VISUAL AUGMENTATION FOR VIRTUAL ENVIRONMENTS IN SURGICAL TRAINING

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Augmented reality is an important tool for surgical training and skills assessment. The use of computer simulation, particularly the reliance on patient specific data for building realistic models both in terms of biomechanical fidelity and photorealism has attracted extensive interests in recent years. For example, by fusing real bronchoscopy video with 3D tomographic data with the same patient, it is possible to generate photorealistic models that allow high fidelity, patient specific bronchoscope simulation. In order to match video bronchoscope images to the geometry extracted from 3D reconstructions of the bronchi, however, robust registration techniques have to be developed. This is a challenging problem as it implies 2D/3D registration with certain degrees of deformation and different physiological responses.

In this thesis, we propose a new $pq$-space based 2D/3D registration method for camera pose estimation in endoscope tracking. The proposed technique involves the extraction of surface normals for each pixel of the video images by using a linear, local shape-from-shading algorithm derived from the unique camera/lighting constrains of the endoscopes. We demonstrate how to use the derived $pq$-space distribution to match to that of the 3D tomographic model. The registration algorithm is further enhanced by introducing temporal constrains based on particle filtering. For motion prediction, a second-order auto-regressive model has been used to characterize camera motion in a bounded lumen as encountered in bronchoscope examination. The proposed method provides a systematic learning procedure with modular training from ground truth data such that information from different subjects are integrated for creating a dynamic model, which accommodates the learnt behaviour.

To cater for airway deformation, an active shape model (ASM) driven 2D/3D registration has been proposed. ASM captures the intrinsic variability of the tracheo-bronchial tree during breathing and it is specific to the class of motion it represents. The method reduces the number of parameters that control the deformation, and thus greatly simplifies the optimisation procedure. Subsequently, $pq$-based registration is performed to recover both the camera pose and parameters of the ASM. Radial Basis Functions (RBFs) are employed to smoothly warp the 3D mesh based on the ASM point correspondences. The method also exploits the recent development of five degrees-of-freedom miniaturised catheter tip electromagnetic trackers such that the position and orientation of the bronchoscope can be accurately determined under dis-occlusion and bleeding artefacts. The accuracy of the proposed method has been assessed by using both a specially constructed airway phantom with an electro-magnetic tracker, and $in vivo$ patient data.
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To my family

Στους γονείς μου
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<tr>
<td>AR</td>
<td>Augmented Reality</td>
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<td>ARG</td>
<td>AutoRegressive</td>
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<td>ASM</td>
<td>Active Shape Models</td>
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<tr>
<td>BRDF</td>
<td>Bidirectional Reflectance Distribution Function</td>
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<tr>
<td>Condensation</td>
<td>Conditional Density Propagation</td>
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<td>CT</td>
<td>Computed Tomography</td>
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<tr>
<td>DoF</td>
<td>Degree of Freedom</td>
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<td>EM</td>
<td>ElectroMagnetic</td>
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<td>FEM</td>
<td>Finite Element Models</td>
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<td>FFB</td>
<td>Flexible Fibreoptic Bronchoscopy</td>
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<td>FFD</td>
<td>Free-Form Deformation</td>
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<tr>
<td>HAMMER</td>
<td>Hierarchical Attribute Matching Mechanism</td>
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<td>HU</td>
<td>Hounsfield Units</td>
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<td>IBMR</td>
<td>Image Based Modelling and Rendering</td>
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<td>ICP</td>
<td>Iterative Closest Point</td>
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<tr>
<td>LUS</td>
<td>Laparoscopic Ultrasonography</td>
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<td>MI</td>
<td>Mutual Information</td>
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<td>MIS</td>
<td>Minimally Invasive Surgery</td>
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<tr>
<td>MR</td>
<td>Magnetic Resonance</td>
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<td>MRI</td>
<td>Magnetic Resonance Imaging</td>
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<tr>
<td>NCC</td>
<td>Normalised Cross-Correlation</td>
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<td>NMI</td>
<td>Normalised Mutual Information</td>
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<td>NURBS</td>
<td>Non-Uniform Rational B-Splines</td>
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<td>PCA</td>
<td>Principal Component Analysis</td>
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<td>PDM</td>
<td>Point Distribution Model</td>
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<tr>
<td>POSIT</td>
<td>Pose from Orthography and Scaling with Iterations</td>
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<td>RSRM</td>
<td>Rotational Symmetric Reflectance Map</td>
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<td>VB</td>
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<td>Virtual Reality</td>
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Introduction

Over the last ten years, there have been major advances in Minimally Invasive Surgery (MIS). Bronchoscopy and laparoscopy are two common procedures in MIS, which are carried out through natural body openings or small artificial incisions. With these techniques, diagnostic accuracy and therapeutic success are improved when compared to conventional techniques, while at the same time patient trauma and the duration of hospitalisation are greatly reduced. With the maturity of MIS in recent years, there has been an increasing demand of patient specific simulation devices for both training and skills assessment. This is due to the fact that the complexity of the instrument controls, restricted vision and mobility, difficult hand-eye co-ordination, and a lack of tactile perception are major obstacles in performing MIS. They require a high degree of manual dexterity and hand-eye coordination from the operator. Flexible fibre-optic bronchoscopy, for example, is normally performed on patients who are fully awake or with light conscious sedation. The procedure can entail considerable discomfort if it is not handled properly. Training according to the traditional apprenticeship scheme is useful but can result in prolonged surgical procedures with increased patient discomfort and a potential risk of further complications. Simulation devices provide an attractive alternative that offers a number of advantages in terms of cost, time and efficiency.

