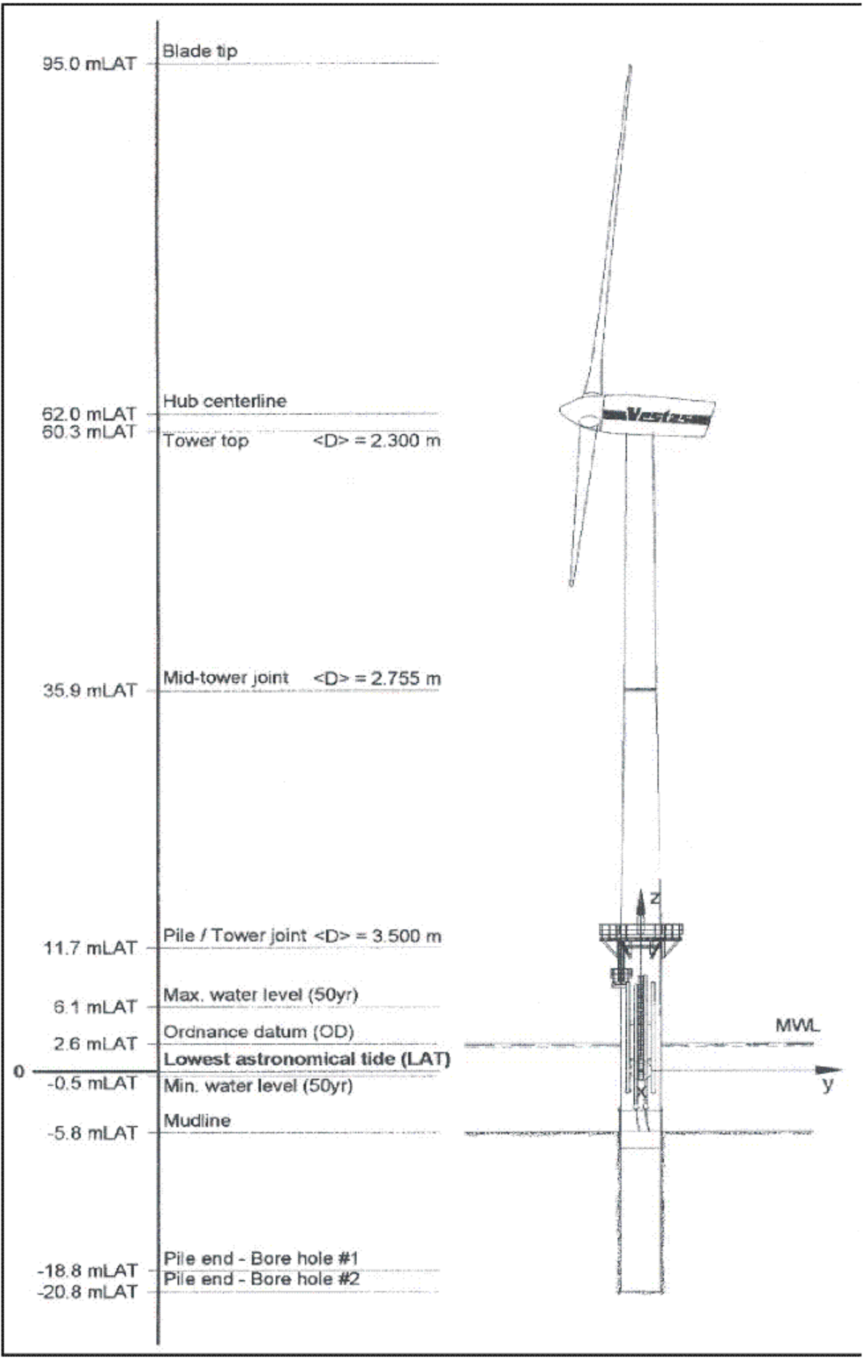
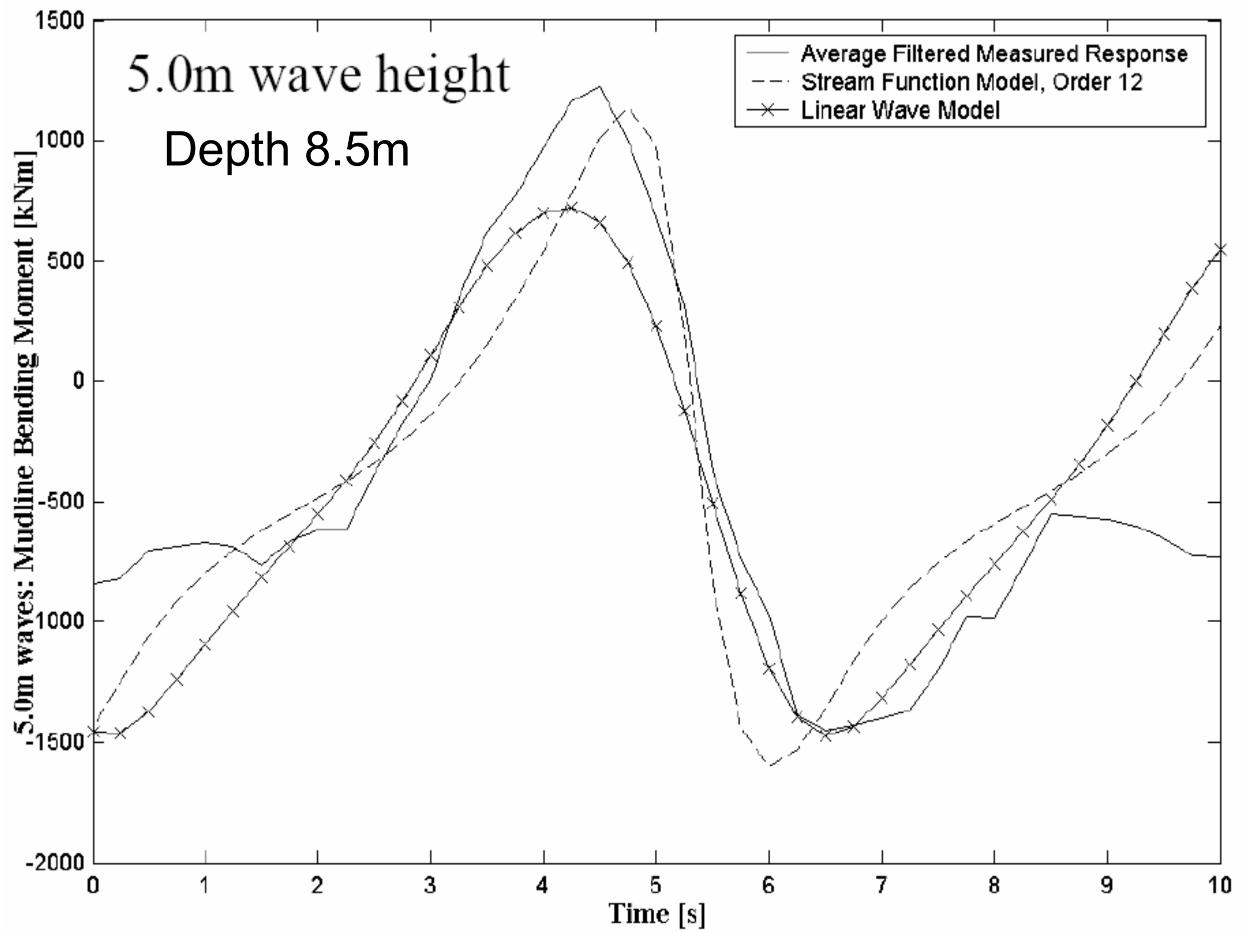
Wave and Current Loads on Offshore Structures

