ON THE THEORY OF
THE HORIZONTAL-AXIS
WIND TURBINE

Otto De Vries
National Aerospace Laboratory NLR, Amsterdam, The Netherlands

INTRODUCTION

The extraction of energy from the wind is an old idea, one used by sailing ships and windmills for many centuries. The development of ancient windmills was based on empiricism and engineering skill. The development of the fluid mechanics, or more specifically the aerodynamics, of windmills (wind turbines in modern usage) is more recent. The study of the aerodynamics of wind turbines was begun after World War I by Betz (1926) and Glauert (1935a) and got a new impulse after the “energy crisis” of 1973–74. Nowadays, it is a worldwide field of research, stimulated and guided by national research programs in the US, Sweden, Denmark, The Netherlands, Great Britain, and many other countries.

Scope of the Present Review

A complete review of the aerodynamic aspects of wind-energy conversion should encompass the following:

1. The characteristics of the natural wind, such as annual wind-velocity distributions, wind shear, turbulence, gusts, effects of local terrain conditions, and siting.
2. The theory of wind-driven turbines, operating in a homogeneous and nonturbulent flow.
3. The influence of the natural wind (turbulence, wind shear) and turbine-induced irregularities (yawing misalignment, blade-tower interaction) on turbine performance and blade loading.
4. The influence of wake interaction and decay in the terrestrial boundary layer in connection with large arrays of wind turbines.

The present review is limited to item (2.) for the case of horizontal-axis turbines. These turbines are singled out because the analysis is now in a transition phase from relatively simple to increasingly sophisticated. For a more extensive survey of the aerodynamic aspects, see De Vries (1979).

We begin with a description of the basic aerodynamic conversion process. It is then shown that the application of the laws of conservation of mass, momentum, and energy leads to a rough indication of the maximum possible energy output of a wind turbine. This is followed by a discussion of calculation methods for wind turbines, from the simple blade-element theory to the more sophisticated vortex and panel methods. The review concludes with some remarks on the experimental confirmation of the theory. This is important, since the ability to predict the characteristics of wind turbines for design purposes is the main motive for developing more sophisticated and complicated calculation methods.

**Types of Wind Turbines**

Many different types of wind turbines have been invented. A distinction can be made between turbines driven mainly by drag forces and turbines driven mainly by lift forces. A distinction can also be made between turbines with axes of rotation parallel to the wind direction (horizontal) and with axes perpendicular to the wind direction (vertical). The efficiency of wind turbines driven primarily by drag forces is low (see, for example, De Vries 1979, Chap. 4, pp. 5–6) when compared with the lift-force-driven type. Therefore, all modern wind turbines are driven by lift forces. The most common types are the horizontal-axis wind turbine (HAWT) and the vertical-axis wind turbine (VAWT; see Figure 1). The latter has the advantage that its operation is independent of the wind direction, whereas a HAWT has to be yawed into the wind direction.

**Aerodynamic Characteristics of Wind Turbines**

The aerodynamic operation of a wind turbine can be characterized by the following overall quantities: the rotor torque $Q$, the rotor drag $D$, the angular velocity $\Omega$, and the power output $P = \Omega Q$. By dimensional analysis, these quantities can be made dimensionless as follows:

\[
\lambda = \frac{\Omega R}{U_0} \quad \text{(tip-speed ratio),} \quad (1)
\]

\[
C_Q = \frac{Q}{(\frac{1}{2} \rho U_0^2 R S_{\text{ref}})} \quad \text{(torque coefficient),} \quad (2)
\]
Figure 1  The main types of lift-driven wind turbines, namely the horizontal-axis wind turbine (HAWT) and the vertical-axis wind turbine (VAWT). Also indicated are the wind velocity $U_0$, the angular velocity $\Omega$, the aerodynamic torque $Q$, and the rotor drag $D$.

