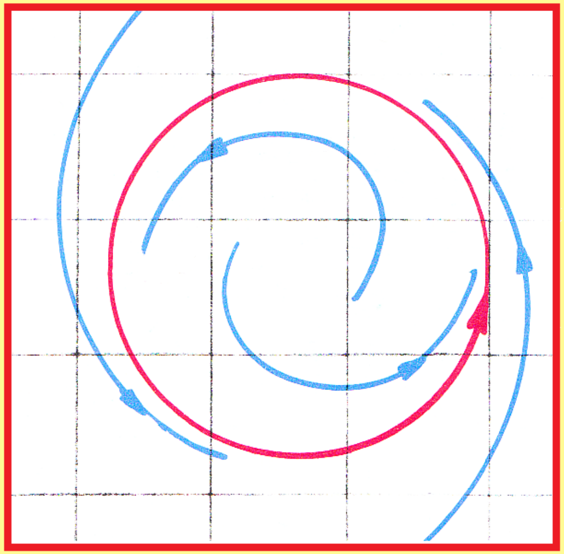


The Attractors of Nonlinear Dissipative Dynamics

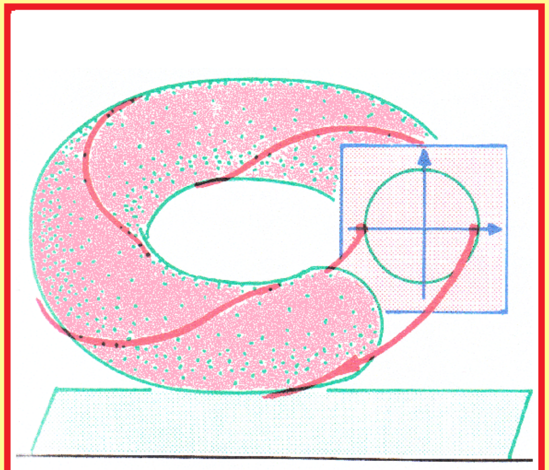
The only attractors in 2D are the fixed point and the periodic cycle. In higher D: quasi-periodic attractor, chaotic attractor.



Point attractor

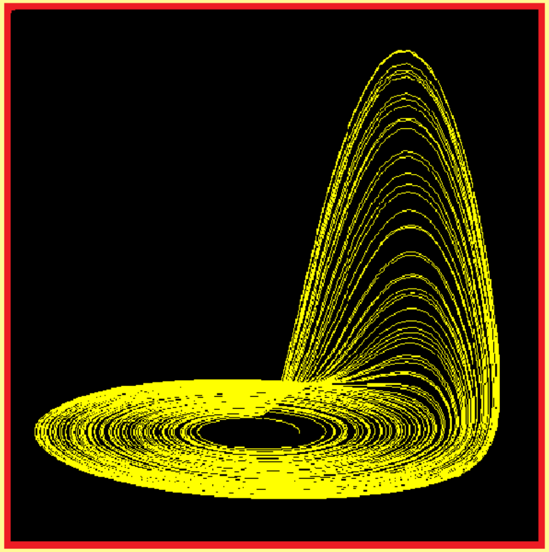


Periodic attractor



Quasi-periodic attr

The above three were thought to be the only attractors until the advent of chaos theory ....



Chaotic attractor

BASINS OF ATTRACTION

When you have a bath, take a good look at the soap bubbles.

Imagine you are looking at a 3D phase space.

Each bubble is a basin of attraction

At its centre is an attractor

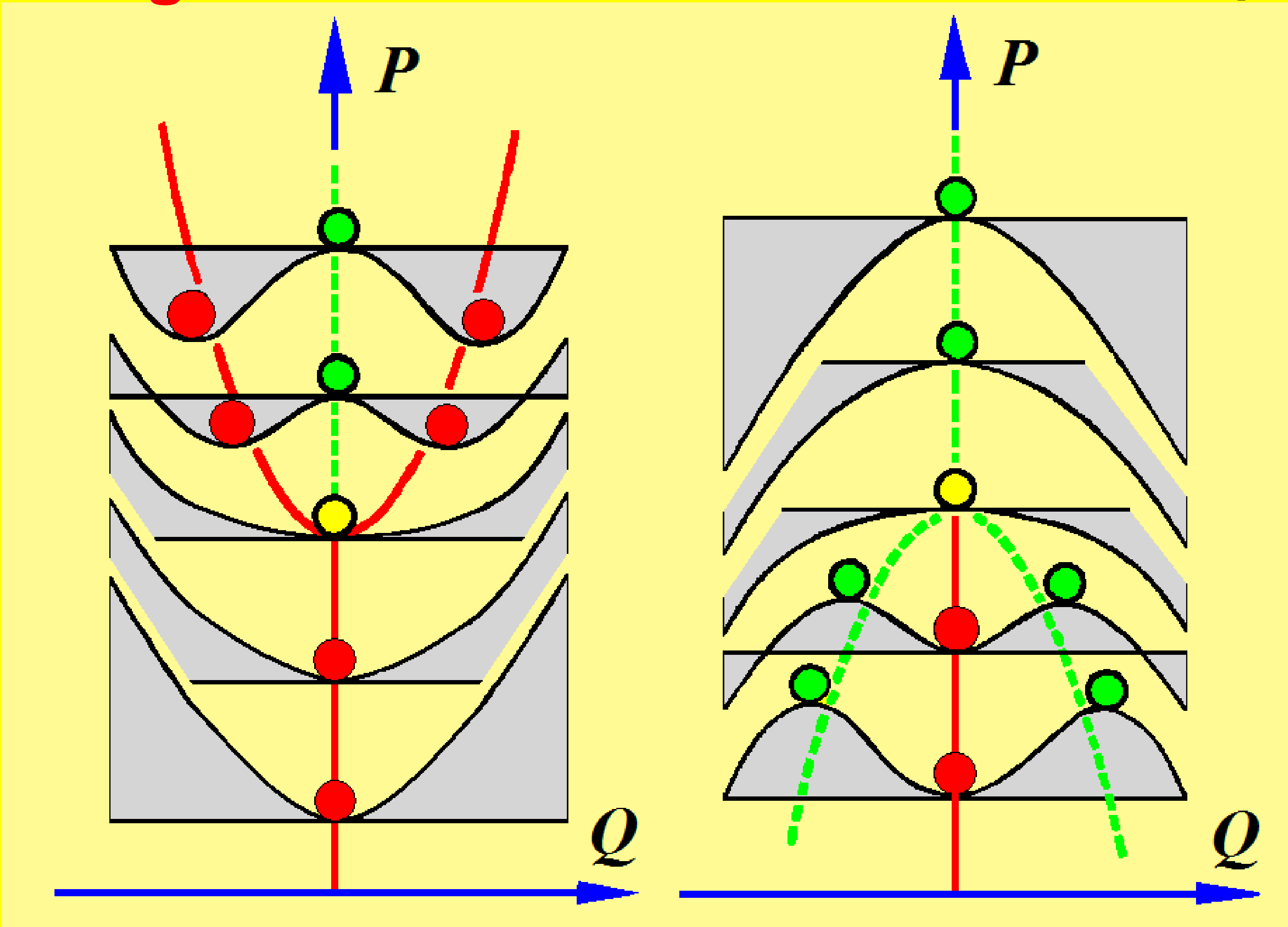
A given system can have many attractors of different types, each sitting in its own basin of attraction.
The attractor chosen depends on the starting conditions.