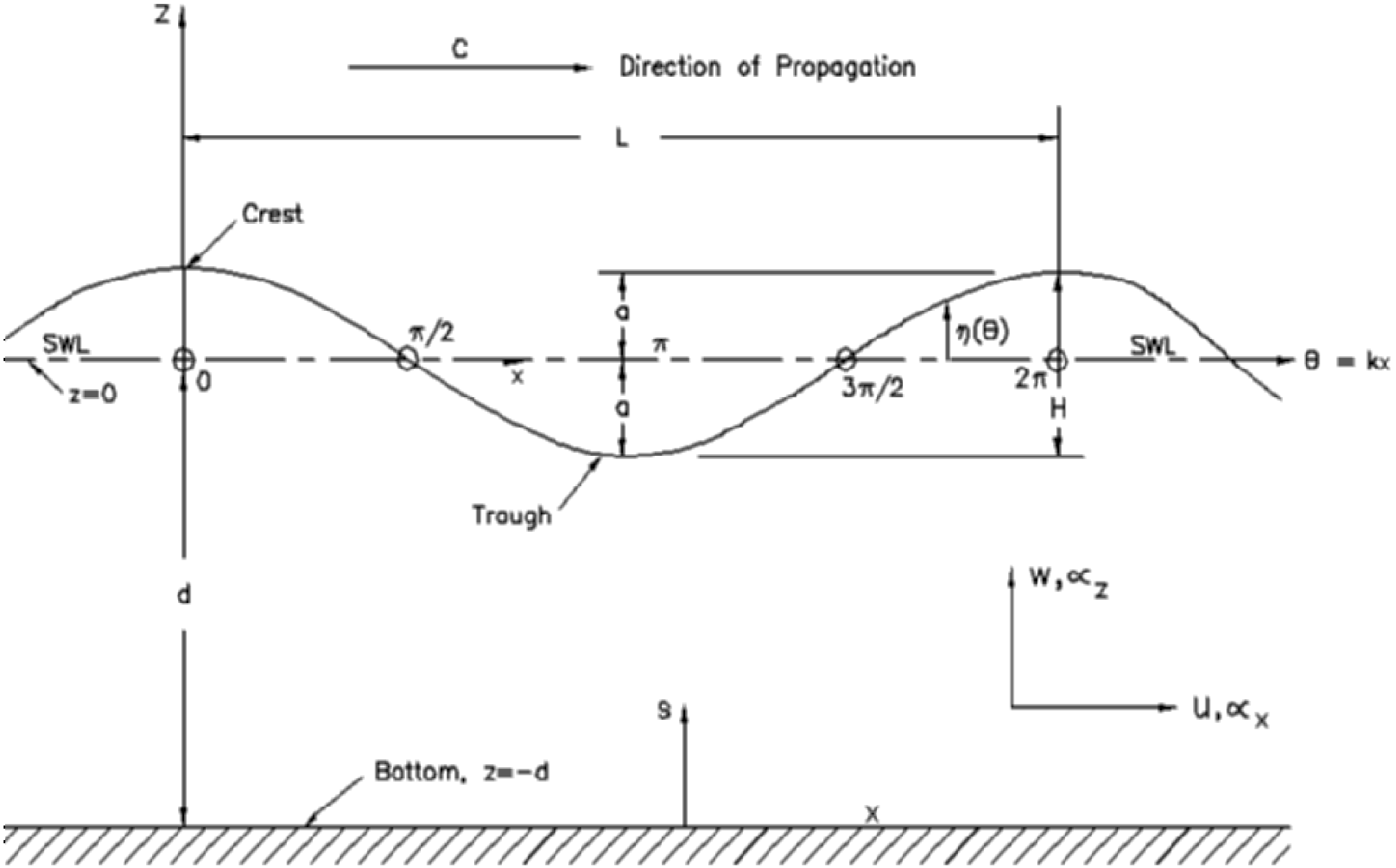
Application of Morison Equation

Wave Load Example





Wave Parameters

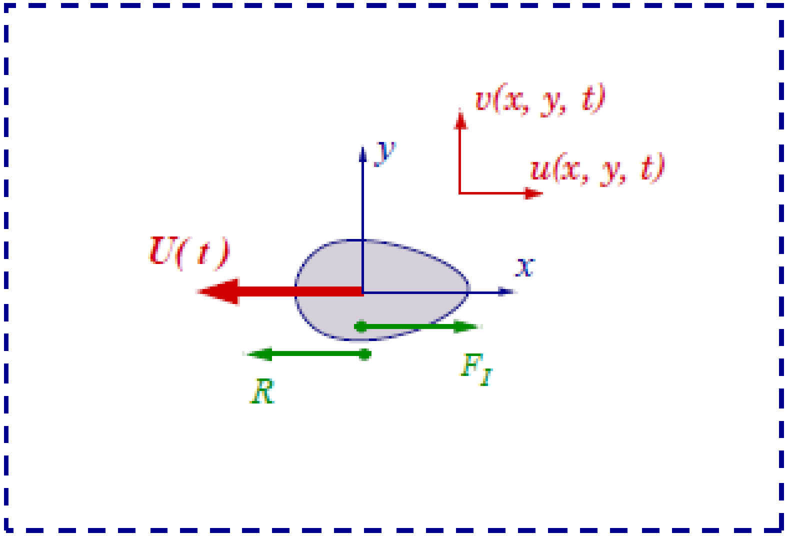


Wave Parameters

Relative Depth	Shallow Water $\frac{d}{L} < \frac{1}{25}$	Transitional Water $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	Deep Water $\frac{d}{L} < \frac{1}{2}$
1. Wave profile	Same As >	$\eta = \frac{H}{2} \cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2}\cos\theta$	< Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right] C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water particle velocity (a) Horizontal (b) Vertical	$u = \frac{H}{2}\sqrt{\frac{g}{d}} \cos\theta$ $w = \frac{H\pi}{T}\left(1 + \frac{z}{d}\right) \sin\theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos\theta$ $w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin\theta$	$u = \frac{\pi H}{T} e^{\left(\frac{2\pi z}{L}\right)} \cos\theta$ $w = \frac{\pi H}{T} e^{\left(\frac{2\pi z}{L}\right)} \sin\theta$

Added Mass and Inertial Forces

Body moves in an inviscid fluid with speed $U < 0$:



- Fluid velocity (u, v) is proportional to body velocity U