$C_P = P/(\frac{1}{2} \rho U_0^3 S_{ref})$  \hspace{1cm} (power coefficient), \hspace{1cm} (3)

$C_D = D/(\frac{1}{2} \rho U_0^2 S_{ref})$  \hspace{1cm} (rotor-drag coefficient), \hspace{1cm} (4)

with $R$ the maximum radius of the rotor, $U_0$ the undisturbed wind velocity, $\rho$ the density of the air, and $S_{ref}$ the cross section swept by the rotor blades ($= \pi R^2$ or $2HR$). It should be noted, however, that there is not yet a generally accepted standardization of symbols in the field of wind-energy conversion.

AERODYNAMIC CONVERSION PROCESS

The basic process of converting kinetic energy from the wind into mechanical energy of the rotor can be described in two ways, namely from the point of view of the rotor (the driving force) and from the point of view of the wind stream (loss of flow energy). Both points of view are, of course, connected by the laws of conservation of mass, momentum, and energy. The most direct way to set up a calculation method for a wind turbine is to consider the driving force.

The Driving Force

The rotor is driven by a component of the lift force. The lift force and the profile drag at each section of the rotor blade depend on the velocity of
the relative flow \( \mathbf{U}_{\text{rel}} \), the chord length \( c \), the angle of attack \( \alpha \), and the dimensionless coefficients \( C_f \) and \( C_d \), which are functions of \( \alpha \) (Figure 2). The section lift and drag (force per unit span of airfoil) can be expressed by

\[
\text{section lift} = C_f c^2 \rho U_{\text{rel}}^2, \tag{5}
\]

\[
\text{section drag} = C_d c^2 \rho U_{\text{rel}}^2. \tag{6}
\]

The section lift can also be related to the circulation \( \Gamma \) around the airfoil, namely

\[
\text{section lift} = \rho \Gamma U_{\text{rel}}, \tag{7}
\]

which implies

\[
\Gamma = \frac{1}{2} C_f c U_{\text{rel}}. \tag{8}
\]

The theory of wing sections is a well-established part of aerodynamics. (For the theory and a compilation of airfoil data, see Abbott & von Doenhoff 1959; for a modern approach, see Eppler & Somers 1978.) The lift force is perpendicular to \( U_{\text{rel}} \) and the profile drag is parallel to it. The lift force can only have a positive driving component (i.e. a component in the direction of motion; see Figure 2) when three conditions are fulfilled, namely \( a) \theta \neq 0, \) \( b) C_f \neq 0, \) and \( c) \theta \) and \( C_f \) have the same sign. The profile drag always reduces the driving force.

When a HAWT operates with \( \Omega \) constant and \( U_0 \) both constant and parallel to the axis of rotation, the flow relative to the rotor blade is steady. When the setting angle between the blade element and the plane of rotation (i.e. \( \Theta - \alpha \); see Figure 2) has the correct value, the driving force on the blade element is positive and constant during the complete revolution of the rotor. This situation is different from the case of a VAWT, where the lift force varies periodically from positive to negative.

![Figure 2](image.png)

*Figure 2*  Airfoil characteristics in steady flow (schematic). \( C_f \) is the section lift coefficient, \( C_d \) the section drag coefficient, \( c \) the section chord length, \( U_{\text{rel}} \) the flow velocity relative to airfoil, \( \alpha \) the angle of attack, and \( \theta \) the angle between \( U_{\text{rel}} \) and the direction of motion (relative flow angle). A: lift loss due to stall.
during a revolution of the rotor. The driving component of the lift force is, however, always positive or zero.