The use of computer simulation has attracted extensive interests in recent years. For most of the current simulation systems, however, the degree of visual realism is severely limited. In endoscope simulations, most systems have used standard polygon rendering techniques with synthetic texture mapping. Texture mapping is usually uniform throughout the whole simulation, and even in cases where special visual effects, such as polyps or inflammation, are provided, they are limited in both accuracy and adaptability. Natural objects, such as the colon or the bronchi show considerable diversity of shape and texture. The problem of generating realistic structure and surface properties has hindered the production of generic
test-case databases. These drawbacks highlight the importance of augmenting virtual endoscopic views with patient specific endoscopic videos.

In this thesis, we aim to rely on patient specific data for building anatomical models of MIS both in terms of biomechanical fidelity and photorealism. In Chapter 2, the problem and major challenges of MIS simulation are discussed. Particular attention is directed to bronchoscope simulation, which is used as an exemplar to highlight the visual, physical, and tactile realities required for MIS simulation. The objective of this chapter is to establish the basic requirements for creating photorealistic bronchoscopy simulation and identify the drawbacks of existing approaches.

One of the key computational issues identified in Chapter 2 for bronchoscopy simulation is how to fuse real-bronchoscopy video with 3D tomographic data of the same patient. In order to match video bronchoscope images to the geometry extracted from 3D reconstructions of the bronchi, robust registration techniques have to be developed. This is a challenging problem as it implies 2D/3D registration with the presence of local deformation. In Chapter 3, existing methods for 2D/3D registration based on intensity and geometric features are reviewed. Both techniques involve optimising a similarity measure that evaluates how close a 3D model viewed from a given camera pose is to the current 2D video frame. Intensity based techniques entail comparing a predicted image of the object with the 2D image without any structural analysis. With this approach, similarity measures such as cross-correlation and mutual information are typically used. Mutual information exploits the statistical dependency of two datasets and is particularly suitable for multi-modal images. Existing methods, however, are based on special illumination conditions that may not match bronchoscope images because they are illuminated by a light source that is close to the tissue surface and are heavily affected by inter-reflections. In this case, the intensity decreases with the square of the distance from the light source and it is essential to adjust the illumination conditions of the rendered 3D model in order for the intensity-based techniques to work. The method is further complicated by specular reflections due to the mucous layer of the tissue surface, which is difficult to model for simulated views. As an alternative, feature based techniques depend on the alignment of corresponding image features, which are relatively robust against changes in lighting conditions. However, using features purely based on visual appearance is not reliable due to the richness of surface texture observed in bronchoscope views, which are absent in 3D tomographic images.

The basic hypothesis of the thesis is that a robust 2D/3D registration can be achieved by exploiting the unique geometrical constraint between the camera and the light source in bronchoscopic procedures. Shape-from-shading techniques utilise this information to
recover surface structure that can be described by \((p,q)\) vectors. This approach directly relates the intensity information available in endoscopic image frames to the 3D CT data. Based on this hypothesis, a novel \(pq\)-space based 2D/3D registration technique is developed. In the specific case of using perspective projection with a point light source near the camera, the use of intensity gradient can reduce the conventional shape-from-shading equations to a linear form, which suggests a local shape-from-shading algorithm that avoids the complication of changing surface albedos. Albedo is a unitless measure indicative of the surface’s reflectivity. We have demonstrated in this chapter how to effectively use the derived \(pq\)-space distribution to match to that of the 3D tomographic model. The validation of the accuracy of the method is based on the positional and angular errors by using 6 DoF Electro-Magnetic (EM) tracking as the reference. The major advantage of the proposed \(pq\)-space based method is that it depends neither on the illumination of the 3D model, nor on feature extraction and matching. Furthermore, the temporal variation of the \(pq\) distribution permits the identification of localized deformation, which offers a means of excluding these areas from the registration process.