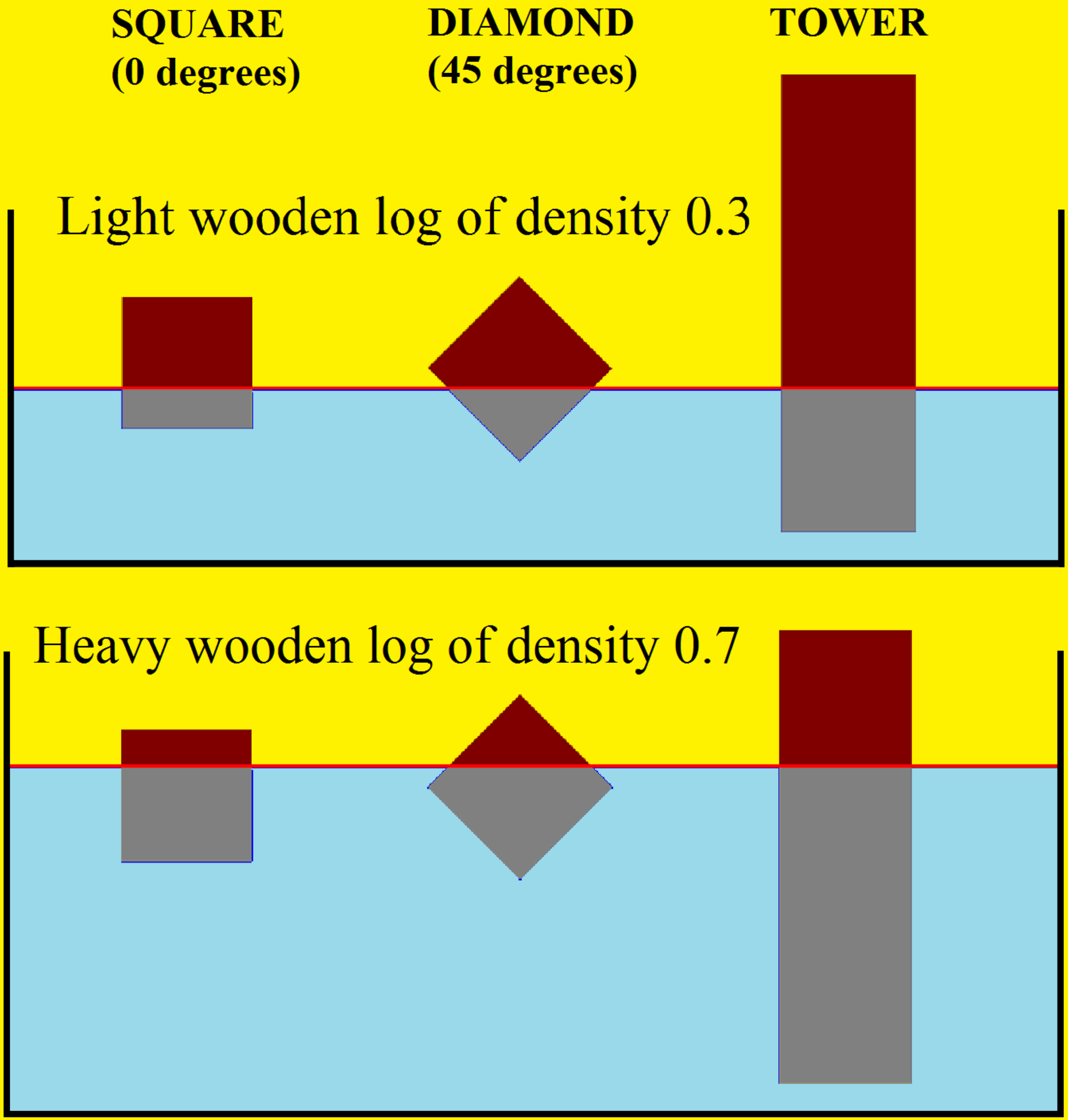
Chaos theory brought us the new 'FRACTAL BASIN'

m14 jig   m29 lv coex

Two bifurcations of an equilibrium path.