- Mass of moving fluid is proportional to fluid density ρ and body volume V
- Average fluid velocity is opposite to U
- Fluid does not move far away from the body

Momentum: $I(t) = -c_m \, \rho V \, U(t)$ where c_m depends on body shape

Steady motion: $I = const \quad \Rightarrow \quad R = 0$

Unsteady motion: $R = dI/dt = -c_m \, \rho V \, dU/dt$

Inertial force on an accelerating body: $$F_I = c_m \, \rho V \frac{dU}{dt} = m_a \frac{dU}{dt}$$

ADDED MASS: $m_a = c_m \, \rho V$; c_m – ADDED MASS COEFFICIENT

Morison Equation

MORISON'S EQUATION: calculates force applied to a body by an uniform unsteady flow

$$D(t) = \frac{1}{2} C_D\, A\, \rho\, U(t)\, |U(t)| + C_M\, V\, \rho\, \frac{dU(t)}{dt}$$

V, A – volume and cross section area of the body;

C_D – drag coefficient;

$C_M = c_m + 1$ – inertial force coefficient, where "1" accounts for a hydrostatic force component in accelerated fluid

Task: Prove that $C_M = c_m + 1$

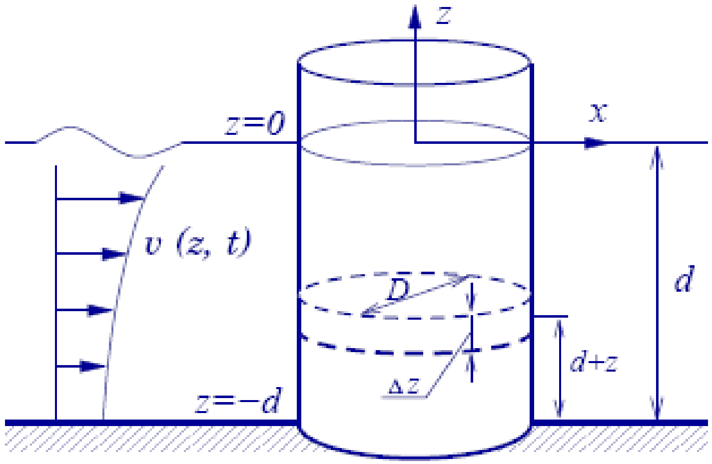
- Typical values of coefficients for a cylinder: $C_D = 1$; $C_M = 2$

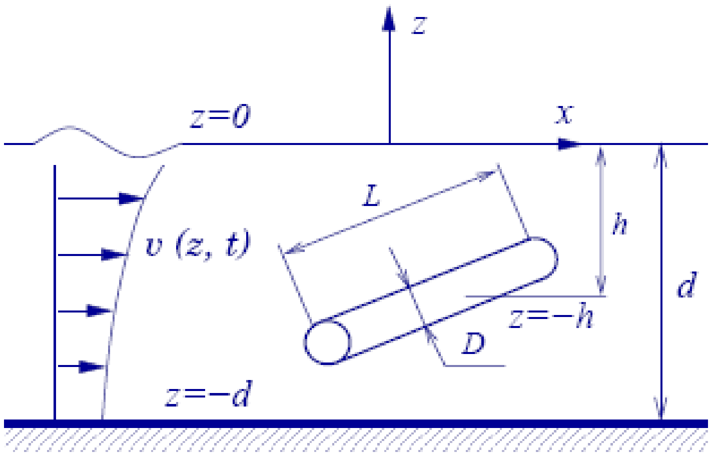
Application to Wave Loads

Deep water:

$$v_x = a\,\frac{g\,k}{\omega}\,\frac{\cosh(k\,(z+d))}{\cosh(k\,d)}\cos(k\,x-\omega\,t);\quad \omega=\sqrt{k\,g\,\tanh(k\,d)};\quad k=2\,\pi/\lambda$$

a– wave amplitude, g– gravity, k– wave number, λ– wave length, ω– frequency





Vertical column:

$$dA = D\,dz;\quad dV = \pi\,D^2\,dz/4;$$
$$U = v(z,t)$$

Integration from $z=-h$ to $z=0$

Horizontal element:

$$A = D\,L;\quad V = \pi\,D^2\,L/4;$$
$$U = v(-h,t)$$

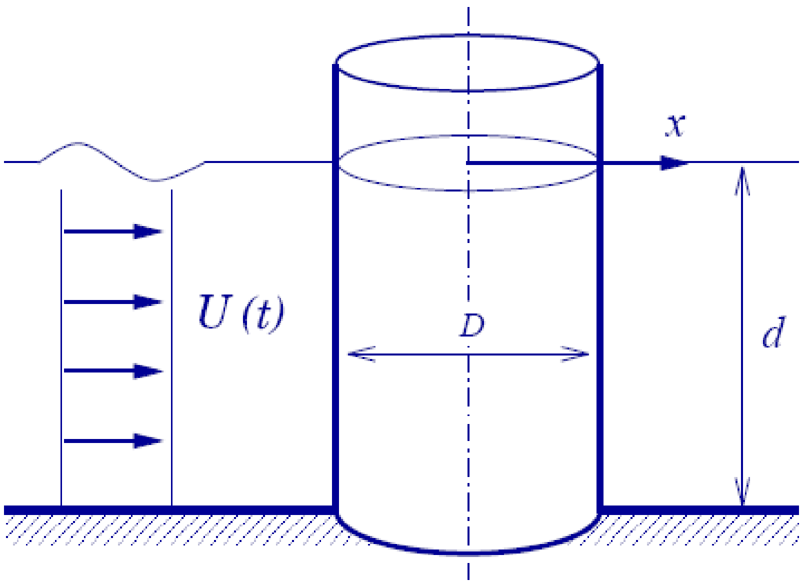
Local parameters, no integration

Application to Wave Loads

Shallow water ($\lambda > 20\,d$):

$$U(t) = a\sqrt{\frac{g}{d}}\cos(k\,x - \omega\,t); \quad k = 2\,\pi/\lambda; \quad \omega = k\,\sqrt{g\,d},$$

a– wave amplitude, g– gravity, k– wave number, λ– wave length, ω– frequency



Velocity does not change with depth

$$A = D\,d; \quad V = \pi\,D^2\,d/4;$$

$$U = U(t)$$

APPLICABILITY: Morison's equation can be applied to waves when $D \ll \lambda$ and flow can be considered as locally uniform

Problem

Horizontal velocity component of a wave propagating in x-direction in water of constant depth d is described by the equation

$$v_x = \frac{a\,g\,k}{\omega}\,\frac{\cosh(k\,(z+d))}{\cosh(k\,d)}\cos(k\,x - \omega\,t)\,,$$

where a is wave amplitude, g is gravity acceleration, $k = 2\,\pi/\lambda$ is wave number, λ is wave length, $\omega = \sqrt{k\,g\,\tanh(k\,d)}$ is frequency of the wave, $z = 0$ and $z = -d$ represent water surface and bottom respectively. A vertical cylindrical oil rig column of $10\,m$ in diameter is placed in $50\,m$ deep water. Calculate the maximal horizontal force and the moment about the bottom mounting applied to the column by a $200\,m$ long wave of $3\,m$ amplitude. The values of drag coefficient and inertial coefficient are $C_D = 1$ and $C_M = 2$. Discuss the applicability of your solution.

Reading

Chakrabarti, Subrata (2005). Handbook of Offshore Engineering, Volumes 1-2. Elsevier.