Since 2D bronchoscope video only provides a localised view of the inner surface of the lumen, the exact 3D location of the structure it represents is ambiguous. Different segments of the airways may well have a similar local structure. Therefore, the use of temporal information to derive the trajectory of the bronchoscope camera in 3D space is important for resolving such ambiguities. Another advantage of using temporal correspondence is that the estimates of the camera’s orientation and position can be used to accelerate the registration process. In previous research, Kalman filtering has been used but it is generally restricted to situations where the probability distribution of the state variables is unimodal. In bronchoscopy, tissue deformation, inter-reflection, and view dependent specularity due to mucosa can influence the accuracy of registration algorithms, and the resultant probability density function of the state vector can be multi-modal. In Chapter 5, a predictive tracking algorithm based on the Condensation algorithm has been developed. The method is designed to cater for the general situation when several competing observations form a non-Gaussian state-density. It uses a stochastic approach that has very few restrictions on the system/measurement models used and the distribution of error sources. An autoregressive algorithm is used as a predictive model, which is based on the fact that during bronchoscope navigation, the motion is restricted to a bounded area and a rapidly moving camera is more likely to slow down or change direction, rather than accelerate further. The method provides a systematic learning procedure with modular training from the ground truth data such that information from different subjects are integrated for creating a dynamical model that can accommodate the learnt behaviour.
The results presented in Chapters 4 and 5 show that the proposed \( pq \)-space based 2D/3D registration combined with temporal tracking are effective, but they can be problematic when large airway deformation is encountered. This, however, is common in practical examinations due to extreme breathing and deformation (such as coughing) of the patient. With the recent advances of miniaturised EM tracking devices, it is now possible to insert these devices into the biopsy channel of the bronchoscope to provide \textit{in situ} measurement of the camera pose of the bronchoscope. In Chapter 6, issues related to the practical use of the EM tracker are discussed. Since airways are highly deformable and their shape is affected by respiratory motion, the use of the EM tracking data must first consider the alignment of the fixed EM coordinates with the moving frame-of-reference of the bronchoscope camera. In this chapter, both respiration and patient movements are considered, and a global and local respiratory motion decoupling technique is proposed.

After global and local frame-of-reference alignment, the positional and orientation data derived from the EM tracker can be used for 2D/3D registration. This significantly enhances the robustness of the technique as temporal tracking is now more immune to discontinuities caused by abrupt airway motions. This provides the scope in exploiting the improved accuracy to examine detailed local deformation of the airways. To address the high degrees-of-freedom involved in bronchial deformation during the 2D/3D registration process, an active shape model has been developed in Chapter 7 to capture the principal modes of airway deformation during respiration such that the registration process can be implemented with a much reduced parameter space to allow for simultaneous registration and deformation tracking.

Finally, limitations of the current technique and possible improvements of the proposed registration framework are discussed in Chapter 8. To our knowledge, this is the first systematic study of 2D/3D registration incorporating airway deformation with comprehensive phantom and patient data validation. The work presented in this thesis has been included in a number of peer-reviewed academic journals and conference proceedings.


Medical Image Computing and Computer Assisted Intervention (MICCAI06), Copenhagen, Denmark, 2006.


Collaboration with other colleagues of the group has also led to joint publications that are related to the project. They include:


Minimally Invasive Surgery (MIS), also known as minimal access surgery, refers to surgical operations that are carried out through natural body openings or small surgical incisions. Bronchoscopy and laparoscopy are two examples of MIS. The first laparoscopic cholecystectomy was performed in 1987 by Dr Phillipe Mouret and it is widely accepted that MIS is the most significant change in surgical practice since the introduction of aseptic technique and safe anaesthesia [1, 2].

MIS has become possible through a combination of several technological advances in optics, illumination systems, insufflation devices, and imaging techniques. In 1954, Harold Hopkins, published in Nature an article reporting the transmission of images along tiny glass fibers and produced the first fibre-scope image. Five years later, he described a method that could efficiently transmit light through a solid quartz rod without introducing heat [3]. The Hopkins rod lens system and the subsequent miniaturisation of video cameras have allowed surgeons to visualise inside the body with clarity that is comparable to that of open surgery. In the early 1970s, flexible endoscope found widespread applications in surgery. At the same time, insufflation devices have also been developed to allow controlled distension of body cavities with CO₂ gas to provide surgeons with improved work-space.

With MIS, improvements in diagnostic accuracy and therapeutic outcome are significant when compared to conventional techniques. Firstly, small incisions lead to less post-operative pain and a reduction in the morbidity due to immobility. Secondly, patient trauma
and the duration of hospitalisation are greatly reduced. Thirdly, the small port-holes of MIS surgery lead to improved cosmetic results. There are also a number of other advantages of MIS, which include improved visualisation of inaccessible areas, minimisation of the risk of adhesive intestinal obstruction, and a decreased inflammatory response.

However, MIS can also cause a number of complications if it is not handled properly. The creation of a gas filled cavity, as well as the prolongation of the operation can carry certain risks [4, 5]. It can also cause significant mental and physical stress to the surgeon by standing in a fixed position for long hours. Most importantly, advanced surgical skills and manual dexterity are required for MIS and they are often associated with a steep ‘learning curve’ [6]. Complications usually occur early in the surgeon’s overall experience of MIS, or when an experienced surgeon is expanding into new procedures.

MIS requires a range of skills that are different from those used in open surgery. This is due to the fact that the complexity of the instrument control, restricted vision and mobility, difficulty in hand-eye co-ordination, and the lack of tactile perception are major obstacles in performing MIS. In MIS, the view of the operative field is displayed on a 2D monitor that is widely separated from the field of action. The two-dimensional view of a three-dimensional field has to be interpreted and synchronized with instrument movement. Depth information has to be extracted from 2D image cues in order to be able to navigate inside the cavity. In laparoscopy, the surgeon also has to adapt to the fulcrum effect, whereby the tip of the instrument moves in a direction that is opposite to the surgeon’s hand around the port. The fulcrum effect causes a fundamental visual-alignment conflict that requires extended practice. Subsequently, a high degree of manual dexterity and visual-motor coordination from the operator is essential to the success of the laparoscopic procedure. Since the acquisition of these skills is fundamental to the success of performing MIS, effective training is important.