Chaos gives a lot of new bifurcations. **Exp strut**



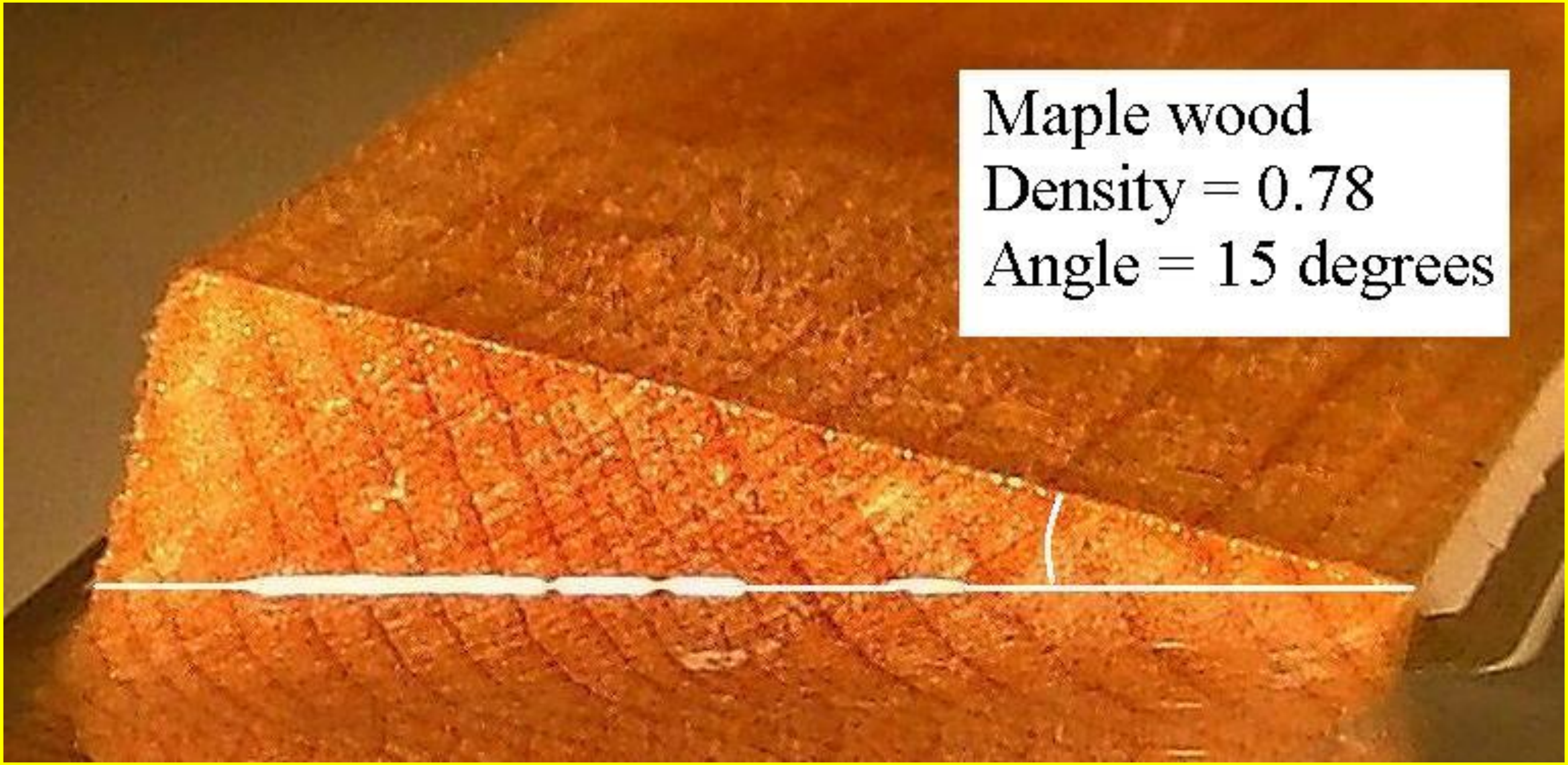


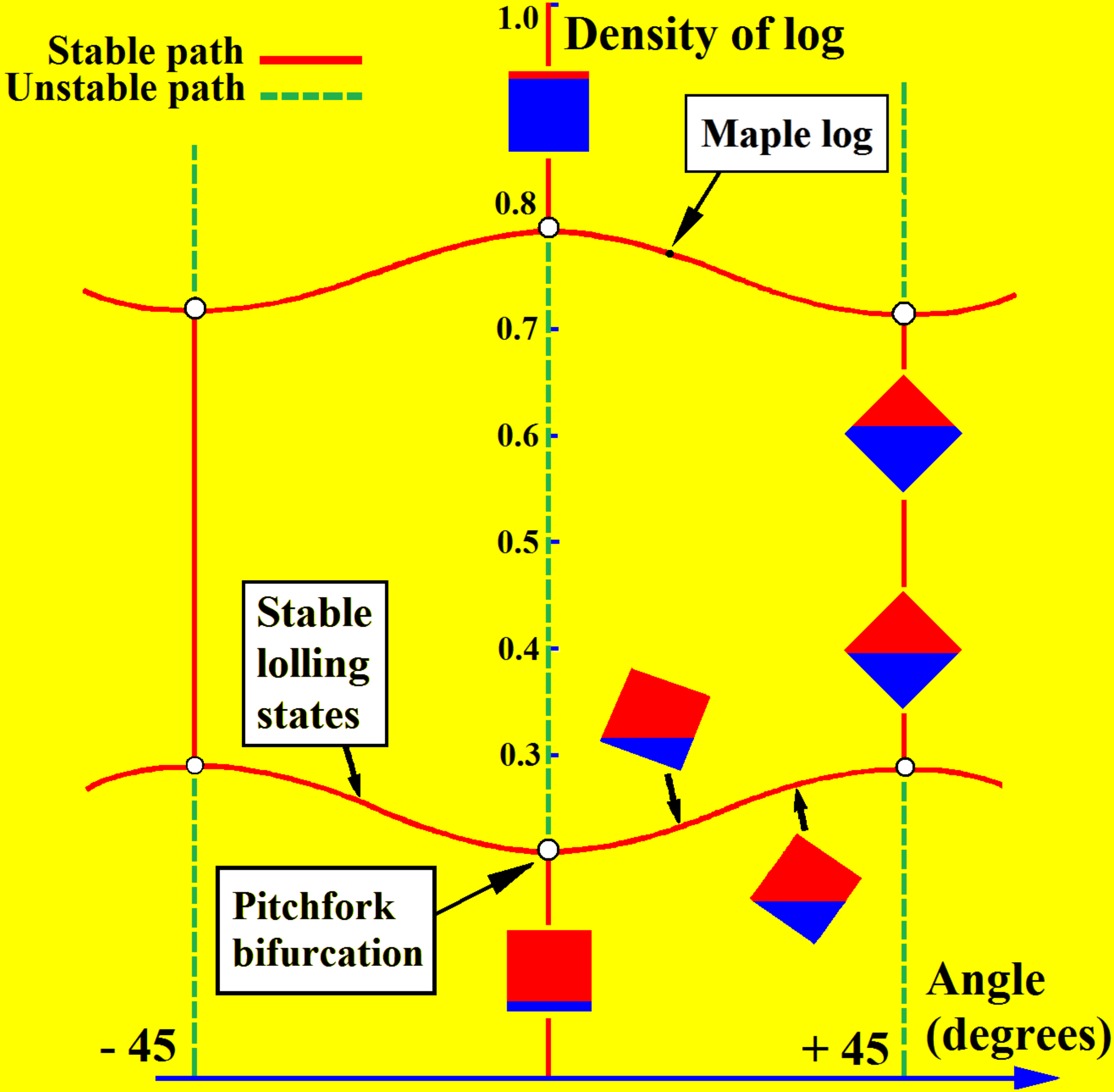
Challenge Question

What would be the floating configuration of a maple-wood log of density 0.78

c015b

The answer!





Stable path
Unstable path
1.0
Density of log
Maple log
0.8
0.7
0.6
0.5
0.4
0.3
Stable lolling states
Pitchfork bifurcation
Angle (degrees)
- 45
+ 45

Population Growth

In ecology we use discrete time for **yearly steps** between breeding seasons.

The more mayflies in a pond, the more offspring we expect next year.

A population that increases by a **fixed ratio** each year will explode!

Applied to humans this result alarmed **Thomas Malthus**.

His *Principle of Population* (1798) influenced Darwin's thoughts on natural selection.

Simple Growth with an Abundance of Food

A discrete dynamical system (called a map)

Suppose a population, *x*, increases by a ratio *a* each year
The rule (map) is then

$$\text{New } x = a\, x$$

Let us start with x = 12 (million, say) and take a = 2
The population in successive years is then

12 ... 24 ... 48 ... 96 ...

It increases exponentially to infinity!

Had we chosen a value of *a* = 0.5 (less than one) the population decreases towards zero as 12 ... 6 ... 3 ...

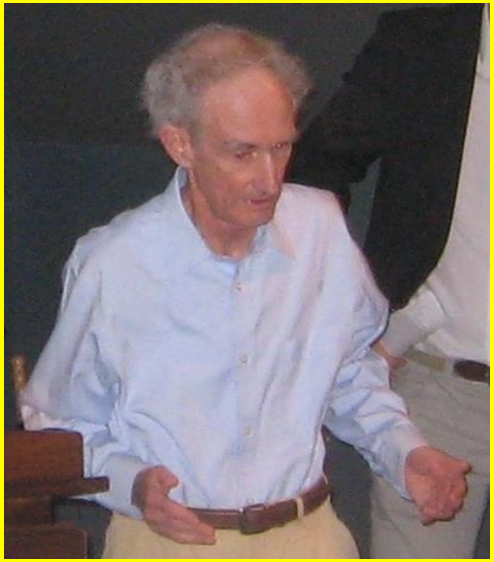
Bacterial growth under microscope [m36 BacGrow]

Chaos in Logistic Map

New $x = a\,x\,(1 - x)$

x is fraction of the maximum population

$$x_{n+1} = r x_n (1 - x_n)$$



An improved model of population growth.

The (1 - x) recognises a constraint of limited food.

One-time President of the Royal Society, Lord Robert May, published a paper in *Nature* (1976).

This showed sensitive to initial conditions ...

THE BUTTERFLY EFFECT !!!