4. Loads and Responses
    - 4.1 Introduction
    - 4.2 Gravity Loads
    - 4.3 Hydrostatic Loads
    - 4.4 Resistance Loads
    - 4.5 Current Loads on Structures
    - 4.7 Wave Loads on Structures
        - 4.7.1 Morison Equation
        - 4.7.2 Forces on Oscillating Structures
        - 4.7.3 Wave Plus Current Loads
        - 4.7.4 Design Values for Hydrodynamic Coefficients

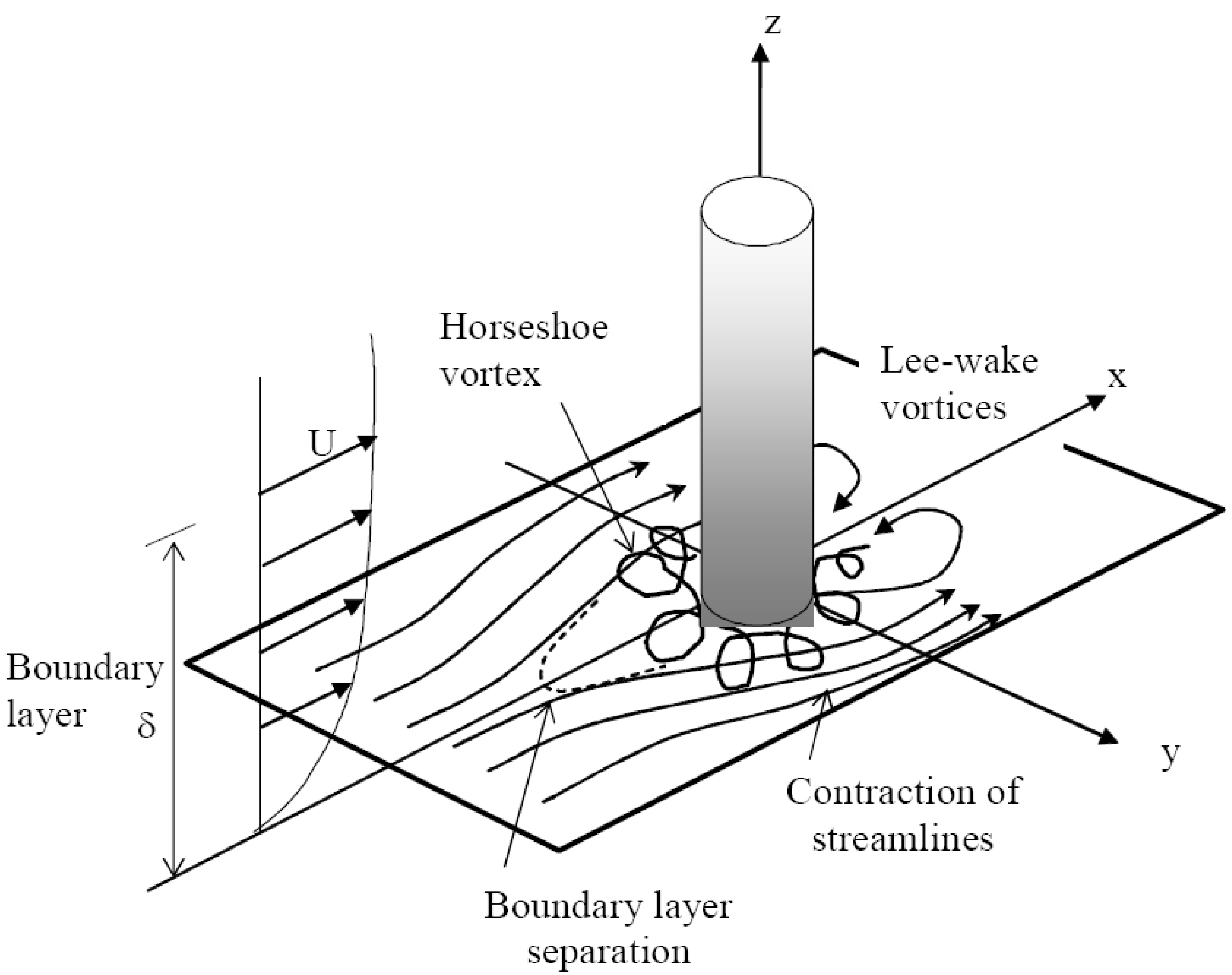
Online version available at:
http://app.knovel.com/hotlink/toc/id:kpHOEV0001/handbook-offshore-engineering

Sorensen, John D. Sorensen, Jens N. (2011). Wind Energy Systems - Optimising Design and Construction for Safe and Reliable Operation. Woodhead Publishing.