**Loss of Flow Energy**

The energy content of an incompressible flow is expressed by the so-called "Bernoulli constant,"

\[ H = p + \frac{1}{2} \rho U^2, \]  

where \( p \) is the static pressure and \( U \) the local velocity. In the case of flow of an incompressible and inviscid fluid, only forces perpendicular to the local velocity are possible, and the equation of motion for a fluid particle is

\[ \frac{DH}{Dt} = \frac{\partial H}{\partial t} + U \cdot \nabla H = \frac{\partial p}{\partial t}. \]  

Equation (10) shows that the energy content of a fluid particle can only be changed by an unsteady static pressure variation working on the fluid particle along its path through the plane of the rotor. In the case of a HAWT, the flow is steady in a coordinate system fixed to the rotor, and \( \frac{DH}{Dt} = 0 \). This apparent paradox can be solved by adding the centrifugal and Coriolis forces to the equation of motion in the rotating coordinate system. Transformation of this modified equation to an earth-fixed coordinate system leads to the well-known turbine equation

\[ \Delta H = \rho U \cdot (\Omega \times r) = \rho U_{tan\infty} \Omega r_{\infty}, \]  

where \( \Delta H \) is the total-head loss across the rotor, \( r \) is the distance to the axis of rotation, \( U_{tan\infty} \) is the tangential velocity component behind the rotor at radius \( r_{\infty} \).

Equation (11) is valid for a single rotor. In the case of two counter-rotating rotors, the equation must be applied to each rotor separately, in the ideal case leading to a value of \( U_{tan\infty} = 0 \) behind the second rotor. Equation (11) is not valid for a VAWT, because in that case no coordinate system can be found in which the flow is steady.

**APPLICATION OF CONSERVATION LAWS**

When a wind turbine is surrounded by a "control" surface, the laws of conservation of mass, momentum, and energy, applied to the volume enclosed by the control surface, yield relations between the time-averaged aerodynamic forces on the turbine inside the volume and the velocities and static pressures on the control surface. The flow inside the volume does not appear in the equations and, therefore, most of the results are
valid for both a HAWT and a VAWT. On the other hand, specific results can be obtained only by making assumptions about the flow inside the volume.

As is shown below, relatively simple assumptions lead to important conclusions about the maximum turbine performance.

**Single-Streamtube Analysis**

Betz (1926) considers only average axial velocities in a streamtube through the rotor disk $S_{\text{ref}}$ (no radial and tangential velocities; see Figure 3) and applies the energy and linear momentum equation to that streamtube. He assumes that only the rotor absorbs energy and he neglects the viscous and turbulent stresses at the control surfaces. This leads to the following equations for $C_p$ and $C_D$:

$$C_p = 4a(1-a)^2 \quad \text{and} \quad C_D = 4a(1-a),$$

(12)
where $a = (U_0 - U_{ax})/U_0$ is the axial induction factor and $U_{ax}$ is the axial velocity through the rotor. Equation (12) shows that the maximum power output is obtained when

$$a_{opt} = 1/3 \quad \text{and} \quad (C_P)_{opt} = 16/27,$$

which is the well-known Betz limit.

An important by-product of Betz’s analysis is a relation between the average axial velocity at the rotor ($U_{ax}$) and the velocity downstream in the wake ($U_{ax\infty}$):

$$U_{ax} = \frac{1}{2}(U_0 + U_{ax\infty}), \quad \text{or} \quad U_{ax\infty} = (1 - 2a)U_0.$$

The conservation of mass then leads to a relation between the downstream cross section of the wake ($S_w$) and the rotor disk area ($S_{ref}$),

$$S_w/S_{ref} = (1 - a)/(1 - 2a),$$

which reveals a physically unrealistic behavior at $a \approx 1/2$.

**Multiple-Streamtube and Angular-Momentum Analysis**

It is often assumed that Equations (12) through (15), derived for a streamtube containing the entire rotor (single streamtube), also apply to an element of this streamtube (multiple-streamtube theory; see Figure 3). This notion becomes especially useful when tangential velocity components are included. In the case of a HAWT, the equation for the angular momentum relates the torque $\delta Q$ on the blade elements in a streamtube element to the tangential velocity $U_{tan\infty}$. The connection with Equation (11) becomes clear by noticing that $\delta P = \Omega \delta Q$ and $\delta P = \Delta H U_{ax} 2\pi r dr = \Delta H U_{ax\infty} 2\pi r_{\infty} dr_{\infty}$ (mass conservation). This presupposes an axisymmetric flow. Glauert (1935a) takes a zero tangential velocity in front of the rotor and introduces the tangential induction factor $a'$ for the tangential velocities at the rotor and downstream (Figure 3):