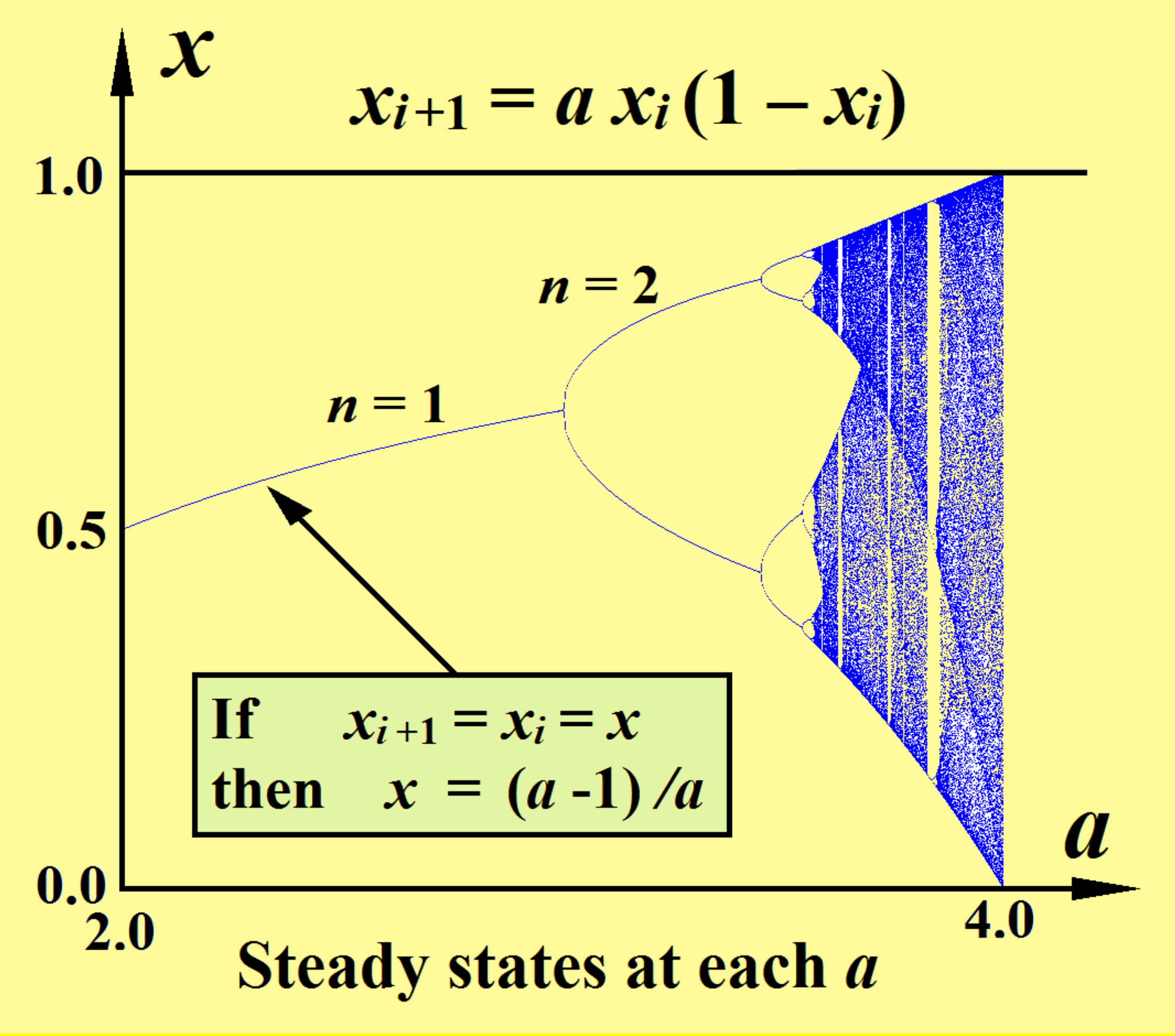
Cascade to Chaos in Log. Map s09 log bif s08 log 45

Steady states plotted, *x*(*a*)

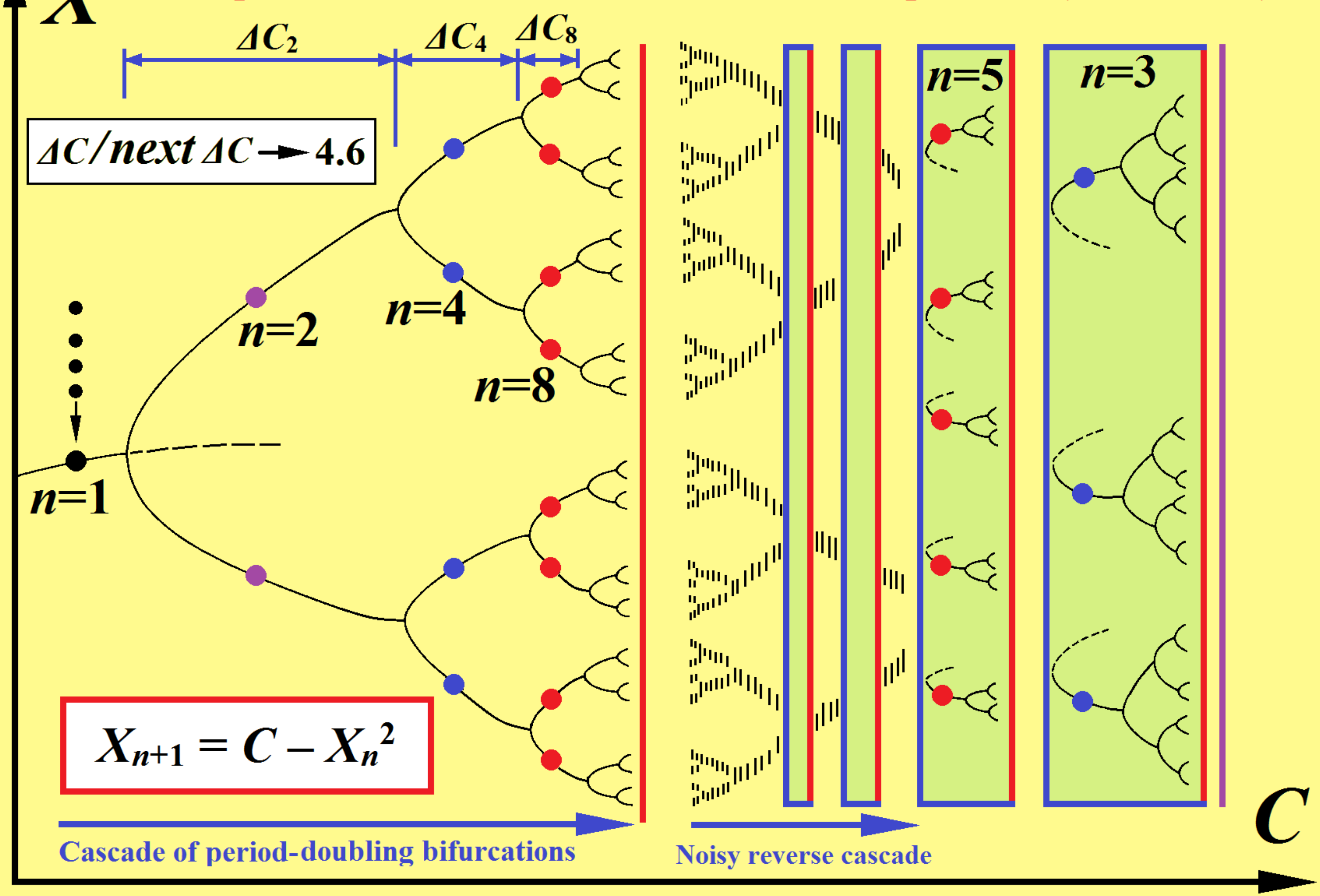
Feigenbaum cascade leads to chaos

Cascades in cascades

Patterns shrink indefinitely



Feigenbaum Scenario for Logistic (variant)



$\Delta C_k / \Delta C_{2k}$ tends to the universal number 4.66920 … as k tends to infinity

Infinite number of periodic windows | Accumulation | Crisis at $C = 2$

Moral of the Logistic Map

Simple systems can have very complex behaviour.