15 Offshore environmental loads and wind turbine design:
impact of wind, wave, currents and ice

Online version available at:
http://app.knovel.com/hotlink/toc/id:kpWESODCS2/wind-energy-systems-optimising

A Note on Scour



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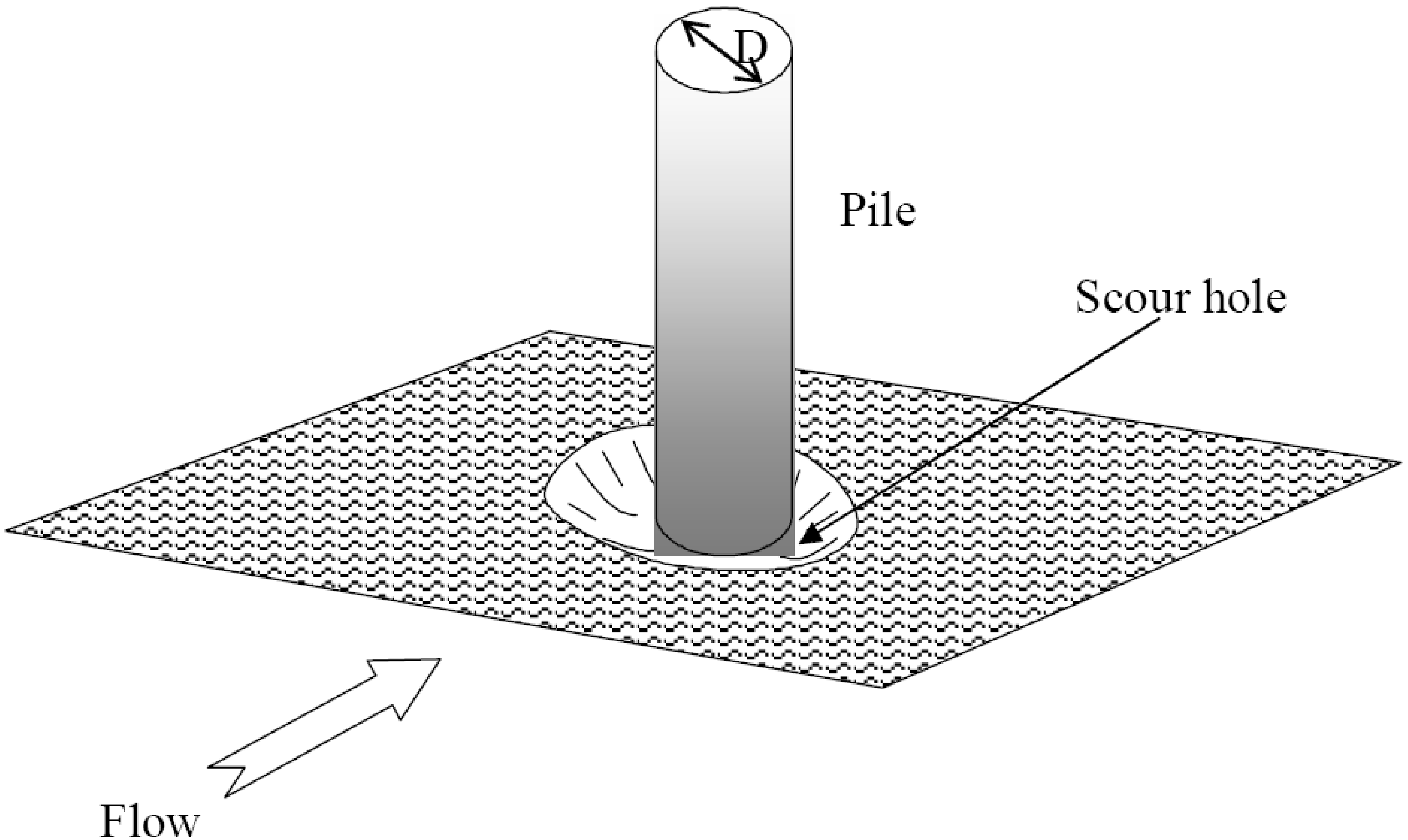

A Note on Scour