$$U_{tan} = a'\Omega r \quad \text{and} \quad U_{tan\infty} = 2a'\Omega r.$$  

This leads, with some additional assumptions, to the relation

$$C_p = (2/\lambda)^2 \int_0^\lambda (1 - a)\lambda X^3 d\lambda,$$

where $X = \Omega r/U_0 = \lambda r/R$ is the local tip-speed ratio. By assuming that the different streamtube elements behave independently of each other, it is possible to optimize the integrand for each $X$ separately. For a fast running turbine ($\lambda \gg 1$), this leads to the result that $a_{opt} \to 1/3$, whereas for a slow running turbine ($\lambda_{opt} \approx 1$) or for elements close to the hub ($r/R \ll 1$), $a_{opt}$ decreases. Substituting the result in Equation (17) yields $(C_p)_{opt}$ as a function of $\lambda_{opt}$, which is presented in Figure 4. The power
loss with respect to the optimum of Betz, \( (C_p)_{\text{opt}} = 16/27 \), can be interpreted as the kinetic energy of the rotation left in the wake. It appears from radial equilibrium that the wake rotation must be connected with a static pressure deficit in the wake, which is neglected by Glauert in the linear-momentum equation. This seems justified in the case of an aircraft propeller in cruise condition, with its small perturbation velocities, but is questionable in the case of a low-\( \lambda \) HAWT (De Vries 1979, Appendix C).

It is noted in passing that in the case of a VAWT, \( \delta Q \) is perpendicular to \( U_0 \) and the axisymmetric flow model cannot be applied. This excludes the possibility of using the angular-momentum equation to estimate in a relatively simple way the energy loss caused by the vorticity shed into the wake.

**CALCULATION METHODS FOR A HORIZONTAL-AXIS WIND TURBINE**

In the preceding discussion, overall estimates of the optimum power output of an isolated wind turbine were made, but it was not possible to determine the optimum shape of the rotor from these general considerations. In addition, it is important to obtain accurate predictions of the
off-design conditions, i.e. $\lambda \neq \lambda_{opt}$, especially in connection with, for example, turbine control. Both kinds of calculations require more sophisticated methods. These methods have been developed, following in the footsteps of aircraft-propeller theory.

Before describing these calculation methods, we survey the different flow states of a rotor in a wind stream.

**Survey of Flow States**

The different flow states of a rotor in a wind stream, shown in Figure 5, have been taken from a presentation by Stoddard (1977). Only the "windmill brake state" $(0 \leq a < \frac{1}{2})$ and the "turbulent-wake state" $(\frac{1}{2} \leq a < 1)$ are of interest for a wind turbine, because in those cases $C_p \geq 0$.

The simple momentum theory fails when $a \geq \frac{1}{2}$, because the trailing vortex structure is unstable at the large wake expansion when $a \to \frac{1}{2}$. Helicopter rotor tests show that the turbulent-wake state can be physically realized, but a good mathematical model is still lacking (see, for

Figure 5 Survey of the different flow states of a horizontal-axis wind turbine, characterized by the average axial velocity $U_{ax}$ through the rotor disk and the rotor drag $D$. A: helicopter experiments; B: Equation (12); C: momentum theory fails; I: propeller state; II: windmill brake state; III: turbulent wake state; IV: vortex ring state.
example, Zimmer 1972). There is some evidence that the effect of the turbulent-wake state starts already at $a = 0.4$, which is rather close to the optimum condition ($a = 1/3$).

At low induced velocities, the wind turbine is often affected by flow separation ($\alpha > \alpha_{\text{max}}$; see Figure 2), which presents a calculational problem. This contrasts sharply with a propeller at cruise condition, which operates at small $\alpha$ when the induced velocities are small. The optimum condition for a wind turbine is thus closely surrounded by the turbulent-wake state and conditions with blade stall, as illustrated in Figure 7 and discussed below.

