Lecture 3 Free space flight, Driven Oscillators and Fractals

NASA space superhighway

Heteroclinic connections between orbits

Stroboscopic Poincaré section

The discovery of chaos by Ueda

Cantor set and Mandelbrot set

Coastline of Britain and Koch island

Use of fractals in cell phones

Zero fuel superhighways discovered and used by NASA with chaos theory. These natural chaotic trajectories are easily deflected to alternative destinations.

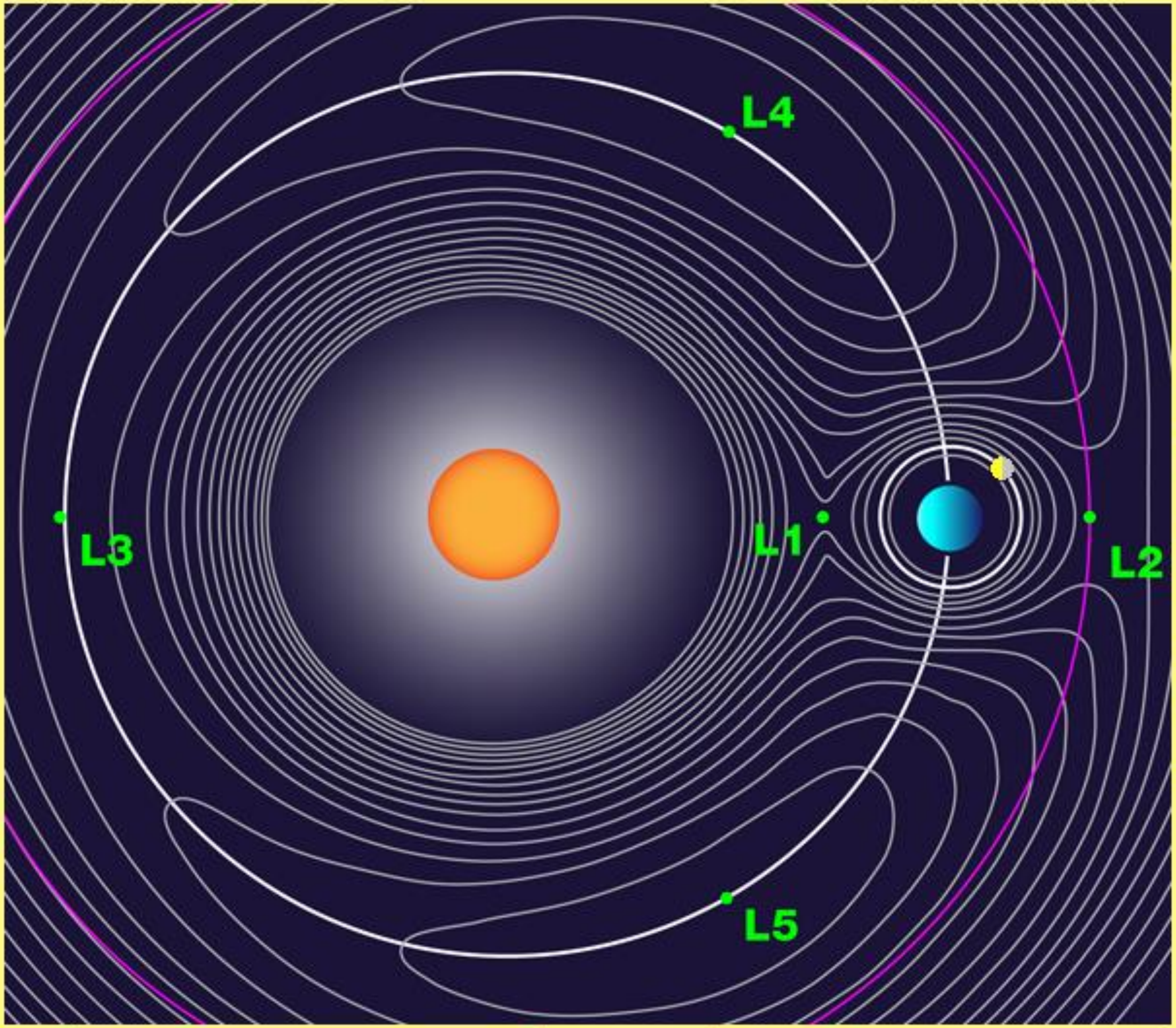
Parking points for spacecraft

Lagrange points for the Sun-Earth-Spacecraft (3 body) in rotating frame

Potential energy contours, including gravitational and centrifugal effects

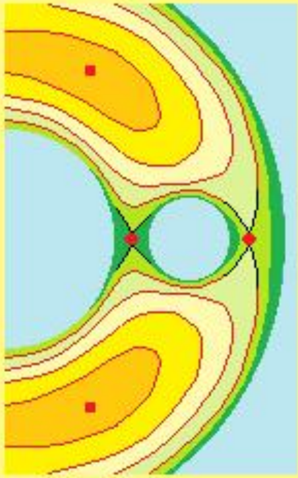
L1, L2, L3: unstable states at energy saddle points