This should be taught in schools!!

Unfortunately text books concentrate on solvable problems, usually linear (small amplitude) ones

Why did it take 300 years from Newton to chaos?

(1) There were no computers or video displays

(2) Researchers were looking for order

(3) Random results were thought to be wrong:

so they ended up in the waste paper basket

Lorenz's Butterfly

The flap of a butterfly's wings in Brazil can set off a tornado in Texas

This is a parable about sensitive dependence on initial conditions

A tiny difference is amplified until two outcomes are totally different

Due to inevitable chaos, long term weather forecasting is impossible

m38 twister

Chaos, Predictability and the Weather

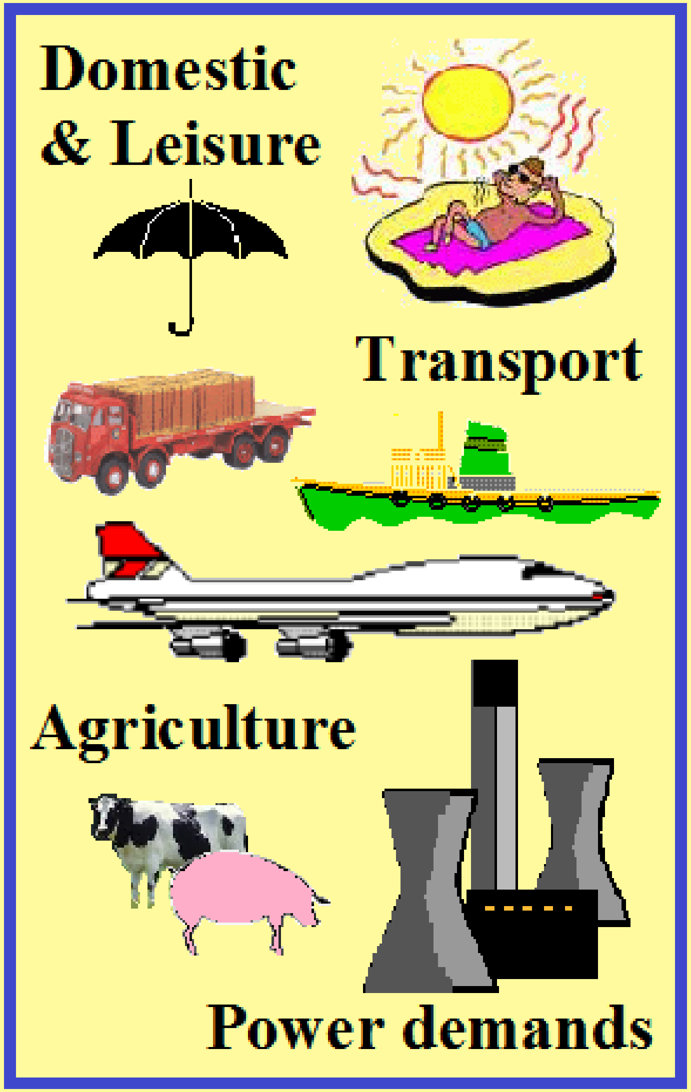
The Met office computer tries to model the atmosphere using the laws of physics.

Earth's atmosphere appears to be a chaotic system.

So to make predictions the current state must be precisely known.

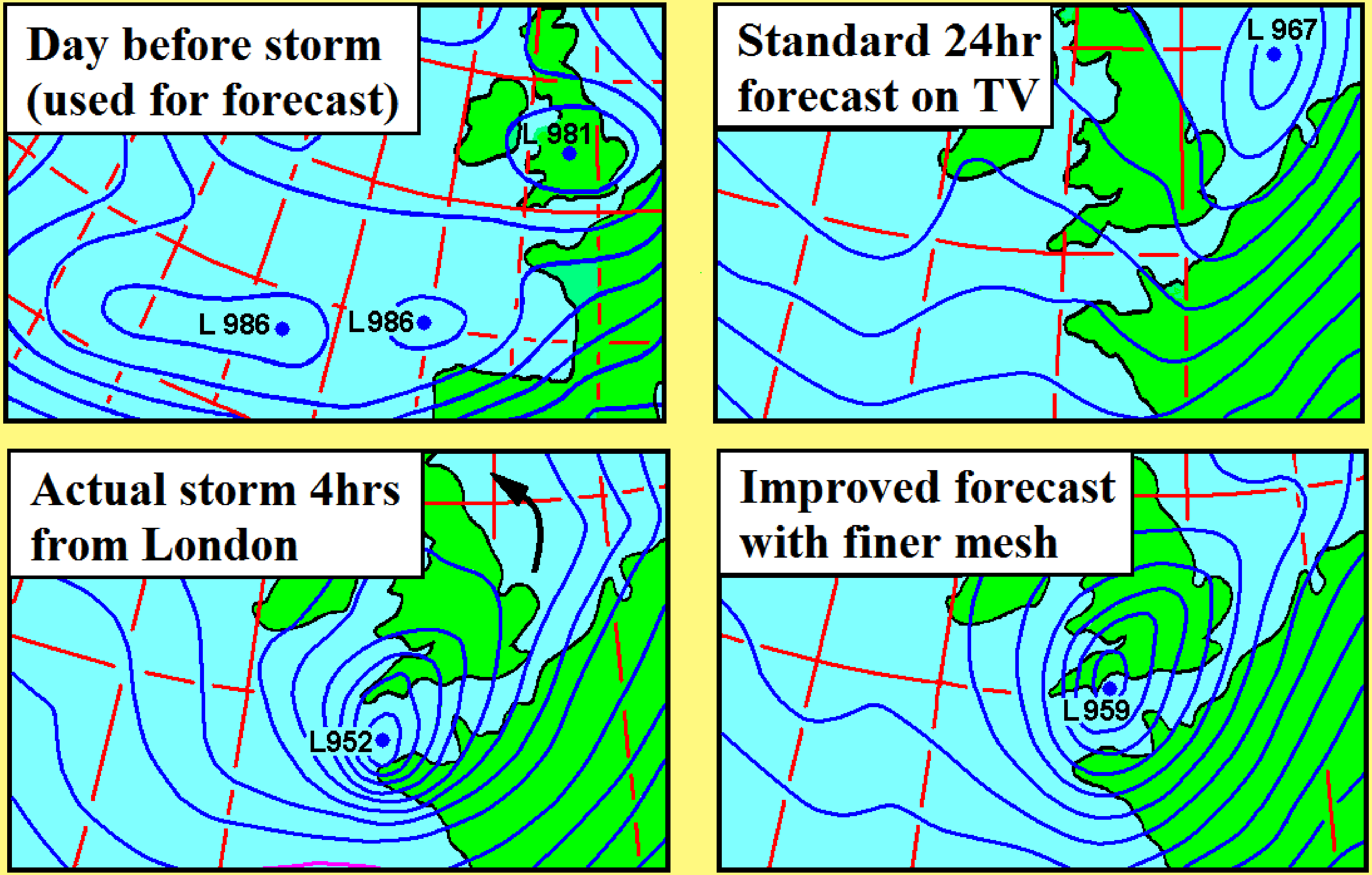
And the model must be perfect.
Neither of these are possible!