**Optimum Rotor Shape**

The application of the conservation laws and some additional assumptions led Glauert to a formulation of the flow conditions in the plane of a rotor at optimum power output [see discussion of Equation (17) and Figure 4]. The simplest way to relate the flow in the plane of the rotor to the geometry of and the forces on the rotor is to use the existing blade-element theory for aircraft propellers, of which an excellent review is given by Glauert (1935b). The analysis does not determine the shape of the blade uniquely, but only the product of the local chord and the local lift coefficient for a chosen $\lambda_{\text{opt}}$ and number of blades $B$. There is a good reason to choose a constant $C_{\ell}$ along the blade span (viz. at $(C_{\ell}/C_{d})_{\text{max}}$). This choice leads to a strong variation of the chord length along the span (taper), as is illustrated in Figure 6. Since $C_{\ell} = \text{constant}$ means $\alpha = \text{constant}$, this also implies that the blade-setting angle $i = \theta - \alpha$ varies strongly along the blade span (twist).

The analysis of Glauert (1935a) does not take the profile drag $C_{d}$ and the effect of a finite number of blades (tip correction factor) into account. This limitation has been recently removed by a number of investigators, such as Wilson & Lissaman (1974), Wilson & Walker (1976), and Griffith (1977). For some different approaches toward the determination of the optimum shape, the reader is referred to Weber (1975), Stewart (1976), and Giordano (1979). The latter bases his approach on a different optimization for an aircraft propeller (Giordano 1974, Hirsch 1948). Whether improved calculation methods lead to different optimum blade shapes is an open question. Nevertheless, the aerodynamic optimum shape certainly is highly tapered and twisted, which is impracticable from the manufacturer's point of view. More practicable shapes require a calculation method for arbitrary blade shapes and a full range of tip-speed ratios. Such methods are discussed next.
Blade-Element Theory

The blade-element theory assumes that the different spanwise elements are independent of each other and that the forces on the blade elements can be determined from the local flow conditions and the corresponding two-dimensional wing-section data (Figure 2). The local flow conditions are estimated by either momentum or vortex considerations, or both. Though originally developed for aircraft propellers (Glauert 1935b), the blade-element theory has been applied to wind turbines by Glauert (1935a), Wilson & Lissaman (1974), Holme (1981), and others.

A typical result of such an application is given in Figure 7. In this figure, the $C_p$ vs. $\lambda$ curve indicated by “C” is calculated with “ideal” airfoil characteristics, i.e. $C_d = 0$ and $dC_\ell/d\alpha = $ constant. The optimum is less than the Betz limit, because of (a) wake rotation, (b) tip losses (i.e. a finite number of blades instead of axisymmetric flow), and (c) hub losses (i.e. the blade terminates at $r_{hub}$ instead of at $r = 0$).
Blade stall affects $C_p$ by both the decrease of $C_f$ and the increase of $C_d$. This is clearly demonstrated by the area "D" and "E" at low $\lambda$. At large $\lambda$, $C_d$ is small ($\alpha$ is small), but nevertheless the $C_p$ loss is large, because this loss is almost proportional to $C_d \lambda^3$. Figure 7 also shows an empirical correction for the turbulent wake state "F," as discussed by Miller et al. (1976) and De Vries & Den Blanken (1981, Appendix C). The example of Figure 7 clearly illustrates that the optimum is closely surrounded by blade stall and a turbulent-wake state.

Figure 7  Performance of a HAWT, calculated with the blade-element theory ($r_{hub}/R = 0.15, B = 2$, blade with taper and twist). A: Betz limit; B: decrease of optimum; C: "ideal" profile characteristics; D: loss due to lift decrease (stall); E: loss due to profile drag; F: increase due to turbulent wake state (empirical correction).
With respect to the $C_D$ vs. $\lambda$ curve, the profile drag has hardly any effect and the lift loss due to blade stall is responsible for the marked decrease of $C_D$ at low $\lambda$. The turbulent wake state might have a large effect at high $\lambda$ and negative blade-setting angles.