L4, L5: energy hill-tops stabilized by coriolis forces



At L1

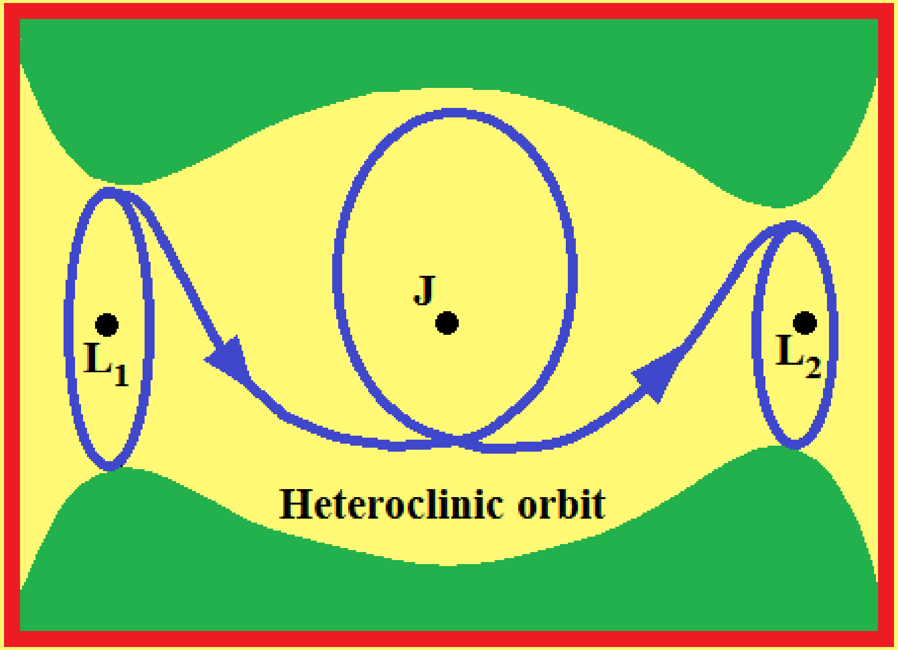
At L2

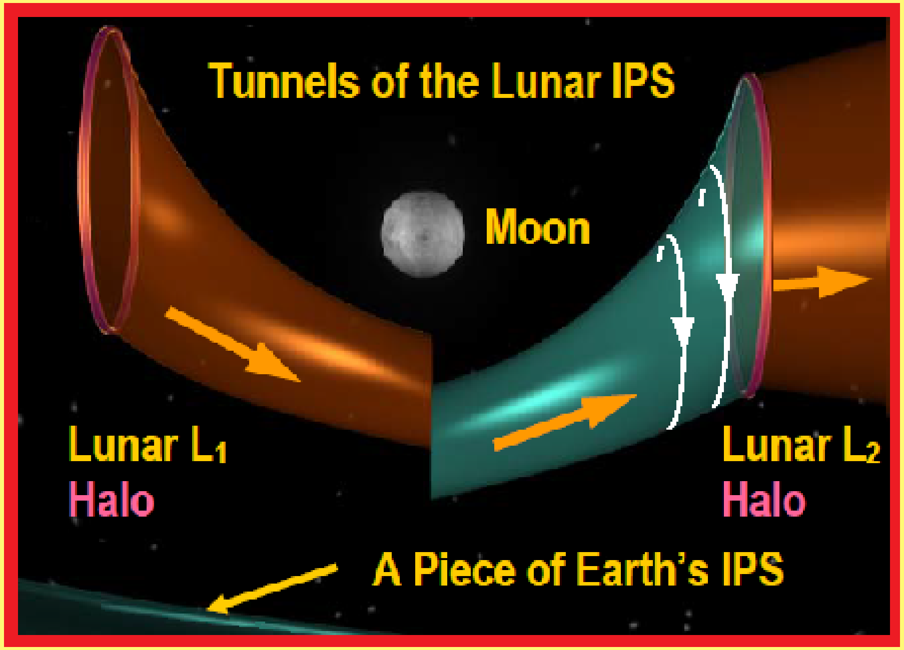


The Interplanetary Transport Network (ITN) IEEE 2002

1990 Japan (Hiten, moon)
2001 NASA (Genesis, solar)
2003 Europe (Smart 1, moon)
2010 China (Change 2, moon)

ITN of the Sun-Earth-Moon created by halo orbits around Lagrange points

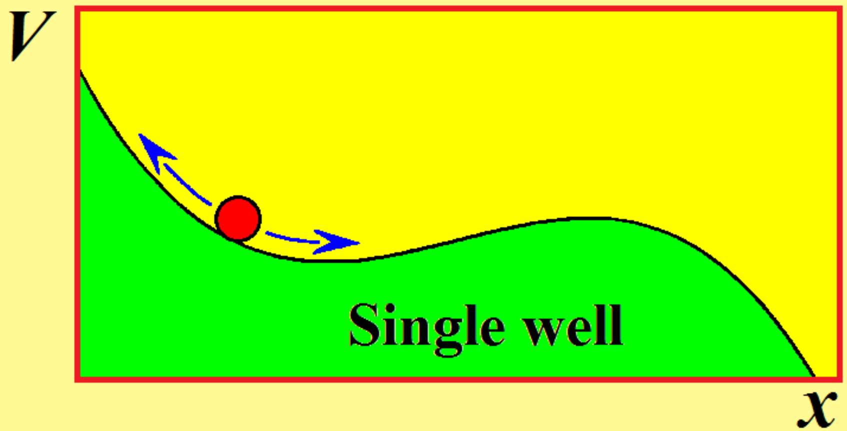


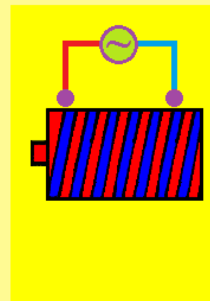


Using Chaos Theory, NASA discovered ITN, created and accessed by halo orbits around unstable Lagrange points of planets and satellites. Using ITN (like comets and asteroids) allows a spacecraft to explore the Solar System using minimal fuel.

The CHAOTIC PATHS are easily deflected to ALTERNATIVE DESTINATIONS.

