Wave and Current Loads on Offshore Structures

Application of Morison Equation
Wave Load Example

5.0m wave height
Depth 8.5m
Wave Parameters

- Crest
- Trough
- SWL (Still Water Level) at z = 0
- SWL at θ = kx
- Direction of Propagation
- L (Wave Length)
- C (Wave Speed)
- η(θ) (Wave Height)
- Bottom, z = -d
- x
- u, αx (Horizontal Velocity)
- w, αz (Vertical Velocity)
## Wave Parameters

<table>
<thead>
<tr>
<th>Relative Depth</th>
<th>Shallow Water ( \frac{d}{L} &lt; \frac{1}{25} )</th>
<th>Transitional Water ( \frac{1}{25} &lt; \frac{d}{L} &lt; \frac{1}{2} )</th>
<th>Deep Water ( \frac{d}{L} &lt; \frac{1}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Wave profile</td>
<td>Same As &gt; ( \eta = \frac{H}{2} \cos \left( \frac{2\pi x}{L} - \frac{2\pi t}{T} \right) = \frac{H}{2} \cos \theta )</td>
<td>(&lt; \text{Same As} )</td>
<td>( \eta = \frac{H}{2} \cos \left( \frac{2\pi x}{L} - \frac{2\pi t}{T} \right) = \frac{H}{2} \cos \theta )</td>
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<tr>
<td>2. Wave celerity</td>
<td>( C = \frac{L}{T} = \sqrt{gd} )</td>
<td>( C = \frac{L}{T} = \frac{gT}{2\pi} \tan \left( \frac{2\pi d}{L} \right) )</td>
<td>( C = C_0 = \frac{L}{T} = \frac{gT}{2\pi} )</td>
</tr>
<tr>
<td>3. Wavelength</td>
<td>( L = T\sqrt{gd} = CT )</td>
<td>( L = \frac{gT^2}{2\pi} \tan \left( \frac{2\pi d}{L} \right) )</td>
<td>( L = L_0 = \frac{gT^2}{2\pi} = C_0 T )</td>
</tr>
<tr>
<td>4. Group velocity</td>
<td>( C_g = C = \sqrt{gd} )</td>
<td>( C_g = n C = \frac{1}{2} \left[ 1 + \frac{4\pi d/L}{\sinh (4\pi d/L)} \right] C )</td>
<td>( C_g = \frac{1}{2} C = \frac{gT}{4\pi} )</td>
</tr>
<tr>
<td>5. Water particle velocity</td>
<td>(a) Horizontal ( u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos \theta )</td>
<td>( u = \frac{H}{2} \frac{gT}{L} \cosh \left( \frac{2\pi (z+d)/L}{\cosh (2\pi d/L)} \right) \cos \theta )</td>
<td>( u = \frac{\pi H}{T} e^{\left( \frac{2\pi z}{L} \right)} \cos \theta )</td>
</tr>
<tr>
<td></td>
<td>(b) Vertical ( w = \frac{H\pi}{T} \left( 1 + \frac{z}{d} \right) \sin \theta )</td>
<td>( w = \frac{H}{2} \frac{gT}{L} \frac{\sinh \left( 2\pi (z+d)/L \right)}{\cosh (2\pi d/L)} \sin \theta )</td>
<td>( w = \frac{\pi H}{T} e^{\left( \frac{2\pi z}{L} \right)} \sin \theta )</td>
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</table>
Added Mass and Inertial Forces

Body moves in an inviscid fluid with speed \( U < 0 \):

- Fluid velocity \((u, v)\) is proportional to body velocity \( U \)
- Mass of moving fluid is proportional to fluid density \( \rho \) and body volume \( V \)
- Average fluid velocity is opposite to \( U \)
- Fluid does not move far away from the body

Momentum: \( I(t) = -c_m \rho V U(t) \) where \( c_m \) depends on body shape

Steady motion: \( I = \text{const} \Rightarrow R = 0 \)