For some critical remarks on the underlying assumptions of the blade-element theory, the reader is referred to De Vries (1979, Chap. 4, pp. 19–20).

**Vortex Theory and Panel Methods**

Instead of estimating the induced velocities from momentum equations and a number of additional assumptions, a more exact model can be formulated, analogous to the theory for a wing with a finite span. In this respect we can mention the “lifting-line” theory of Prandtl (1918), the “lifting-surface” theory of Weissinger (1947), and the “panel” method of Hess (1972) and others (Figure 8). The flow around the wing is assumed to be irrotational everywhere (except in a thin layer of trailing vorticity), and the velocity can therefore be calculated from a perturbation potential $\phi(x, y, z)$:

$$U = U_0 + \nabla \phi(x, y, z).$$

We assume in Equation (18) the following boundary conditions: (a) zero normal velocity on the wing surface and the trailing vortex sheet, (b) zero

---

**Figure 8** Increasing complexity in lifting-wing theory. (a) lifting-line theory (wing chord $\rightarrow 0$), (b) lifting-surface theory (wing thickness $\rightarrow 0$), (c) panel method. A: bound vortex; B: trailing vortex sheet; C: wing contour panel with singularity distribution.
pressure difference across the vortex sheet, and (c) smooth flow at the trailing edge of the wing (Kutta condition). It is often admissible to fulfill the boundary conditions for the trailing vortex sheet in a flat plane parallel to $U_0$ instead of at its actual position.

The flow around a steadily rotating HAWT in a steady wind stream $U_0$, parallel to the axis of rotation, can be considered as irrotational to first order. The velocities in an earth-fixed coordinate system can be calculated from an unsteady perturbation potential, namely

$$U = U_0 + \nabla \phi_u(x_u, r_u, \eta_u, t),$$

(19)

where $U_0$ is the wind velocity, $x_u, r_u, \eta_u$ are cylindrical coordinates, and $t$ is time. In a rotating coordinate system fixed to the rotor, the flow is steady and rotational, but the perturbation velocities are still irrotational because this cannot be changed by a mere coordinate transformation. Thus

$$U_{rel} = U_0 + \Omega \times r + \nabla \phi(x, r, \eta),$$

(20)

where $\phi = \phi_u$ and $\eta = \eta_u - \Omega t$. Equation (20) demonstrates that although the flow in rotating coordinates is steady and rotational, the perturbation velocities can be calculated from a potential, in a way analogous to the airplane wing in rectilinear flight. However, as is illustrated in Figure 9, there is one important difference between wings and rotor blades, namely the shape of the trailing vortex sheet. In the case of negligible perturbation velocities, the shape is determined by $U_0$ and $\Omega \times r$, but at large perturbations ($a = 1/3$), the shape of the trailing vortex sheet is determined by the induced velocities and cannot be given a priori. Therefore, the shape has to be determined by an iteration procedure ("relaxed-wake analysis") or has to be assumed beforehand ("fixed-wake analysis").

The potential-flow problem is solved by replacing the rotor blades and vortex sheets by singularity distributions (vortices, sources, doublets),

Figure 9  Vortex sheets behind a wing in rectilinear flight and behind a rotating HAWT (one blade shown). A: wing or blade with bound vorticity; B: trailing vortex sheet.
determining the strength of these distributions by fulfilling the boundary conditions, and then calculating the velocity field.

The lifting-line theory (Figure 8a) is the simplest approximation, because here the rotor blade is reduced to a single vortex line [the bound vortex $\Gamma(r)$; see Equations 7 and 8], and a sheet of continuous vorticity ($\gamma = -\partial \Gamma / \partial r$), or with the vorticity lumped into a number of discrete vortex lines. From this vortex system, we can calculate $U_{rel}$ and $\alpha$ at the bound vortex. These values then determine the circulation by using Equation (8) and the aerodynamic characteristics of the airfoil.

The lifting-surface theory (Figure 8b) allows the possibility of including some effects of the finite chord length (see, however, van Holten 1975 for a remark on its applicability in a rotating system).