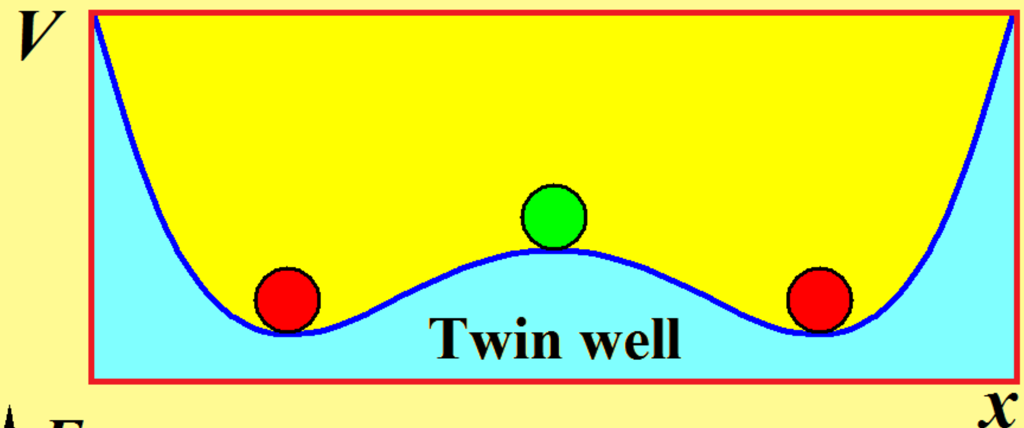
Periodically driven Oscillators

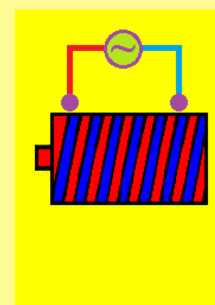




AC supply to electromagnet gives horizontal force on a ball of

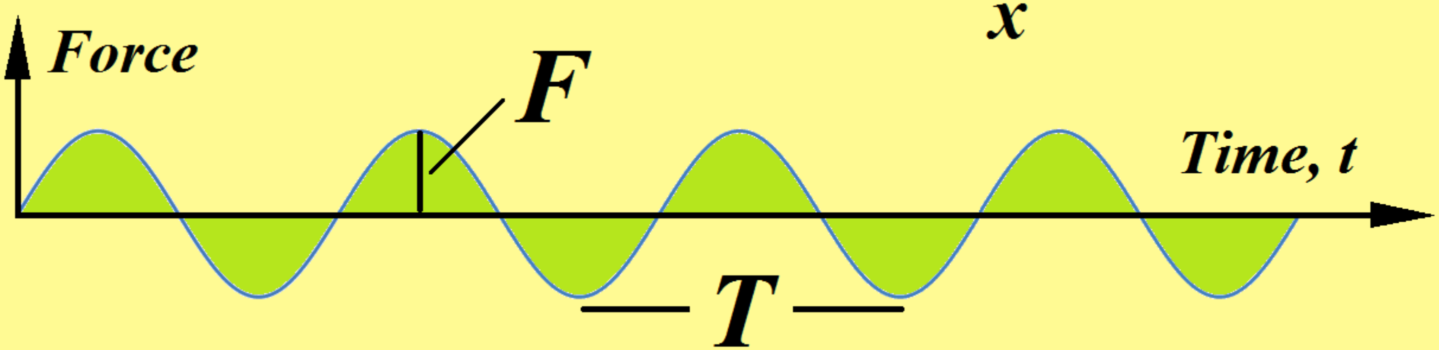
$$F \sin(\omega t)$$





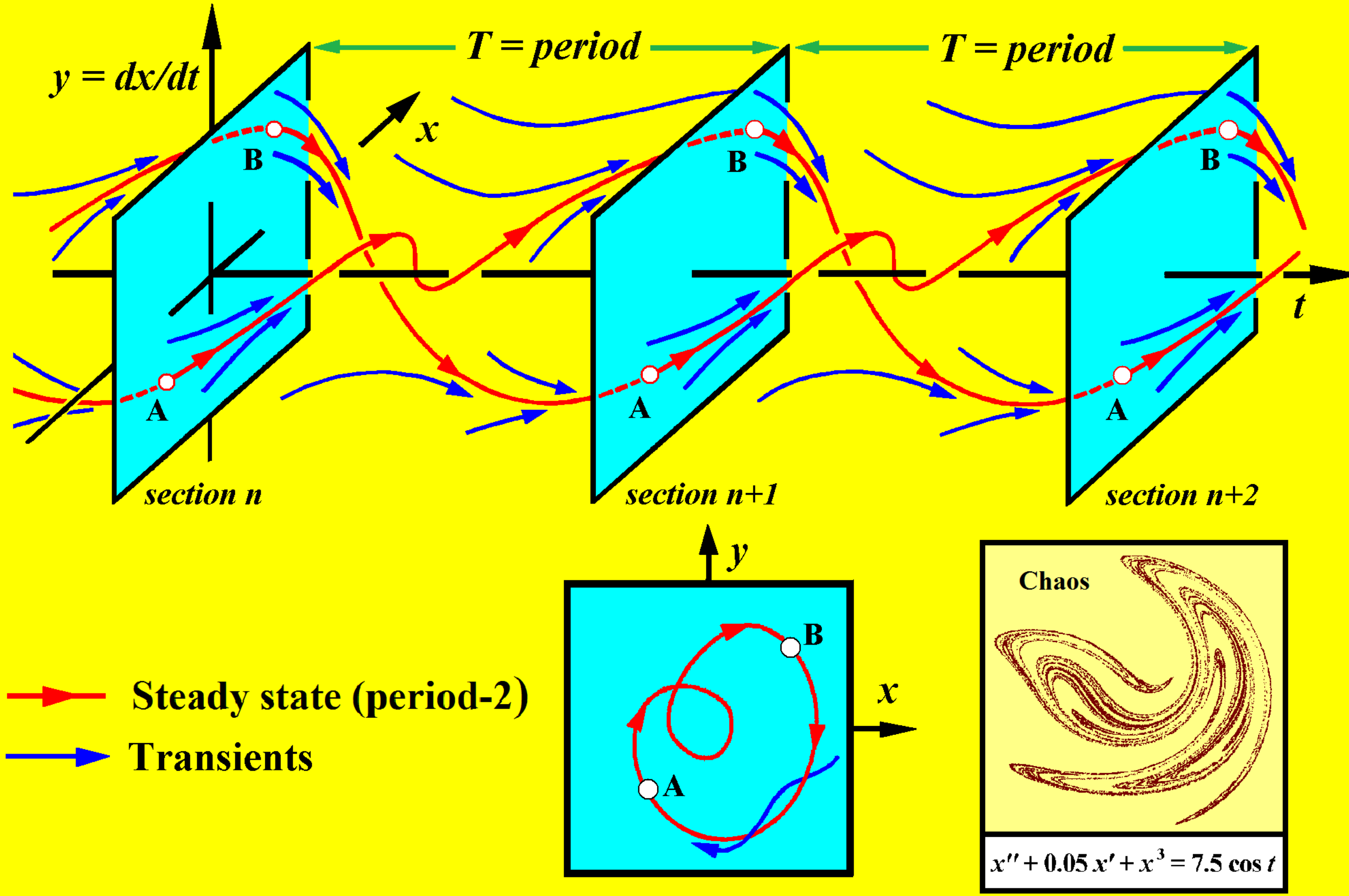
Periodic time of forcing is

$$T = 2\pi/\omega$$



m27 lv twin

Stroboscopic Poincare Sections for a periodically driven oscillator



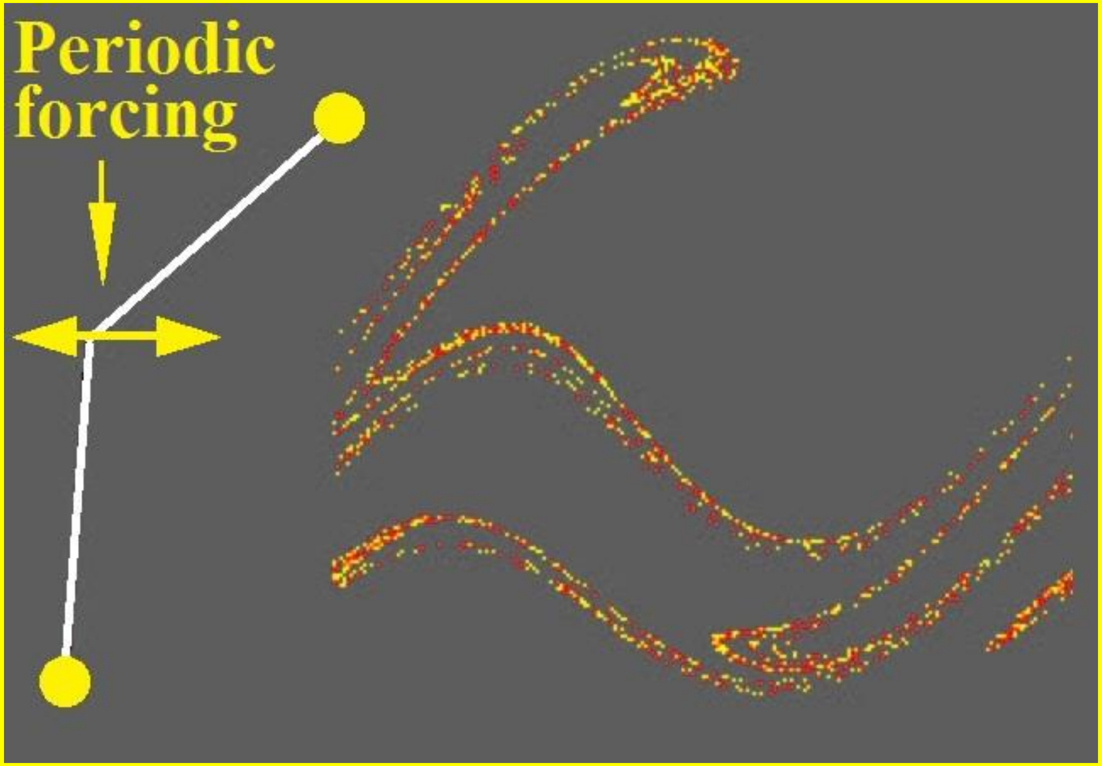
Driven Pendulum: Chaos

Chaotic tumbling of a damped and driven pendulum

In a stroboscopic section a strange attractor appears

If the pendulum has a slightly different start its motion will diverge.

But overall its section remains the same



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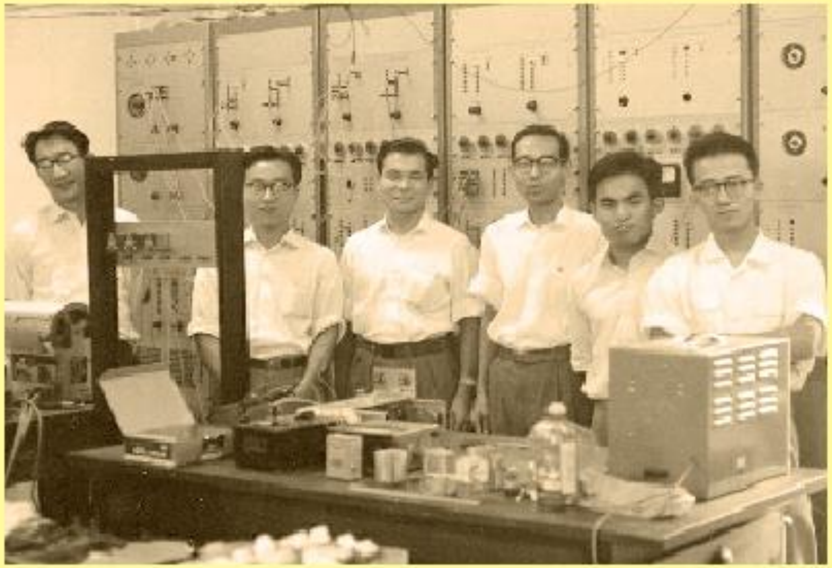
ORDER in CHAOS

Research students of Prof Hayashi using his world-beating analogue computer in Kyoto in the 1960's