2.1 Elements in MIS

To design an effective surgical training scheme, it is important to acquire knowledge concerning the basic skills that must be trained and assessed. Methods of identifying component skills in a complex domain include consultation with experts and task analysis. While experts can provide information on strategies, key skills, critical steps, and common errors, surgical experience mainly consists of procedural knowledge in the form of
perceptual-motor or spatial skills and information management capabilities. These aptitudes cannot be fully described verbally and can only be initiated by performing the task repeatedly. It is important to note that the skills of an experienced surgeon are not always easy to identify, since complex actions learnt at a subconscious level are difficult to be broken down into component parts. Consequently, it is not trivial to generate task-oriented analyses that capture the complexity of relationships among perceptual, motor, and spatial skills. This is further complicated by their compounding effect with external factors such as the experience of assistants and the quality of equipment used. Currently, there is a general lack of a solid theoretical base for the teaching and assessment of surgical competence in MIS.

In general, acquiring the skills for MIS is more difficult than learning open surgical procedures [7]. MIS is more dependent on spatial abilities, since there is little perceptual information available. Direct vision is replaced by a video image and it requires the ability to appreciate depth from a 2D image using subtle visual clues. Because instrument positions are continuously changing, so does the relationship between visual and instrument coordinates. Dexterity is diminished and kinaesthetic feedback of the interaction forces between instruments and tissues is also reduced. Tactile sensation, which is especially useful to gauge hidden lesions or vessels embedded in fat, is unavailable. Consequently, there are fundamental changes in the required perceptual-motor skills. Endoscopic procedures also require the ability to create 3D mental models while viewing a 2D image [8]. Spatial orientation, complex visuo-spatial organisation, and other perceptual abilities are intimately involved in the effective performance of endoscopic procedures [9].

Spatial cognition is the study of how humans acquire, store, retrieve and process knowledge of the spatial properties of objects, events and places in the world. Spatial properties include location, movement, extent, shape and connectivity [10]. Spatial ability plays a significant role in surgical skills training [11]. Several studies have shown a strong correlation between standardized tests on spatial ability and performance ratings on a variety of tasks in open surgery and MIS. Surgeons develop a mental image of the 3D anatomy based on a surface view or cross-sections of X-rays, Computed Tomographic (CT), Magnetic Resonance (MR) or Ultrasound images. Based on this model and their experience and prior knowledge, they plan their strategy accordingly.

The performance of general motor skill is affected by three major factors - 1) cognitive abilities, 2) perceptual-motor skills, and 3) information processing and decision making. Cognitive abilities, such as perceptual awareness, may be more relevant during the early stages of motor learning, and psychomotor abilities may become important in later stages.
when cognitive problems have been solved. The relationship between visual perception and motor abilities is the focus of skill assessment, since it is related to the speed and accuracy, as well as the coordination of the movements for performing MIS.

Perceptual-motor skills, also known as psychomotor skills, include, but are not limited to, the following elements [12]: (1) multi-limb coordination, which is the ability to coordinate the movements of a number of limbs simultaneously; (2) control precision, i.e., the ability to make highly controlled and precise muscular adjustments; (3) response orientation, i.e., the ability to select rapidly where a response should be made, as in a choice-reaction-time situation; (4) reaction time, i.e., the ability to respond rapidly to a stimulus when it appears; (5) speed of arm movement, i.e., the ability to make a gross rapid arm movement; (6) rate control, i.e., the ability to change speed and direction of responses with precise timing in following a continuously moving target; (7) manual dexterity, i.e., the ability to make the skilful, well-directed arm-hand movements that are involved in manipulating objects under speed conditions; (8) finger dexterity, i.e., the ability to perform skilful, controlled manipulations of tiny objects involving primarily the fingers; (9) arm-hand steadiness, i.e., the ability to make precise arm-hand positioning movements where strength and speed are minimally involved; (10) wrist, finger speed, i.e., the ability to move the wrist and fingers rapidly; (11) aiming, i.e., the ability to aim precisely at a small object in space; (12) visual acuity, i.e., the ability to see clearly and precisely; (13) visual tracking, i.e., the ability to follow a moving object visually; and (14) hand-eye coordination, i.e., the ability to perform skills requiring vision and the precise use of the hands.