The panel methods (Figure 8c) are the most sophisticated, because they discretize the rotor blade and vortex sheet by a large number of (chordwise and spanwise) surface elements (panels) with a singularity distribution on the surface or inside the blade volume. The method approximates the pure potential-flow problem, and the effects of viscosity on $C_e$ and $C_d$ have to be included by semi-empirical methods and possibly boundary-layer calculations. This is in contrast with the blade-element and lifting-line theories, in which two-dimensional airfoil data, measured in a wind tunnel, are used.

Panel methods for rotor systems have been developed for helicopters (Foley 1976, Rao & Schatzle 1977) and also for ship propellers (van Gent 1975), but the development of panel methods for wind turbines has only recently begun (see, for example, Suciu et al. 1977 and Preuss et al. 1980).

The main problem in the calculation of a HAWT is the effect of the expanding wake on the induced velocities. The author knows of no systematic studies on this effect. In this respect the studies on helicopter wakes are interesting, although they are not directly applicable to wind turbines, as follows from the discussion of (among others) Landgrebe & Cheney (1972).

EXPERIMENTAL CONFIRMATION OF THE THEORY

The blade-element theory for a HAWT has several theoretical shortcomings, which could be overcome by more advanced calculation methods. The computer codes, however, become more complex and also more time-consuming. Therefore, one must consider carefully whether or not the development of more sophisticated theories is worthwhile with respect to the total R & D costs of large wind-energy conversion systems.
The necessity for the more sophisticated theories has to be judged from a comparison between the calculated and measured performance of HAWTs. Surprisingly, many of the recently built HAWTs have been designed on the basis of aerodynamic theories developed between the years 1920–40, and hardly any experimental data are available.

**Wind-Tunnel Tests Versus Field Tests**

Reliable aerodynamic measurements on wind turbines in field tests are both difficult and time-consuming due to the stochastic character of the wind. Wind-tunnel tests on scaled models seem preferable because of the controlled test conditions and the rapid compilation of the desired data. Field tests are, of course, still indispensable for testing the control system, etc., under real atmospheric wind conditions.

A complete simulation of the full-scale conditions in a wind tunnel (apart from turbulence and wind shear) is obtained when the following dimensionless quantities are the same for the full-scale turbine and for the wind-tunnel model: (a) the tip-speed ratio ($\lambda$), (b) the Reynolds number ($Re_t$), and (c) the tip Mach number ($Ma_t$). As is discussed below, these conditions cannot be realized except in a compressed-air wind tunnel, but this introduces additional difficulties.

For a reliable comparison with calculated results, it is not necessary to obtain the full-scale $Re_t$, but values above a certain minimum have to be reached, e.g., $Re_t > 3 \times 10^5$. The Reynolds number is defined by and can be estimated from

$$Re_t = \frac{U_{rel}c}{v} \equiv Ma_t \frac{R}{a} \frac{(a/v)}{(c/R)} \frac{(r/R)}{,}$$

where $Ma_t = \frac{U_{rel}}{a} \equiv \frac{\Omega R}{a}$ is the tip Mach number, $a$ is the velocity of sound in air, and $v$ is the kinematic viscosity of air. Equation (21) shows that for a given rotor shape [$c/R = f(r/R)$] and for given properties of air ($a, v$), $Re_t$ depends on $Ma_t$ and the size of the rotor $R$.