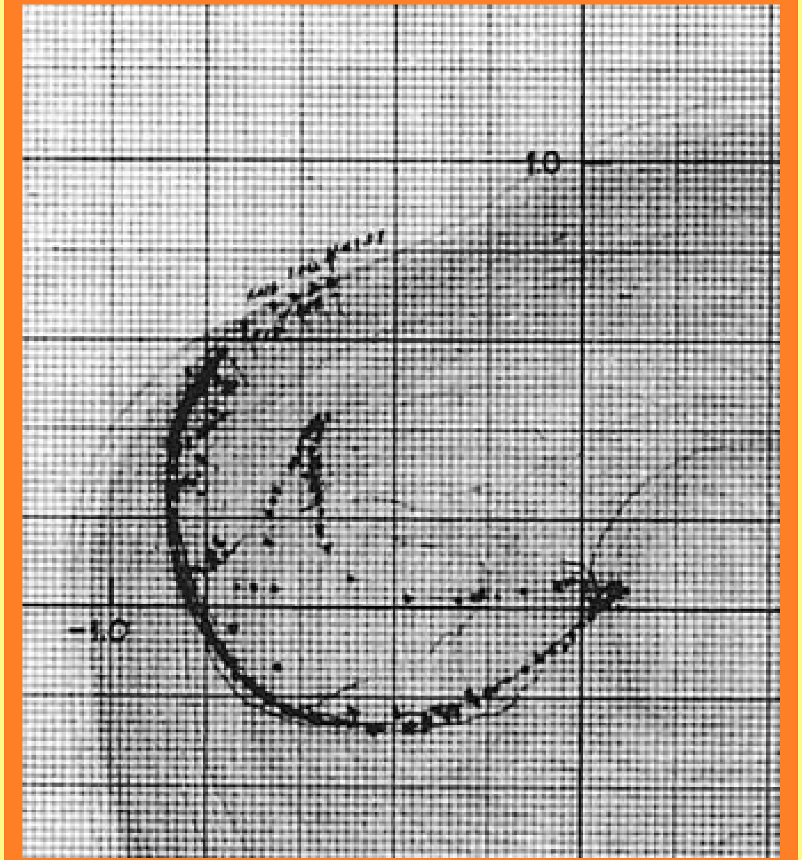


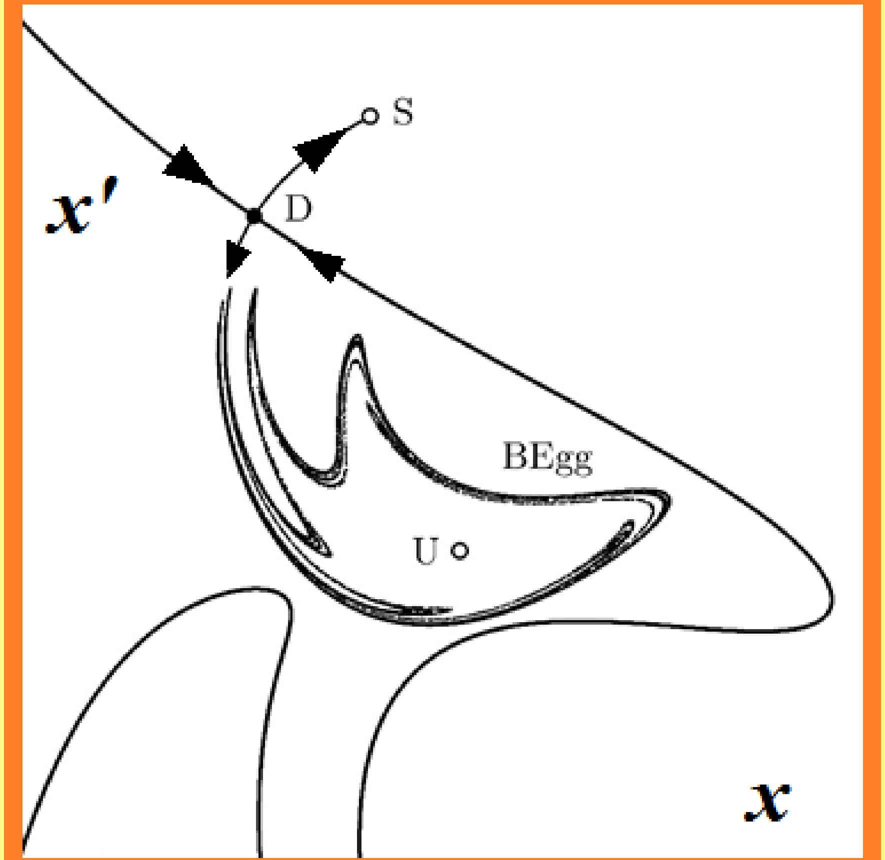
Yoshi Ueda, on the far right, detected what we now know as 'chaos' in 1961 (Lorenz paper, 1963) But his boss, Hayashi, insisted that he ignore it.

This discovery was not published until 1970.

Yoshi in San Francisco in 1990 with me and Bruce Stewart from Brookhaven Lab. We published widely as a 3-man research team.

Yoshisuke UEDA (27 Nov 1961) 1st glimpse of chaos : the 'Broken Egg Attractor'

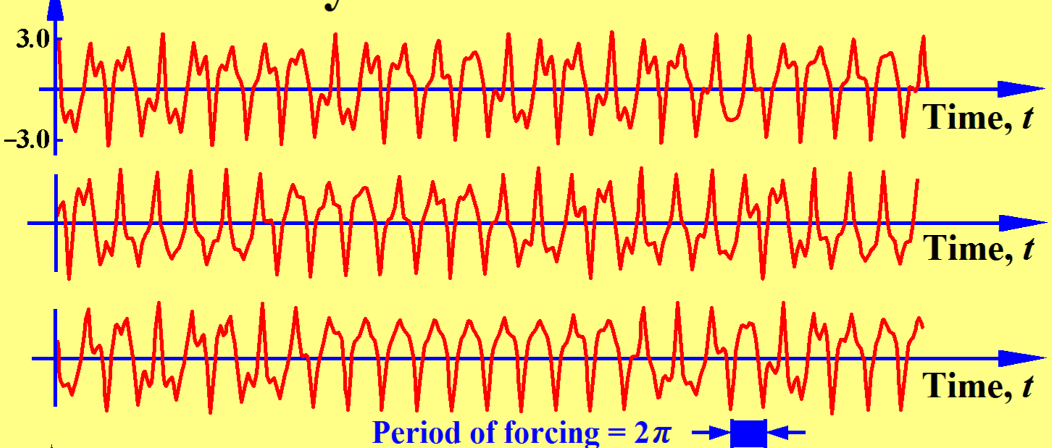




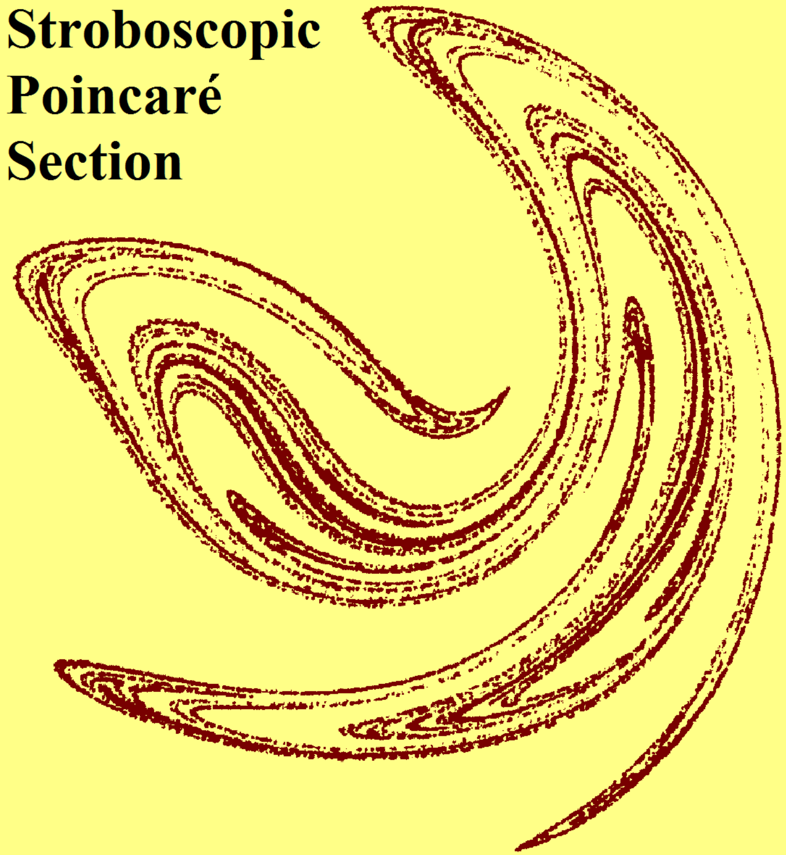
$$x'' - \mu\,(1 - \gamma\, x^2)\, x' + x^3 = B \cos \omega t$$