Basic psychomotor and information processing aptitudes with relation to learning fibre-optic endoscopy with the video-endoscope are presented by Dashfield et al. [9]. Psychomotor abilities, such as manual dexterity and hand-eye co-ordination, however, appeared to be determinants of trainees’ initial proficiency in endoscopy, but did not appear to be determinants of trainees’ rates of progress during early fibre-optic training. Psychomotor tests, such as adaptive tracking tasks, which measure hand-eye co-ordination and dexterity, are particularly relevant. However, the ability to process and use a sequence of fast-moving images is not a key aptitude when performing traditional fibre-optic endoscopy. It is conceivable that spatial orientation, complex, visuo-spatial organisation and other perceptual abilities are intimately involved in the skilful performance of fibre-optic endoscopy. There are evidences that the mean half-life for fibre-optic nasotracheal endoscopy learning is approximately nine endoscopies, and subsequently an average trainee needs to perform at least 45 endoscopic sessions before he/she approaches the asymptote or ‘expert time’, when the learning process may be considered almost complete [9].
In laparoscopy, perceptual and information processing abilities are more accurate predictors of operative skills among surgical residents than psychomotor aptitude. Psychomotor abilities may account for up to one-third of the variations in early endoscopy performance. It is still poorly understood how surgeons learn and adapt to the unusual perceptual motor relationships in MIS. The perceptual (both visual and haptic) cues that surgeons use are complex and some of these cues, such as subtle lighting changes or differences in tissue consistency, are difficult to identify and reproduce.

The notion of a competent surgeon combines other excellences in anatomy, physiology, pathology, communicative skills, and decision making [13]. It has been estimated that decision making accounts for 75% of the successful completion of a surgical operation. The importance of decision making is further highlighted when the surgeon is faced with complications that are rare in routine procedures. In other words, the surgeon needs to carry out unsupervised operations that they had never performed before. Such skills cannot be learnt by example, since they may only appear once in a lifetime. Other essential elements of the effectiveness of a training scheme include a positive mental attitude and the ability to focus one’s attention. Team dynamics and the attainment of a cohesive group are also important factors to consider.

2.2 Surgical Education and Skills Assessment

Constructing a structured programme that provides the means to develop the necessary skills to independently perform MIS is by no means a trivial task. The learning process is inherently a complex procedure that involves a number of stages. The acquisition of psychomotor skills generally requires three steps:

- Cognition - in the sense of perceptual awareness;
- Integration - the task of comprehension of mechanical principles;
- Automation - which includes, speed, efficiency and precision.

There are also various different styles of learning such as concrete experience, abstract conceptualisation, active experimentation, and reflective observation. It has been reported that deficiencies in the teaching and learning of motor skills are unlikely to be corrected, unless there is some mechanism to provide a reliable and systematic feedback.
It has been noted that retention of motor skills appears to be most dependent on the degree to which the skills are perfected, rather than on variables such as the environment. This implies that many of the basic skills required for surgery can be acquired away from the operating theatre [7]. The ability to practice independently to the real procedure is important when surgical tasks need to be performed repeatedly in order to be perfected. Variability in practical experience is important for learning motor skills [12]. This includes variations of the characteristics of the context in which the learner performs the skill, as well as variations of the skill he or she is practising. There is evidence that successful future performance of a skill depends on the amount of variability the learner experiences during practice. Existing research shows that a greater error rate during the initial learning stage results in a better latter performance. However, similarly to musical education, repetitive performance of a specific task is necessary in certain circumstances and it cannot be replaced by training in a variety of skills alone.

The amount of practice influences the amount of learning, although the benefit is not always proportional to the time required. The spacing or distribution of practice can affect both performance and learning of motor skills. Practising skills during shorter sessions may lead to better learning. Furthermore, surgical skills can be practiced as wholes or in parts. Practicing the whole procedure can result in a better feeling for the flow and timing of all the component movements of the skill. On the other hand, practising the skill by parts reduces the complexity of the skill and allows the learner to emphasize on performing each part correctly before putting the whole procedure together.

Mental practice is also effective for learning skills, especially when combined with physical practice. It has been shown that electrical activity in the musculature involved in a movement as a result of the subject’s imagining of an action suggests that the appropriate neuromotor pathways are activated during mental practice. This process increases the likelihood that the subject will perform the action appropriately and reduces the demands on the motor control system as it prepares to perform the procedure. Mental practice can be beneficial especially in assisting the ability to consolidate strategies as well as to correct for errors.

In a surgical environment, it might be relatively easy to learn most steps of a procedure by observation and participating. In every procedure, however, there are a few key steps that are more likely to be performed incorrectly, resulting in complications. The significance of these steps might not be obvious, even to an experienced surgeon, until situations arise such as unusual anatomy or uncommon manifestations of disease. The value of a surgical simulator is analogous to the value of a flight simulator. In current practice, pilots are
certified to fly by dealing with simulated situations, such as wind shear or engine emergencies, that happen only once in a lifetime. A surgical simulator should train surgeons for the principal pitfalls that underlie the major technical complications. Such training and assessment could be used by medical schools, health administrations, or professional accrediting organizations to enforce standards for granting surgical privileges and for comparing patient outcomes with surgeon skills.