For actual HAWTs, $Ma_t$ is limited to 0.25 or 0.30 because of blade strength (centrifugal loads) and noise production. For a wind-tunnel test, $Ma_t$ cannot be increased too much; otherwise, compressibility effects occur that are not present on the actual HAWT. Therefore, the size of the rotor is the only parameter left to secure a sufficiently high $Re_t$. The small size of the test sections of most low-speed wind tunnels, aggravated by wake-blockage effects in closed test sections (Alexander 1978, De Vries & Den Blanken 1981, Appendix B), reduces the maximum size of the models and in that way the attainable $Re_t$. How far open-jet or slotted-wall test sections are free of wake-blockage effects is still a point of discussion. Tests in large low-speed wind tunnels seem to be the safest solution.
Small-Scale Wind-Tunnel Test Results

An example of a comparison between results of a small-scale rotor test in a wind tunnel (viz. a rotor with a diameter of 0.75 m in a closed test section of $3 \times 2$ m) and results of calculations with a blade-element theory are given in Figure 10 (De Vries & Den Blanken 1981). The

![Figure 10](image-url)

**Figure 10** Comparison between results of small-scale rotor tests in a wind tunnel and calculations with the blade-element theory (De Vries & Den Blanken 1981).
Reynolds number used during these tests was \( \text{Re}_c \approx 3.5 \times 10^5 \) at \( \text{Ma}_r \approx 0.55 \). This illustrates the small margin available for attaining sufficiently high values of \( \text{Re}_c \) in a normal low-speed wind tunnel.

Moreover, the wake-blockage corrections were large. Zero blade pitch angle was chosen at the calculated optimum condition, whereas a negative blade pitch meant an increased angle of attack of the blade elements. Also the spanwise distribution of \( \Delta H, U_{ax}, \) and \( U_{tan} \) was measured close behind the rotor disk. This appeared to be a useful tool for the interpretation of discrepancies between test results and calculations.

The large discrepancy between the measured and calculated \( (C_P)_{opt} \) is due to the strong underestimation of the profile drag. This is not yet fully explained, but it might be connected with the small size of the model. The good agreement between measured and calculated \( C_D \) suggests that the lift is well predicted. A comparison with Figure 7 indicates that the discrepancy of \( C_p \) as well as \( C_D \) at low \( \lambda \) stems from an underestimation of the lift beyond the stall.

The main conclusions of the investigation were the following:

1. The blade-element theory predicts the overall characteristics well, notwithstanding differences in the spanwise distribution of \( \Delta H, U_{ax}, \) and \( U_{tan} \). It is expected that the anomaly in the profile drag for attached flow disappears in large-scale tests at higher \( \text{Re}_c \).
2. The two-dimensional profile data are no longer applicable at stalled rotor conditions. The \( C_p \) values remained unexpectedly high beyond the stall.
3. The tip-correction methods developed for propellers seem inadequate in the case of wind turbines.
4. The experiments did not extend far enough into the turbulent wake state to verify the validity of the empirical relation used in the calculation method.

### Full-Scale Tests

It is a remarkable fact that experimental data on the aerodynamic performance of HAWTs are so very scarce (Rohrbach 1976). There are some field-test results (see, for example, Smedman-Högström 1978 and Gustavsson & Törnkvist 1978) but they show appreciable scatter, and the inaccuracies in \( C_p \) vs. \( \lambda \) are masked by presenting the data in the form of \( P \) vs. \( U_0 \). Moreover, \( C_D \) is not measured, which means an important diagnostic element is missing in the evaluation of the calculation method. Though sufficient as acceptance tests for the HAWT as an electricity-generating device, the field-test data are insufficient for use as a basis for deciding whether or not to develop more sophisticated calculation methods.
CONCLUDING REMARKS

The situation described above for the HAWT contrasts sharply with the situation for the VAWT with curved (troposkien) rotor blades. For that type of turbine, comprehensive test results already exist (South & Rangi 1975, Blackwell et al. 1976, Sheldahl & Blackwell 1977, Worstell 1979, Sheldahl 1981). For a stimulating discussion of whether or not the VAWT is aerodynamically inferior to the HAWT, the reader is referred to Maydew & Klimas (1981).

It is hoped, that in the near future more complete and reliable data on the aerodynamic performance of the HAWT will become available to stimulate and motivate the further development of its aerodynamic theory.

Literature Cited


Giordano, V. 1974. The circulation distribution for an optimum propeller according to Betz and to the solution of Goldstein. l’Aerotechnica Missile e Spazio, No. 2, pp. 112-26 (In Italian)