Unsteady motion: \( R = dI/dt = -c_m \rho V dU/dt \)

Inertial force on an accelerating body: \( F_I = c_m \rho V \frac{dU}{dt} = m_a \frac{dU}{dt} \)

\textbf{Added mass:} \( m_a = c_m \rho V \); \( c_m \) – added mass coefficient
Morison’s equation: calculates force applied to a body by an uniform unsteady flow

\[ D(t) = \frac{1}{2} C_D A \rho U(t) |U(t)| + C_M V \rho \frac{dU(t)}{dt} \]

\( V, A \) – volume and cross section area of the body;
\( C_D \) – drag coefficient;
\( C_M = c_m + 1 \) – inertial force coefficient, where ”1” accounts for a hydrostatic force component in accelerated fluid

Task: Prove that \( C_M = c_m + 1 \)

• Typical values of coefficients for a cylinder: \( C_D = 1; C_M = 2 \)
Application to Wave Loads

Deep water:

\[ v_x = a \frac{g \kappa}{\omega} \frac{\cosh(k(z + d))}{\cosh(kd)} \cos(kx - \omega t); \quad \omega = \sqrt{k g \tanh(kd)}; \quad k = \frac{2 \pi}{\lambda} \]

- \( a \)– wave amplitude, \( g \)– gravity, \( k \)– wave number, \( \lambda \)– wave length, \( \omega \)– frequency

![Diagram of water column](image.jpg)

Vertical column:

\[ dA = D \, dz; \quad dV = \pi D^2 \, dz / 4; \]
\[ U = v(z, t) \]
Integration from \( z = -h \) to \( z = 0 \)

Horizontal element:

\[ A = DL; \quad V = \pi D^2 L / 4; \]
\[ U = v(-h, t) \]
Local parameters, no integration
Application to Wave Loads

Shallow water \((\lambda > 20d)\):

\[
U(t) = a \sqrt{\frac{g}{d}} \cos(kx - \omega t); \quad k = \frac{2\pi}{\lambda}; \quad \omega = k \sqrt{gd},
\]

\(a\)– wave amplitude, \(g\)– gravity, \(k\)– wave number, \(\lambda\)– wave length, \(\omega\)– frequency

Velocity does not change with depth

\[
A = Dd; \quad V = \pi D^2 d/4;
\]

\[
U = U(t)
\]

**Applicability:** Morison’s equation can be applied to waves when \(D \ll \lambda\) and flow can be considered as locally uniform
Horizontal velocity component of a wave propagating in $x$-direction in water of constant depth $d$ is described by the equation

$$v_x = \frac{a g k}{\omega} \frac{\cosh(k (z + d))}{\cosh(k d)} \cos(k x - \omega t),$$

where $a$ is wave amplitude, $g$ is gravity acceleration, $k = 2 \pi / \lambda$ is wave number, $\lambda$ is wave length, $\omega = \sqrt{k g \tanh(k d)}$ is frequency of the wave, $z = 0$ and $z = -d$ represent water surface and bottom respectively. A vertical cylindrical oil rig column of 10 m in diameter is placed in 50 m deep water. Calculate the maximal horizontal force and the moment about the bottom mounting applied to the column by a 200 m long wave of 3 m amplitude. The values of drag coefficient and inertial coefficient are $C_D = 1$ and $C_M = 2$. Discuss the applicability of your solution.
Reading


4. Loads and Responses
   4.1 Introduction
   4.2 Gravity Loads
   4.3 Hydrostatic Loads
   4.4 Resistance Loads
   4.5 Current Loads on Structures
   4.7 Wave Loads on Structures
      4.7.1 Morison Equation
      4.7.2 Forces on Oscillating Structures
      4.7.3 Wave Plus Current Loads
      4.7.4 Design Values for Hydrodynamic Coefficients

Online version available at:


15 Offshore environmental loads and wind turbine design:
   impact of wind, wave, currents and ice

Online version available at:
A Note on Scour
A Note on Scour