Clinical performance requires additional cognitive skills, abilities and behaviours that are not adequately reflected in objective measures of academic performance. In order to assess skills learning, it is important to realise that ‘learning’ is not directly observable. Inferences about the quality of learning are based on the performance of the trainee. “Learning is defined as a change in the capability of a person to perform a skill that must be inferred from relatively permanent improvement in performance as a result of practice or experience” [12]. Subsequently, ‘learning’ is measured based on the following criteria:

1. Improvement - performance of the skill shows improvement over a period of time. Note that learning is not necessarily limited to improvement in performance. When it is not appropriate, it can lead to ‘bad habits’ that result in poor performance of the trainee.
2. Consistency - as learning progresses, performance becomes increasingly more consistent. Subsequently, the acquired new behaviour is not easily disrupted by minor changes in personal or environmental characteristics.
3. Persistence - the improved performance capability is marked by an increasing amount of persistence.
4. Adaptability - the improved performance is adaptable to a variety of performance context characteristics.

It is common to use the ‘learning curve’ as an indication of adequate training. This curve shows the progressive improvement of a trainee over time or number of cases. However, a concrete and complete definition of what consists the ‘quality axis -Y’ of the learning curve has not been fully achieved [6]. The ‘X axis’ of the learning curve is also ambiguous. Furthermore, the learning curve usually monitors ‘improvement’ and ‘consistency’, whereas ‘persistence’ and ‘adaptability’ are generally ignored.

The performance of a trainee is a result of both his/her innate abilities and the training quality. The objective assessment of the innate psychomotor skills of a surgeon is a challenging research issue, as well as a controversial topic with both political and social implications [14]. Under the existing technology, an objective evaluation of the innate
abilities of a trainee is rather vague and leads to a controversy among the profession. It also raises questions such as if psychomotor deficiency is innate then it should be considered as a learning disability, and hence protected from the current legislation [14]. A number of researchers [15] support that training assessment should apply more in terms of detecting areas where surgical skills have not been learned efficiently rather than as a method of determining who will be allowed to progress in training. This is an interesting debate but it is out of the scope of this work.

Nevertheless, the need to objectively assess the surgical skills of the trainees before they are allowed to perform on patients is evident. However, it is rather unclear how to construct a valid strategy for evaluation. A test should be standardised, objective, consistent, sensitive and specific. The sensitivity of a test is defined as the proportion of the trainees that are correctly identified to meet the requirements. The specificity of a test is defined as the proportion of the trainees that are correctly identified that do not meet the specified requirements. Under the traditional apprenticeship surgical training, the performance of the trainee is evaluated subjectively from an expert according to the outcome of the operation. A number of studies use an objective evaluation scheme by introducing an examination board [16, 17]. The operations are either videotaped or observed directly. A common problem with this approach is that it requires a large examination team, which implies a high cost and further overload to the surgeons’ existing workload.

2.2.1 Traditional Surgical Education in MIS

Traditional surgical education is based on the apprenticeship scheme. Such educational programs depend on the time of the physicians and the flow of subjects. In most cases, they require five to seven years to complete and involve a relatively high cost, while there is a means of technical evaluation of the trainees’ skills until they perform on patients. Teaching in the operating theatre also takes time and this is in conflict with the need for fiscal restraint and a move towards more civilised hours of work for doctors in training. There is now little opportunity for either reflection or practice during a procedure and this has created the need for formal training outside the operating theatre.

Alternatively, animals or cadavers can be used in surgical training. Typically, anesthetised dogs, pigs or rats are used for training. Training based on animals is constrained by the inability to simulate pathology and their significant variation to human anatomy [18]. Further disadvantages include the concern about the transmission of infectious diseases and
the inherent costs involved. Nevertheless, ethical and humane reasons either prohibit or limit their use for routine training purpose and only used when there are no other alternatives [13]. Similarly, the use of cadavers is limited due to the restricted availability.

Synthetic models made of plastic or latex materials allow the trainee to practice basic skills, such as suturing and dissection, in a laboratory environment without the concern of patient safety and transmission of infectious diseases. However, they cannot give the appropriate physiological response, and thus do not provide students with constructive criticism. The realism is generally poor and therefore these models are only used during preliminary stages of the surgical education. The need to maintain a collection of different pathological cases and the fact that these models are usually non-reusable, can result in a prohibitive cost for most medical schools.

Typically, a surgeon gets certified by passing the Board Examination, which usually consists of a multiple-choice test and an oral examination [19, 20]. Surgery is a high-risk and high-cost environment where eminent technical skills, rapid decision making, and crisis management are of significant importance. In addition, advances in surgical technology have permitted the performance of procedures with increased complexity. Therefore, examinations that evaluate theoretical knowledge only are inadequate for assessing trainees’ abilities in the operation room. There are studies showing that the surgical outcome is significantly worse on the first procedure performed by an inexperienced surgeon as the management of many of the complications cannot presently be taught.

2.3 Medical Simulators and Virtual Reality

Medical simulators provide an attractive means of overcoming the inherent difficulties and limitations of traditional surgical training techniques. Apart from their relatively low cost, they have the potential to provide a comprehensive educational curriculum with a complete database of usual as well as rare and highly hazardous pathological cases. This significantly reduces the time required for the completion of the educational program [21]. They offer the opportunities for informative feedback, repetition, and correction of errors during a well-defined task with an appropriate difficulty level, allowing for practice in a safe environment. These training procedures would otherwise require numerous subjects and hours of supervision [22]. They also give the user a control over the learning process, such
that a trainee with a slow learning curve can safely achieve the same level of expertise compared to the more advanced practitioners.