Ueda (1980)	$x'' + 0.05\,x' + x^3 = 7.5\cos t$

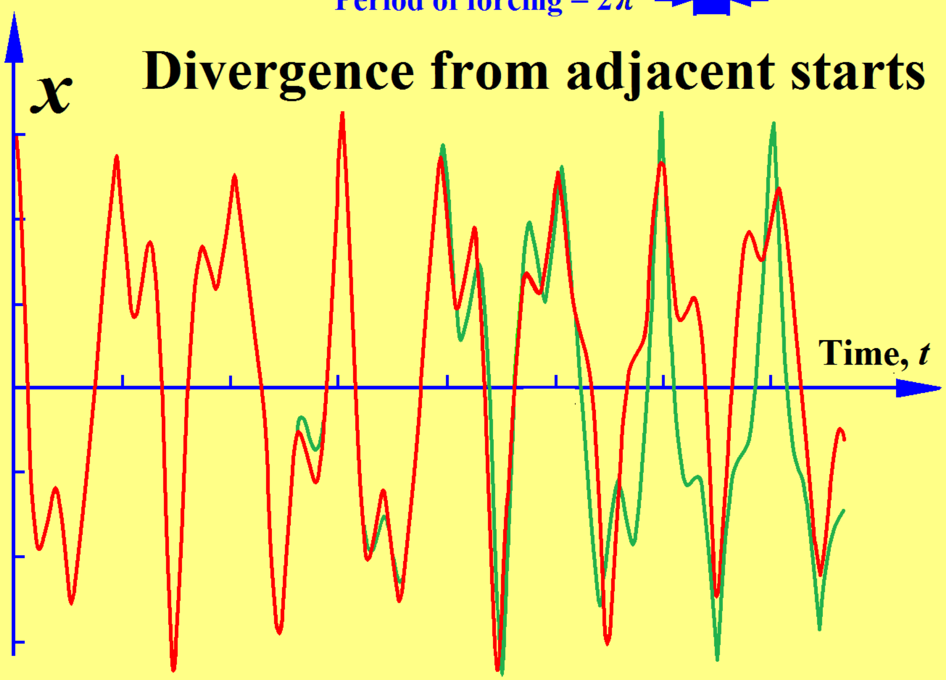
Steady state chaotic attractor



Stroboscopic Poincaré Section

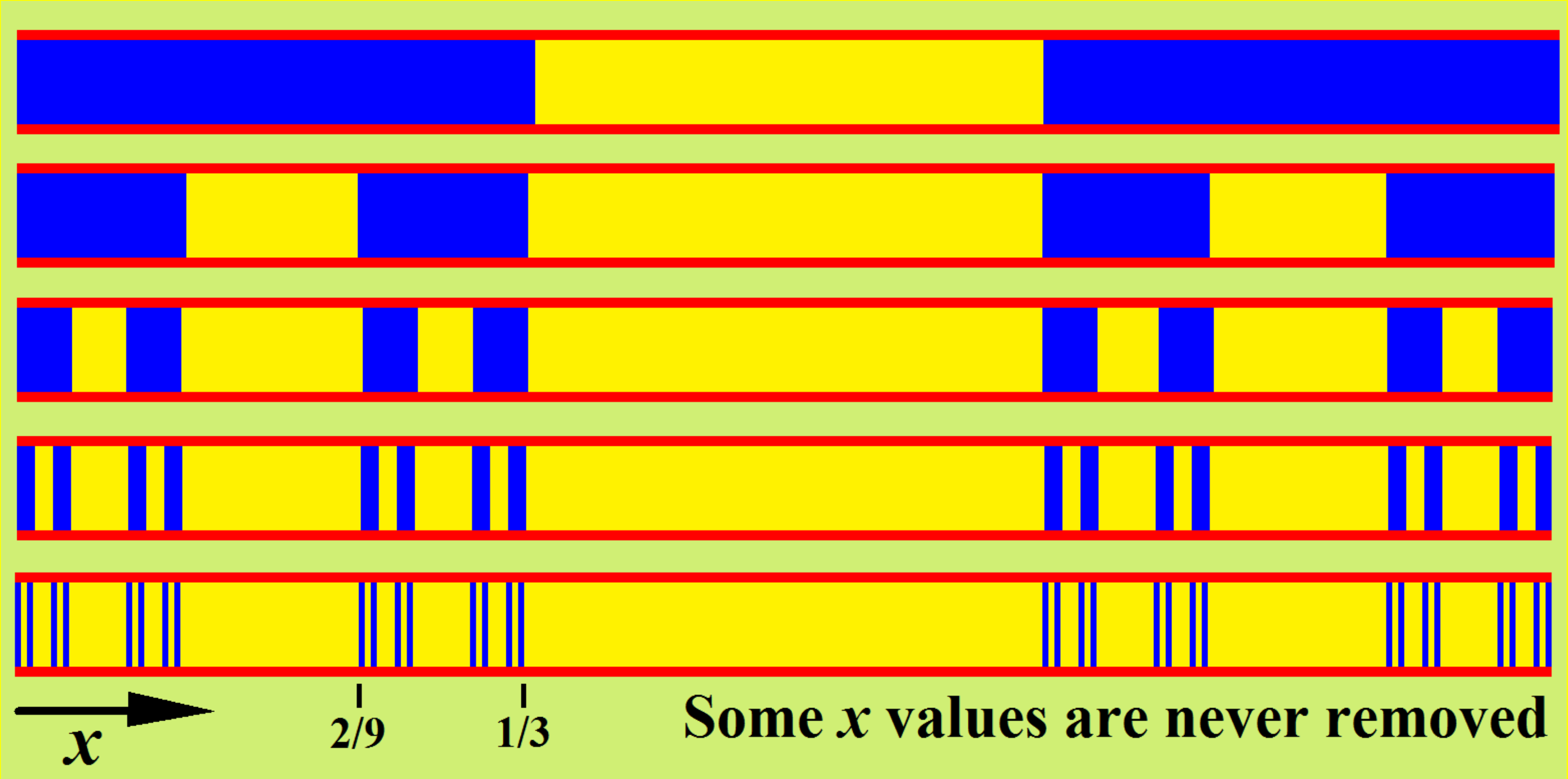


Divergence from adjacent starts



Transient to the attractor

Simple example of a Fractal: Cantor Set



Start with a line (0 to 1) and take out the middle-third

Then take out the middle-third of the remaining lines

Repeat this process for ever, to get the Cantor dust !!

Cantor Set is what is left after the infinite process (removed sets are 'open')

It has strange properties

Removed = $1/3 + 2(1/3)^2 + 4(1/3)^3 + \ldots = 1$

The whole length is removed, but an infinite number of points remain

A sheet has dimension D=2, a line has D=1, a point has D=0

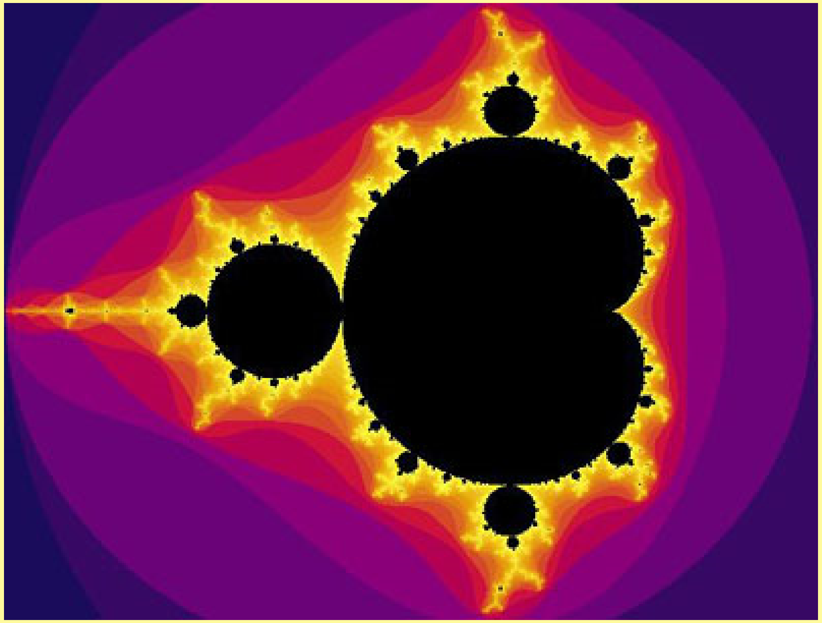
The Cantor set has the fractional value D=0.6309...   (points are awkwardly spaced)

Fractal Mandelbrot Set

New a = Old $(a^2 - b^2 + C)$

New b = Old $(2ab + D)$

C and D are held fixed during each calculation.