An effective predictability barrier for the atmosphere is at about 14 days.



Forecasts are estimated to save the UK £1 billion annually

Met Office now uses multiple runs from different starts



Michael Fish (1987): the storm which destroyed 15 million trees. Cyclone was predicted after the event.

Parable of Chaos

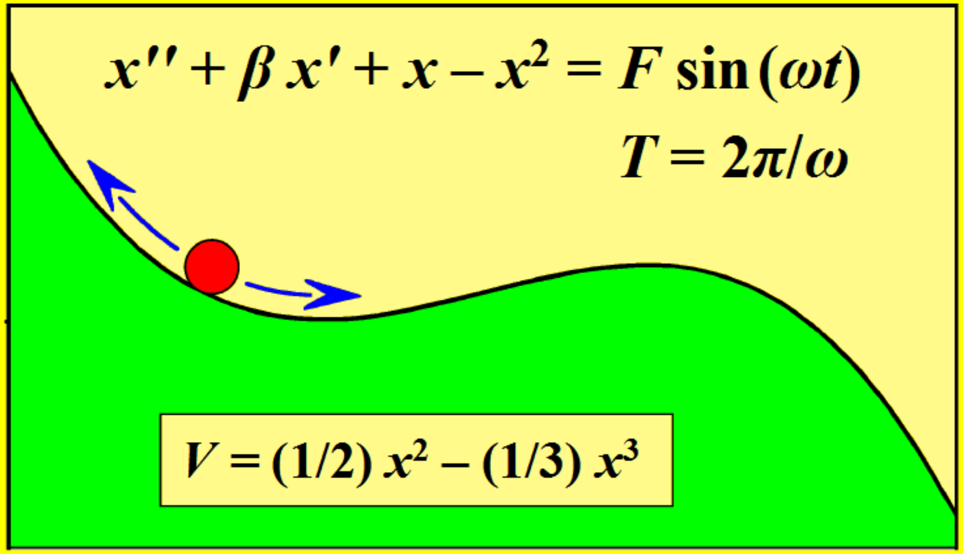
For want of a nail
the shoe was lost.
For want of a shoe
the horse was lost.
For want of a horse
the rider was lost.
For want of a rider
the battle was lost.
For want of a battle
the kingdom was lost.
And all for the want
of a horseshoe nail.
(Proverb, 14th Century)