One important characteristic of computer-based medical simulators is that they have the potential to integrate in one system numerous elements that are essential to MIS training and assessment [23]. General knowledge as well as psychomotor skills can be taught through a well-designed computer simulation. For example, when a trainee performs fiberscope bronchoscopy, it is equally important to have a mental model of the anatomy of the airways and the relation between the pre-operative CT images, as well as the manual dexterity to comfortably navigate inside the tracheo-bronchial tree. Surgical simulators can be used to repeatedly practice individual skills for simple tasks, as well as for more complicate procedures. The variability of the tasks they offer is potentially superior to the traditional training practice. They can also be tuned to provide feedback as well as objective assessment and suggest a different training program according to individual needs.

In addition, they have the potential to objectively evaluate the surgical skills of the trainees and compare them with the performance of the experienced surgeons. Indeed, there are a number of studies that use virtual reality based experiments to assess laparoscopic psychomotor skills [24-26]. Although, the current methodology is still in its infancy, it is widely accepted that this approach removes bias and subjectivity. Gallagher et al. assessed the psychomotor skills of a group of trainees and experienced laparoscopic surgeons using a simulator (MIST), which is common in laparoscopic training and assessment [25]. Typically, the position of the hands [13] or the tips of the instruments [27] are tracked in order to construct objective measures of the performance. Metrics based on common sense and practical experience, such as the number of movements made, distance travelled, speed of movement, and overall time are used to classify the participants into different categories according to their expertise.

Medical simulators may also be a valuable tool in understanding the development of perceptual-motor skills and their relationship to cognitive abilities in surgery. Because of the complexity of the tasks, surgeons appear to use different strategies and rely on various degrees of spatial and psychomotor processes. Studying the surgical performance in virtual environments can potentially elucidate these strategies and test the underlying hypotheses [11, 28]. Cognitive models of different strategies, both declarative and procedural, can be built based on trainees’ interaction with these environments. Subsequently, the structure of complex procedures can be broken down to explicit components to reveal the most efficient way of improving performance.
2.3.1 Current State-of-the-Art

Surgery involves many different processes and dissimilar tasks. This diversity, along with the rapid changes in technology and the continuing emergence of new procedures, has led to apparent differences in the type and capabilities of surgical simulators. Currently, surgical simulators can be classified according to their simulation complexity and application target. For example, the work by Liu et al. [21], as well as Satava et al. [29] classifies simulators as needle-based, minimally invasive, and open surgery. However, the application of Virtual Reality (VR) in medicine is not restricted only to simulation but it is also extended to general medical education, image-guidance, and pre-operative planning. Since these areas have many common requirements, the technological advance in either of them is usually broadly applicable.

In general, needle-based simulators are focused on teaching relatively straightforward tasks, which involve the insertion of small bore instruments, such as catheters and guide-wires. They are characterised by a relatively simplistic computational model that allows for visual and haptic feedback. Examples in this category include intravenous catheterization, pericardiocentesis and diagnostic peritoneal lavage. However, most of the available surgical simulators target MIS procedures. This is due to the nature of the application, which requires advanced psychomotor skills and complex instrument control. Open surgery is more difficult to simulate because it requires larger incisions in the body and the surgeon has direct visual and tactile contact with the region of interest. In open surgery the visual field view, the range of haptic feedback, and the freedom of motion are considerably larger compared to MIS procedures. Extensive advances in haptics, real-time deformation, biomechanical modelling, and visual rendering are required before open surgery can be simulated realistically.

2.3.1.1 VR in Surgical Simulation

VR is an artificial environment created with computer technology that allows the user to interact within. These systems have been extensively used in computer games and 3D visualisation to imitate reality. Human-machine interaction includes visual displays, tracking, tactile feedback, force feedback, and auditory displays. Usually a sense of 3D is conveyed by mimicking binocular vision to give a stereoscopic effect. Tracking technology
is usually used to track the position of the head and update the images accordingly. However, tracking can also be used to report the position of the hands, tools, and the cameras. Tactile and force feedback has been incorporated in many VR systems to provide a feel of the surface texture and material characteristics. Auditory feedback is also used to convey additional information, e.g., patient’s discomfort due to inappropriate manipulation of the inserted instruments.

Flight simulators are among the most successful VR systems used today. Their success and widespread use indicate that the current technology can also be used in medicine. An overview of surgical simulators can be found in the work of Satava et al. [29]. Early surgical simulators provided interactivity in a simplistic environment based on drawings of the underlying anatomy. One example is the cholecystectomy simulator called KISMET which was originally designed to be used with a telesurgery system.