Set $C = 3$, $D = 4$ for now.

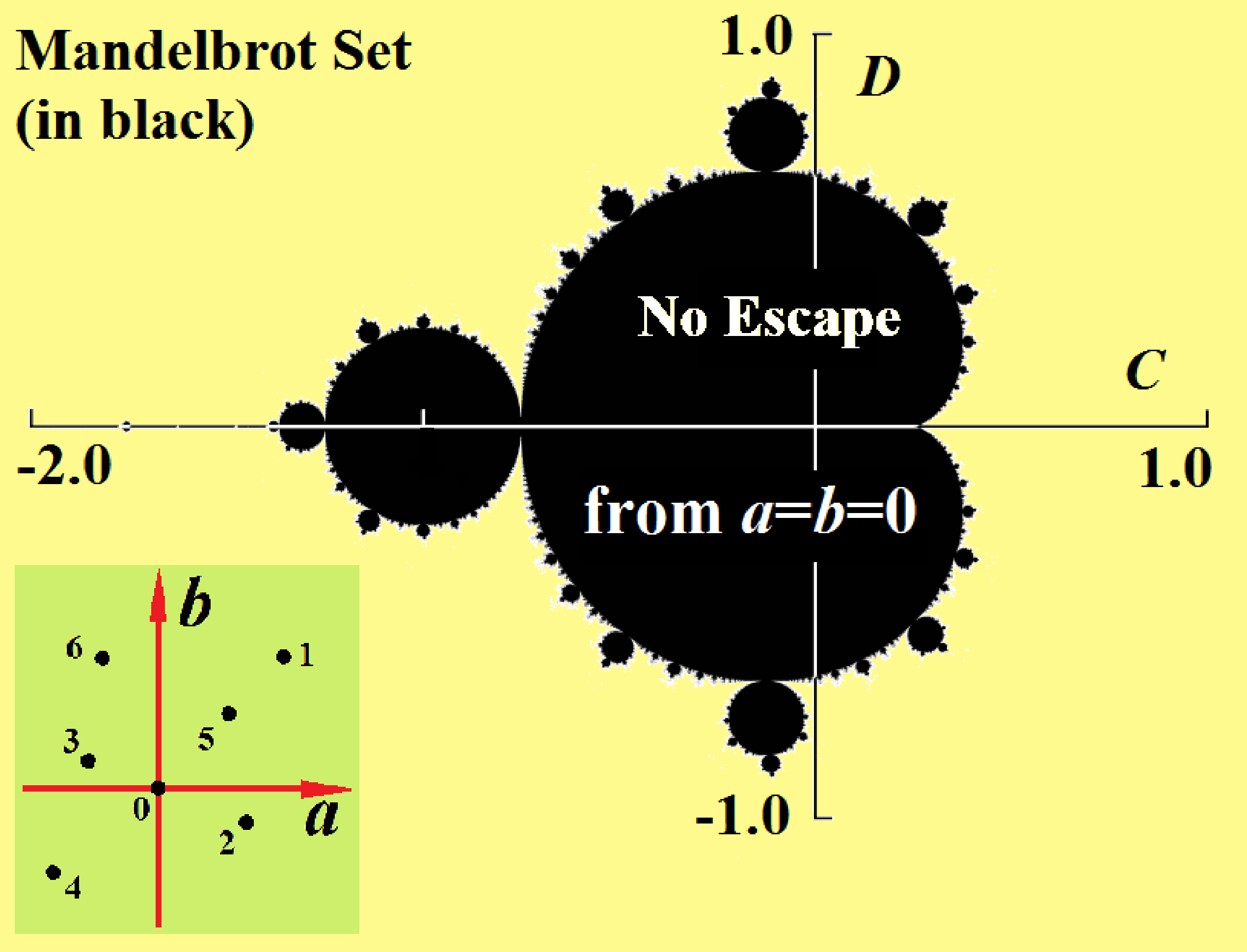
The set is determined by 'runs' from $a = 0$, $b = 0$

	a	b	Next a	Next b
Start	0	0	3	4
2nd point	3	4	9-16+3 = -4	24+4 = 28
3rd point	-4	28	-765	-220
4th point	-765	-220		

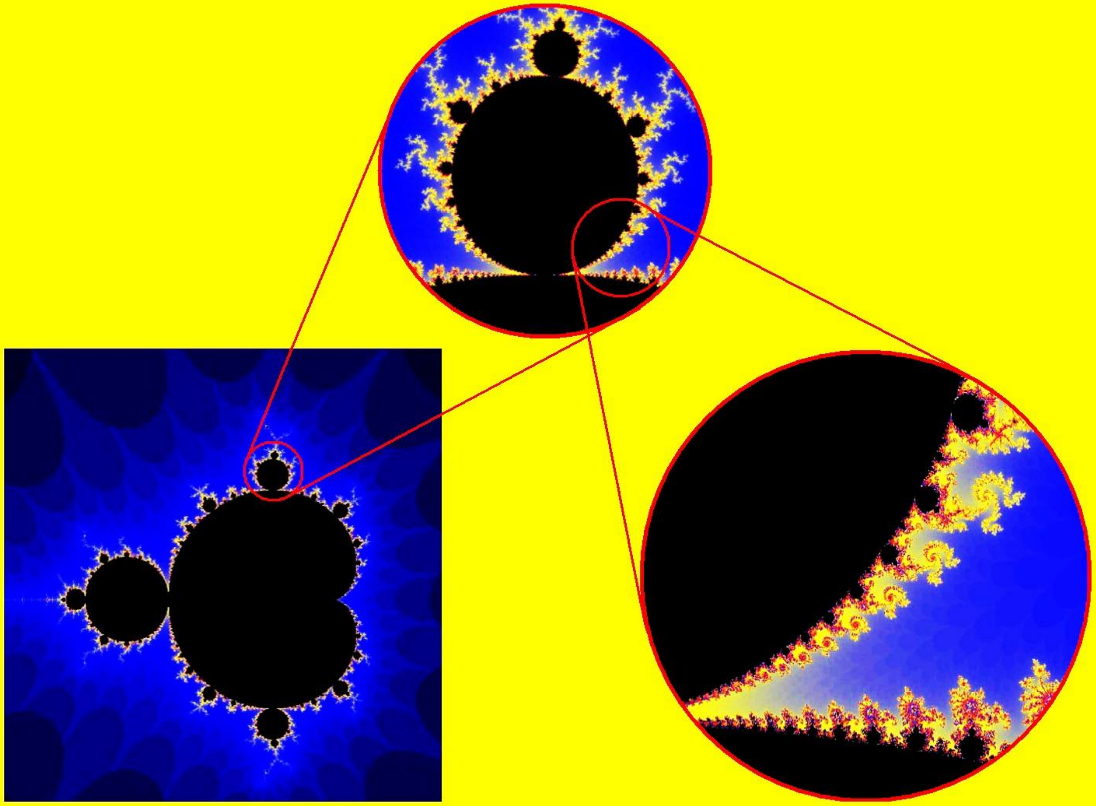
In complex numbers $f(z) = z^2 + c$ starting $z = 0$

Definition of the Mandelbrot Set

At some values of *C*, *D*, a run will diverge to infinity
At others, it will converge (to points, chaotic orbits, etc)
Values of *C*, *D*, giving convergence constitute the SET.