Proverb from the 14th century

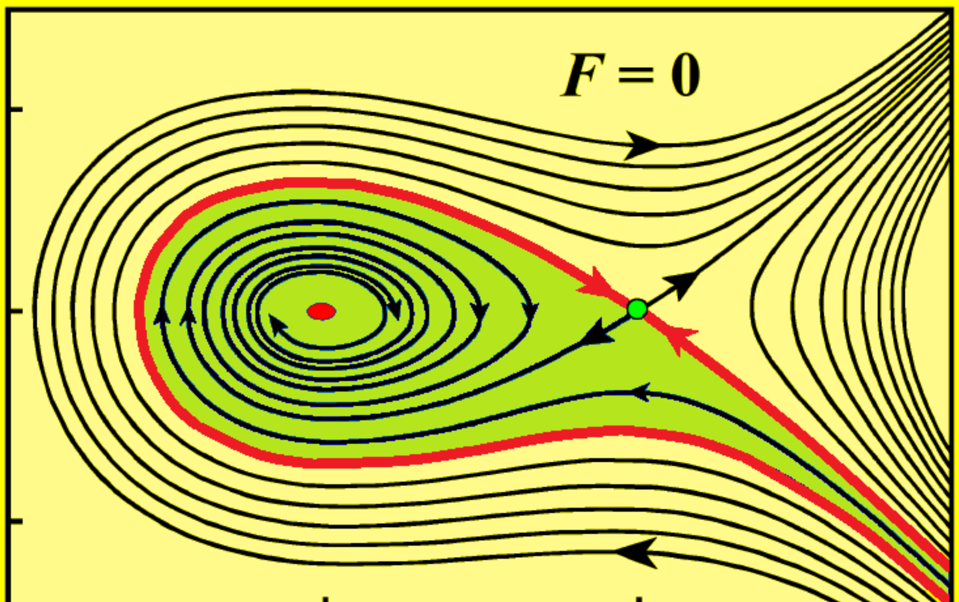
Capsize in beam seas

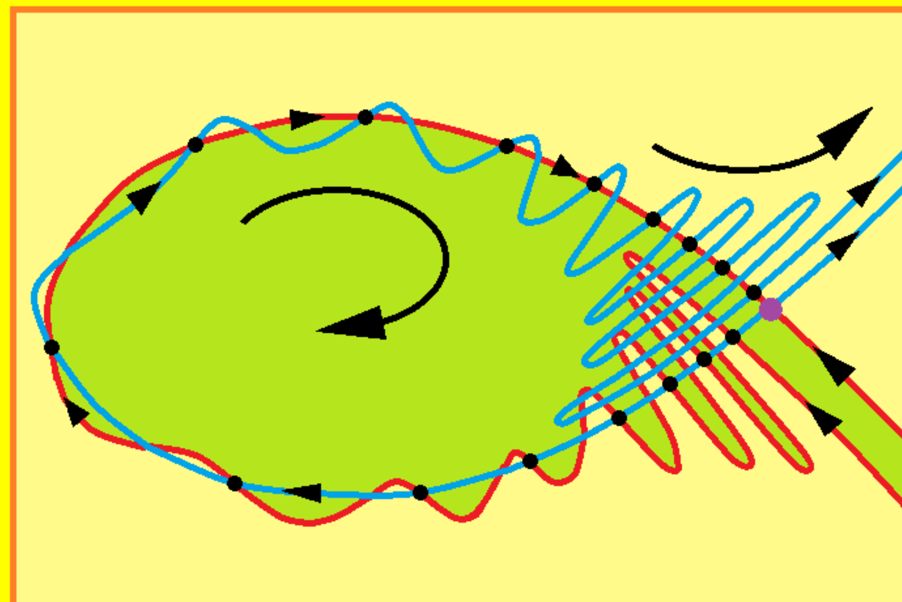
m02 frig

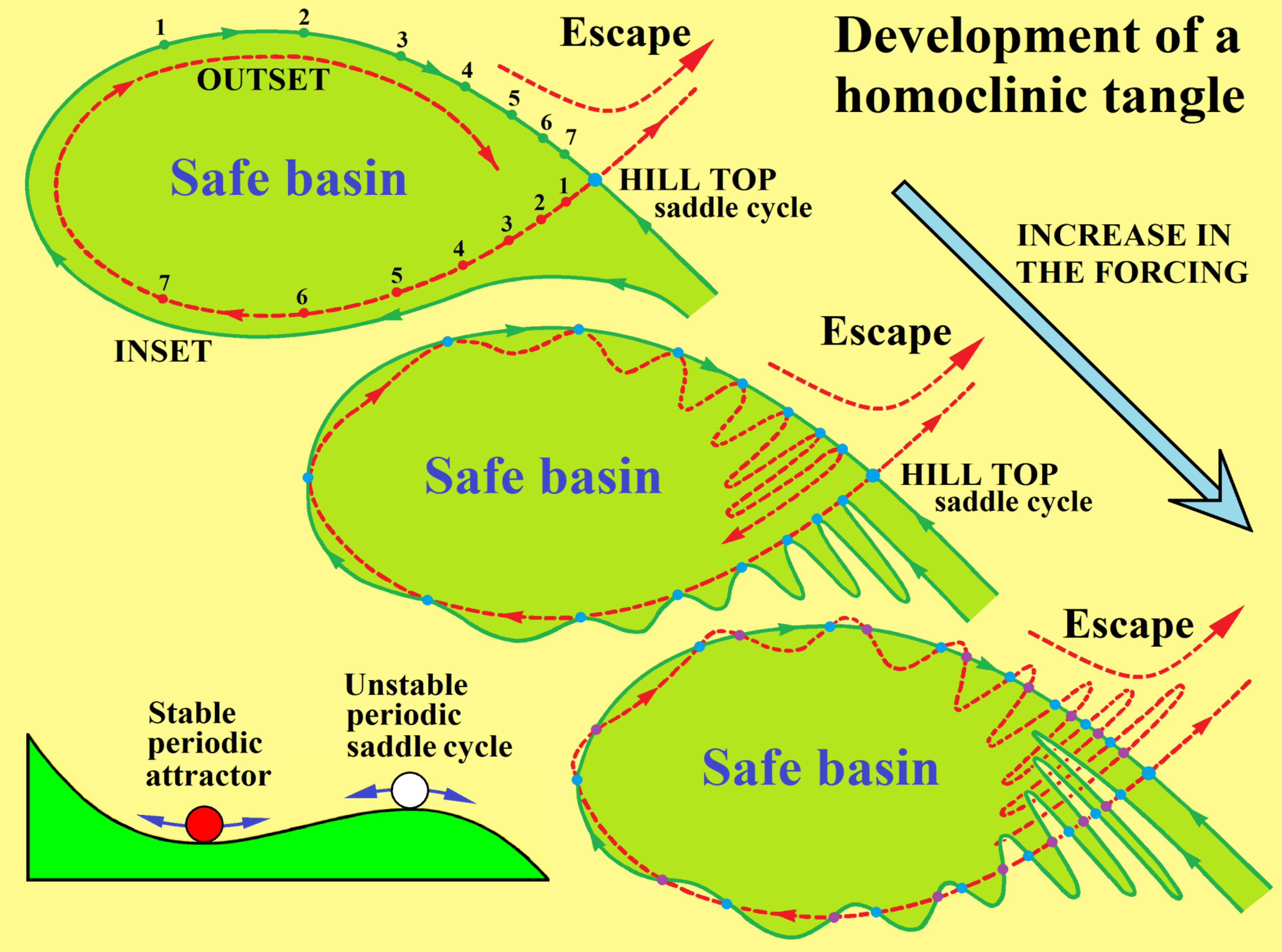
Poincaré despaired on realizing that the 3-body problem contained a 'tangle'



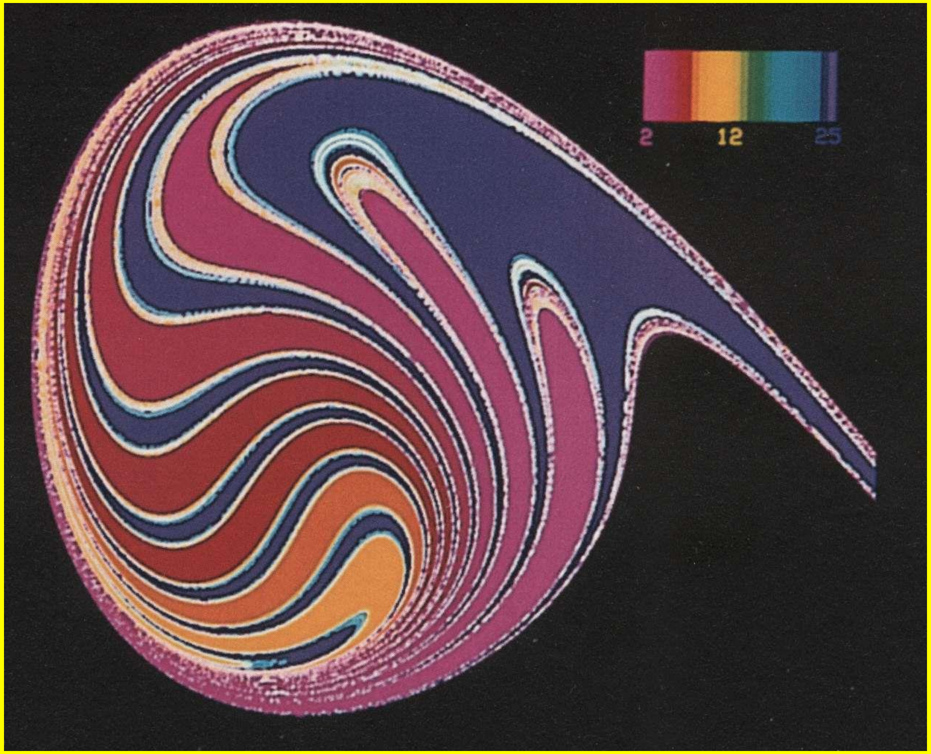
When $F \neq 0$ phase-space is 3D, and we need a Poincaré section (stroboscopic sampling). This gives a dot-map and when $F > F_C$ the inset and outset intersect an infinite number of times. Near the **homoclinic tangle** will be chaos and infinitely many periodic orbits.