Successive magnification of the Mandelbrot Set



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Points outside the black SET are coloured according to the rate of divergence
Mathematically, the sequence of shrinking patterns never ends

Barnsley Fern Fractal

m21 fern

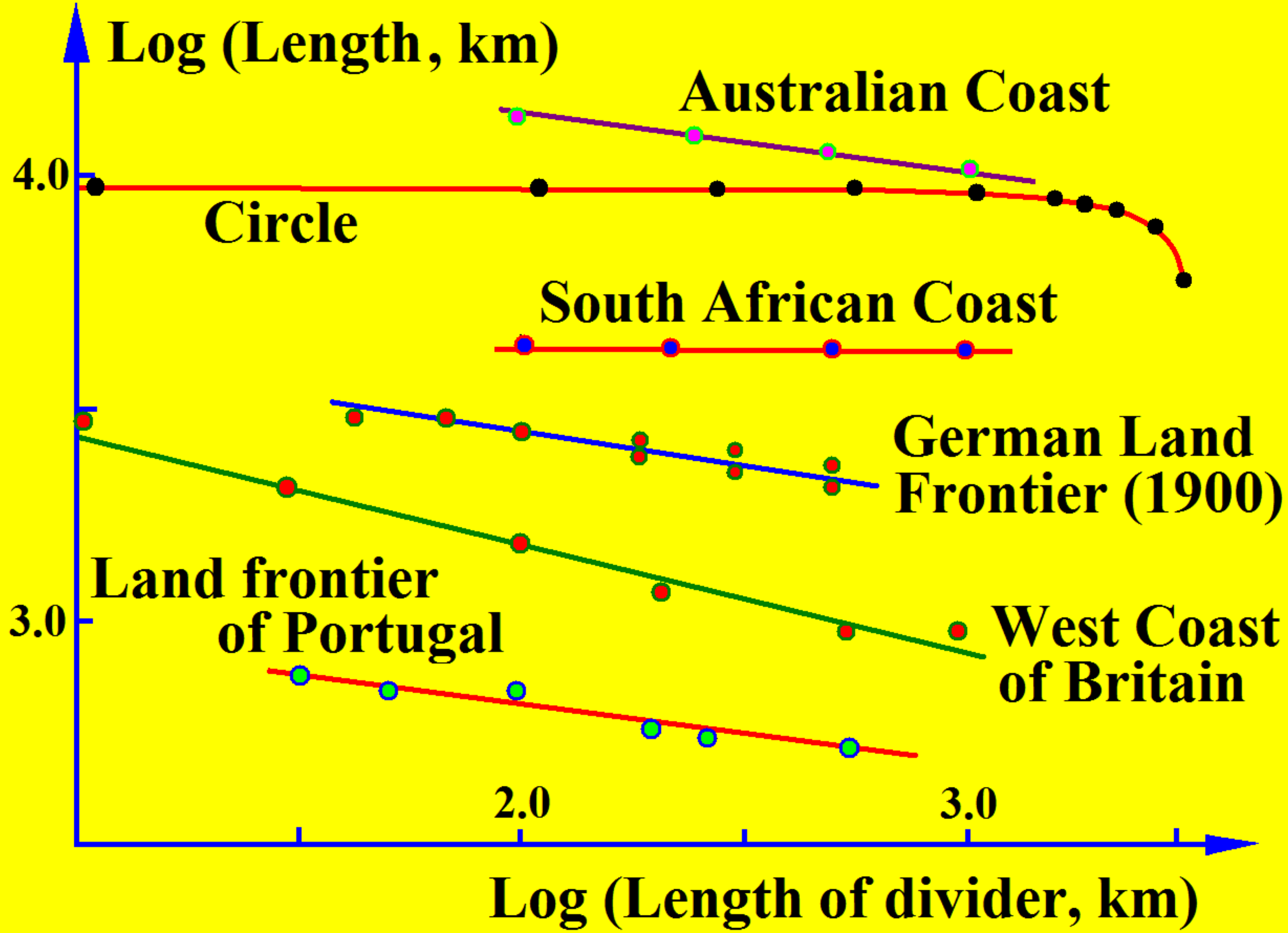
Fractal Coastline of Britain

The more detailed a map, the greater is the length estimate

On the coast itself, a string would wind around every puddle, stone, crack and molecule

A coast is effectively a fractal. Using a divider of length A, coastline tends to infinity as A goes to zero. This is not the case for a circle.

Empirical Data on Coastlines



A log-log plot of the estimated length, *L*, versus the length, *A*, of the divider

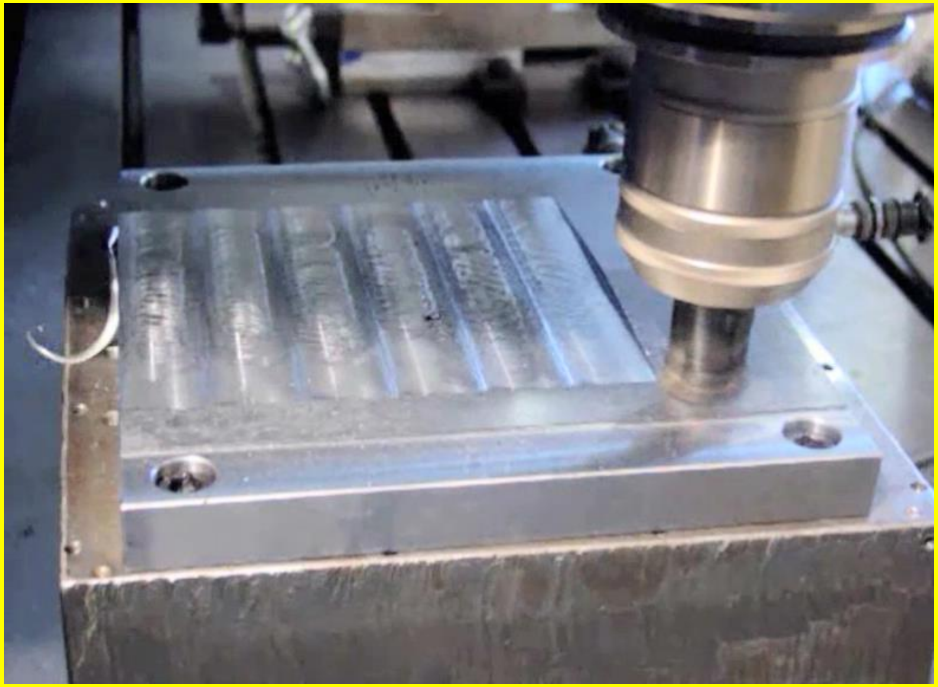
Fractal Dimension and its Applications

Strictly a fractal uses the limit as measurements tend to zero. So the real world only exhibits approximate fractals

There are ways of estimating a so-called fractal dimension, by (for example) covering with squares.

A coastline typically has a dimension of about 1.2

Fractals are needed for modelling jagged objects like machined surfaces



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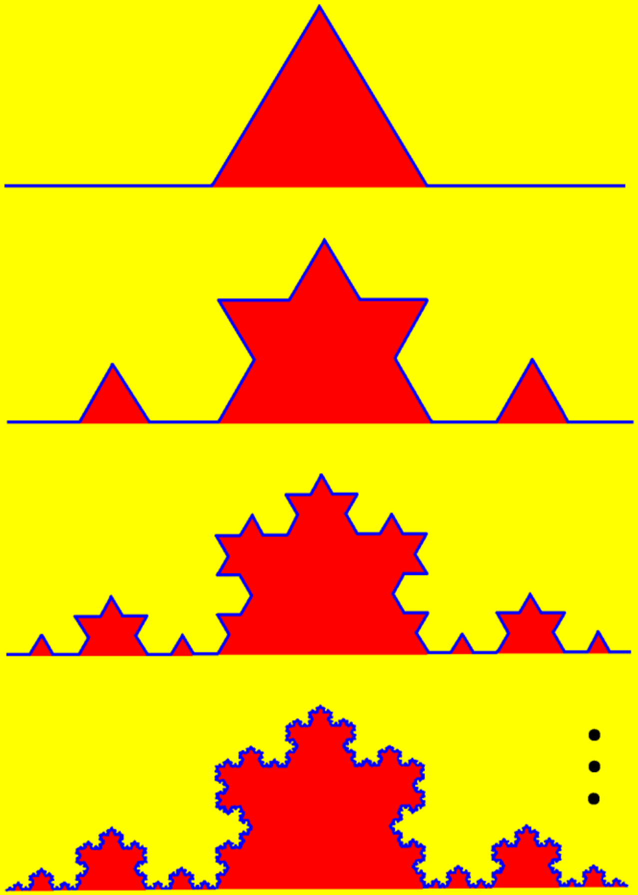
Koch's Island or Snowflake

Making an infinite fractal line:



Length = 1

Length = (4/3)

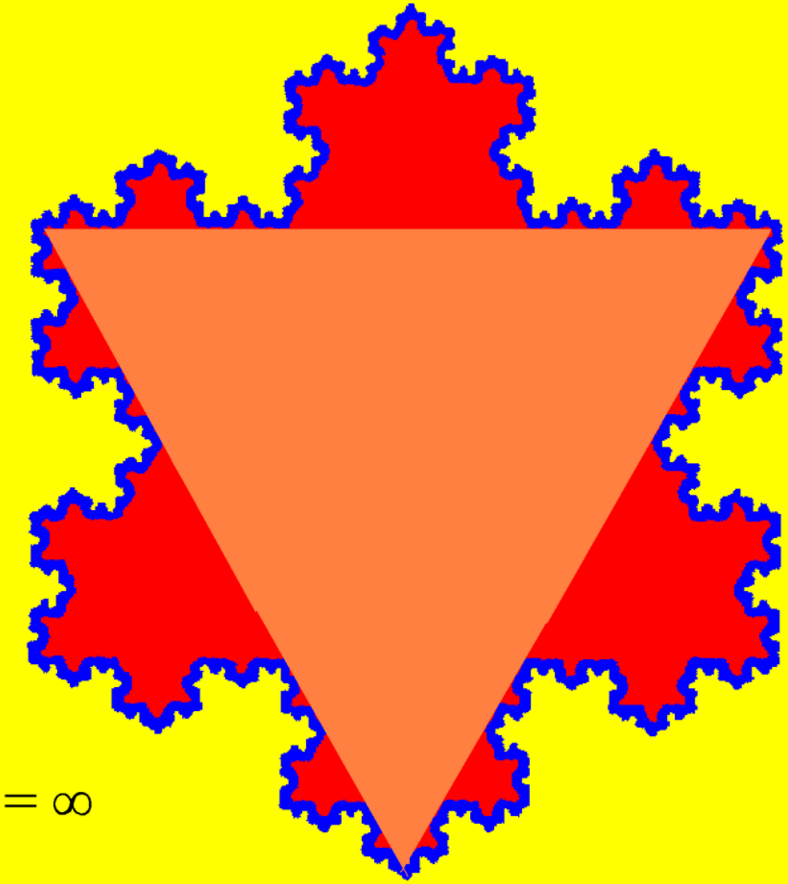
Length = $(4/3)^2$

Length = $(4/3)^3$

⋮

Length = $(4/3)^\infty = \infty$

Using three fractal lines, we can construct an island with an infinite coastline:



To keep your grandchildren quiet!

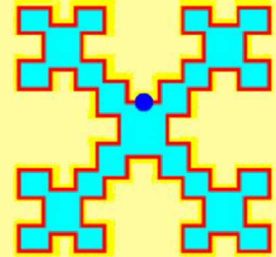
m19 koch

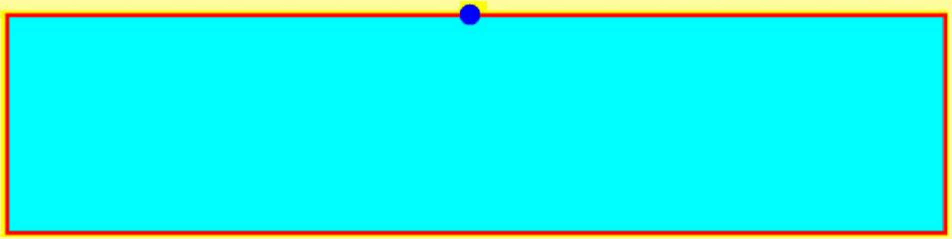
Fractal Antennas for Cell-Phones

Fractal are used for miniature broad-band antennas and arrays for next generation of cell-phones and navigation systems

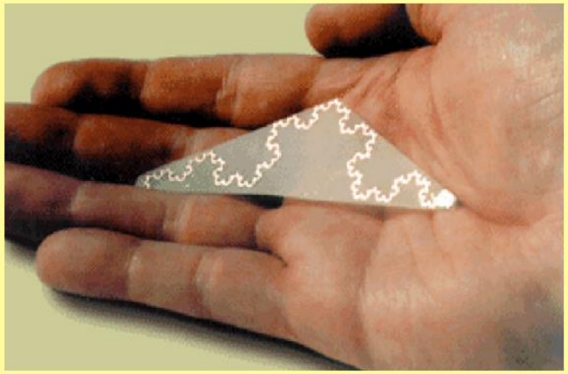
Hardware example of fractal geometry

Fractal Antennas for Cell-Phones, etc





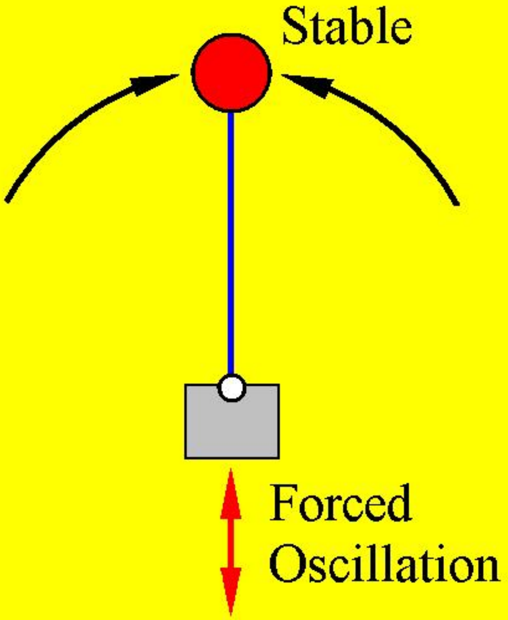
Fractal loop and folded dipole with same gain

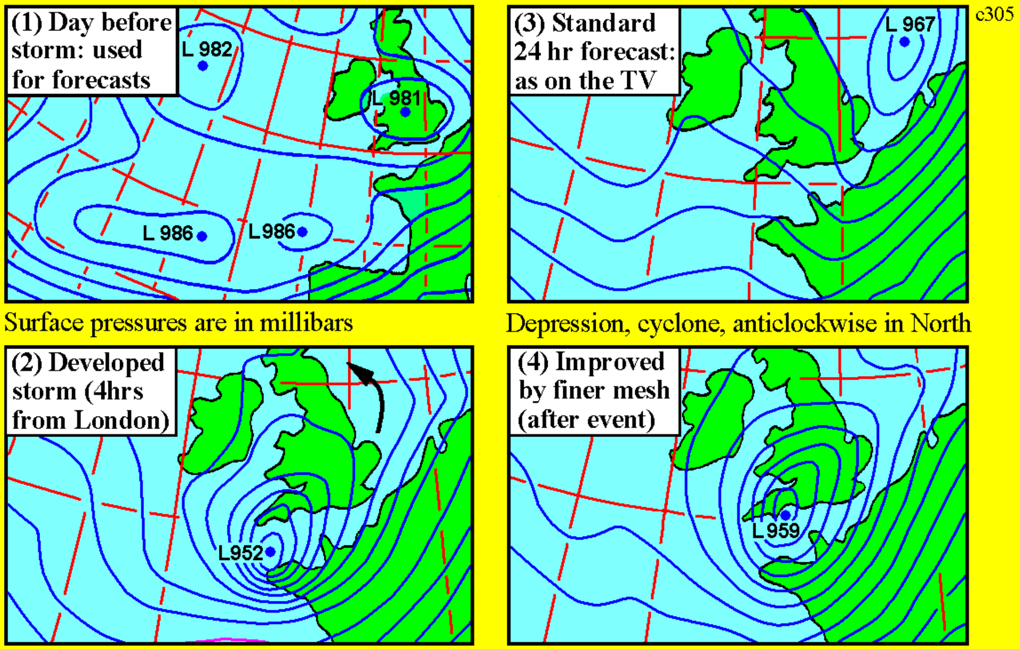


Hardware product

Fabrication facility

Next lecture ...





Chaos theory hits weather forecasting: the great storm of Oct 1987 destroyed fifteen million trees