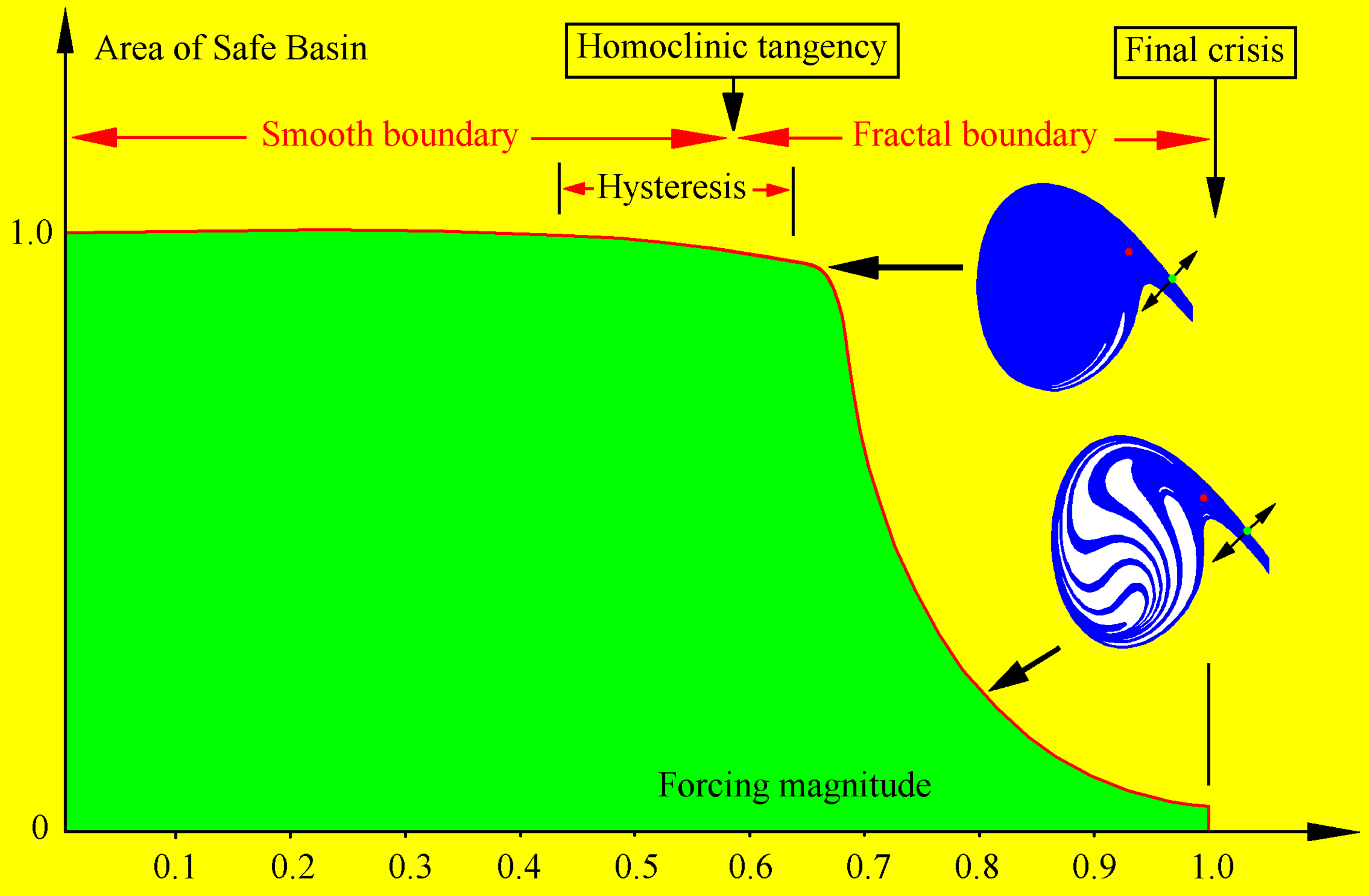



Development of a homoclinic tangle
Escape
OUTSET
Safe basin
HILL TOP
saddle cycle
INSET
1
2
3
4
5
6
7
INCREASE IN THE FORCING
Escape
Safe basin
HILL TOP
saddle cycle
Escape
Safe basin
Stable periodic attractor
Unstable periodic saddle cycle

Fractal Basin Erosion



- As the driving increases, fractal fingers created by homoclinic tangling make a sudden incursion into the safe basin: integrity is lost
- Colours show escape time, measured in driving periods
- Simulation (made by Prof Joseph Cusumano)
- m13 cusu



Area of Safe Basin
Homoclinic tangency
Final crisis
Smooth boundary
Fractal boundary
Hysteresis
1.0
0
Forcing magnitude
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Transient Capsize Test

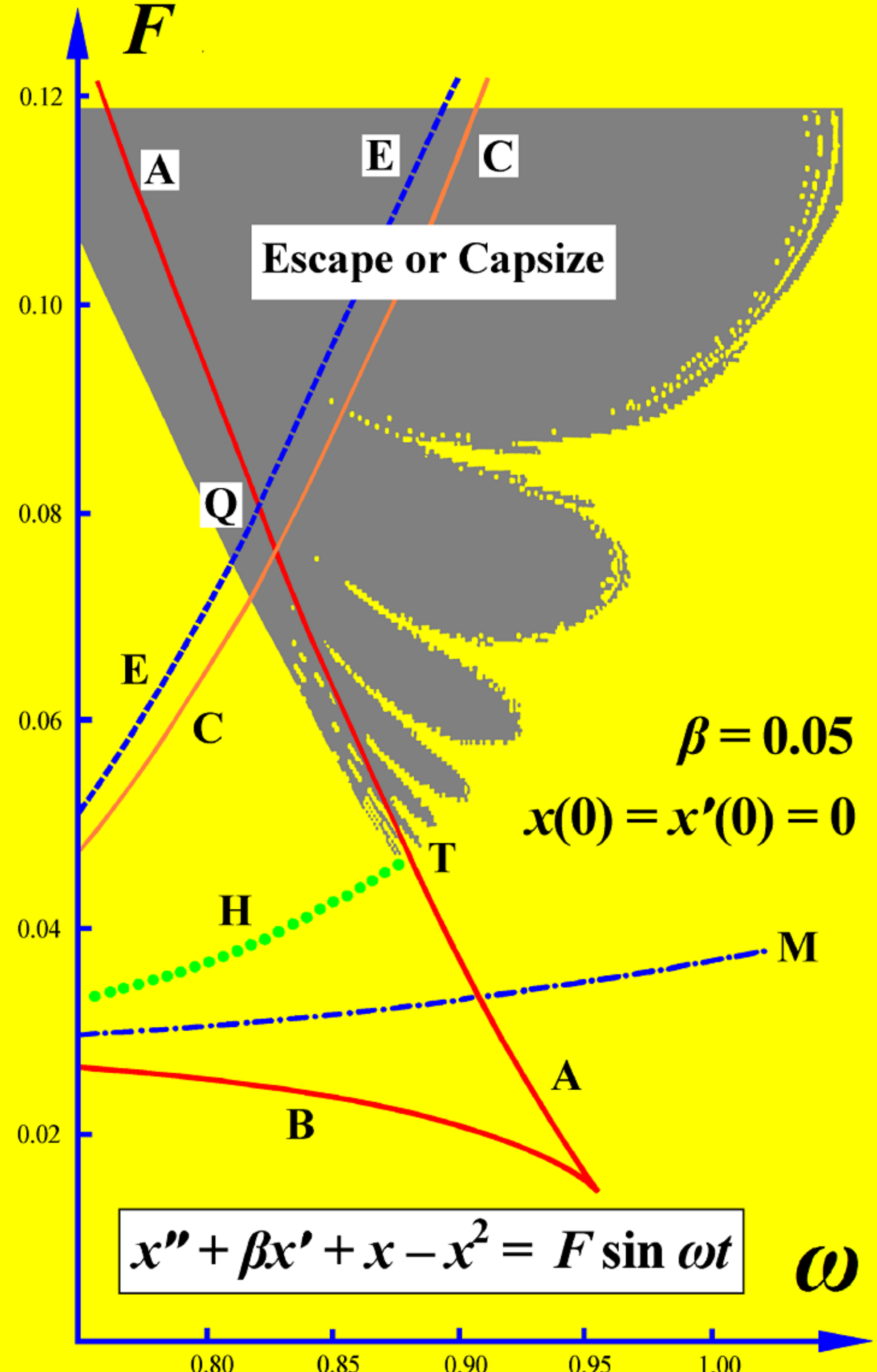
Test a resting model by suddenly switching on the waves.

This is an economical way of assessing its capsizability.

This can generate a diagram as opposite where:

F represents wave height

ω is the wave frequency



Next lecture ... improve your sport!