In the current simulation systems, medical imaging data is used to reconstruct the 3D anatomy. They provide an accurate anatomical representation that incorporates tissue properties. Several simulators have been developed for a range of surgical procedures, such as hysteroscopy, laparoscopic cholecystectomy, bronchoscopy, gastrointestinal endoscopy, sinus surgery, limb trauma, and intravenous insertion. For example, in colonoscopy, one of the most well known simulation systems is the Simbionix simulator [30], which uses real endoscopic procedures to extract lumen texture, which is later applied uniformly to the 3D geometric model of the colon. Global and local deformation is also present and the entire configuration of the colon changes according to tissue instrument interaction. Similar systems have been also developed for bronchoscopy procedures [31], but their visual fidelity and physiological realism have so far been limited. Both texture and physiologic response are simplistic and repetitive, and thus cannot reproduce the complexity and the level of difficulty that is present in a real bronchoscopic procedure.

2.3.1.2 Augmented Reality in Surgical Simulation

In order to enhance the available in vivo information with pre-operative data and prior knowledge during real surgical procedures, Augmented Reality (AR) has increasingly been used in MIS. Usually, this requires merging pre-operative data with in situ - in vivo information. AR is important because it enhances the user’s perception and interaction with the real world. For example, AR systems have been developed to superimpose ultrasound images of a foetus in the mother’s womb [32]. AR systems have a number of possible applications in medical visualisation, planning, as well as training.
Accurate registration between the real and virtual world is a crucial issue for AR. Human vision is very sensitive to even small mis-registration errors or synchronisation lags. For this reason, tracking systems are of great importance in AR systems. In indoor applications, hybrid tracking is usually employed, which involves both magnetic and video sensors. Systems delays, also known as time latency, are often the largest source of registration errors. Prediction models are usually employed to reduce these effects by combining multiple tracking models. AR systems generally require extensive calibration to produce accurate registration. This includes measurement of camera parameters, sensor offsets, and field distortions.

### 2.4 Patient Specific Simulation

Although the majority of the surgeons believe that there is a need for training outside the operating room, the medical profession has been slow to incorporate simulation techniques into surgical training programs on a widespread basis. Their expectations are not fulfilled by the current state-of-the-art in simulation technology. Concerns include visual and haptic fidelity as well as inadequate validation of effectiveness and potential requirements for frequent system upgrades [33, 34]. Thus far, validation is limited to qualitative results due to a lack of the ground truth data. In current virtual simulators, there is also a lack of reliability, especially in terms of skill assessment. For example, various simulation systems indicate that the difference between experienced surgeons and novices is diminished after the first three-to-five trials [25], but in practice the first twenty trials have a significant rate of complications and errors. This indicates that most of the techniques developed so far are not adequate to classify surgeons and identify all the factors that contribute to a successful performance. This is due to the lack of visual and physiological realism, which is important for decision making in real operating environments.

To reach the goal of a truly realistic virtual simulator, it is necessary to create an environment that is not possible to distinguish between real and virtual [19]. In AR, different degrees of realism are classified and they include: (1) visual reality, photorealism; (2) physical reality, which presents tissue deformation and realist reaction to the underline forces; (3) physiological reality, such as bleeding upon cutting, bruising and peristalsis; (4) tactile reality, the trainee senses the forces during operation [13]. This implies a transformation from generic to patient-specific models that have a realistic sight, touch or feel even when they are dissected. Moreover, they should provide full procedural training.
and not just accomplish the requirements of developing simple motor skills [34, 35]. Patient specific models built from medical imaging data offer significant scopes for the practice of complex procedures prior to working on the patient directly.

2.4.1 Challenges in Surgical Simulation

Robert Mann proposed the first medical VR system as early as in 1965. His aspiration was to develop a rehabilitation application to allow surgeons to try out multiple surgical approaches for a given orthopedic problem [19]. This approach requires not only the model to be patient-specific but also to provide an accurate representation of the tissue-instrument interaction over time. Whilst early simulators have often been criticized for being too simplistic, recently technical advances have allowed the implementation of such a vision [35, 36]. This section presents the technological progress in the four basic components that constitute a realistic surgical simulation environment including visual, biomechanical, physiological, and haptic realism. It also investigates what has yet to be achieved in surgical simulation and the role of patient specific simulation models.

2.4.1.1 Photorealism – Visual Fidelity of Complex Structures

Visual perception is the primary information available to surgeons especially in MIS procedures. Surgeons depend on scene appearance to determine tissue properties, instrument pathways, and necessary interactions. Usually, disease identification is based on distinctive texture appearance that guides the biopsy procedure. It is not surprising that visual realism, also referred to as photorealism, plays a major role in MIS training [37 2005].

Visual realism is usually described as the ability to represent a virtual surgical view in a way that cannot be distinguished from a real surgical view. However, for the purpose of MIS training procedures, it can also be represented as an information channel. A virtual environment that provides less information than in real situations would not allow the trainee to develop the appropriate skills to process the missing information. On the other hand, a virtual environment that provides too much information than is normally available would oversimplify the procedure. Subsequently, the trainee would rely on this extra information and not develop alternative methodologies and tactics to acquire the missing perceptual data in a real case scenario. For example, impairment of visual realism, due to
homogenous texture mapping and a lack of an appropriate illumination model can lead to a misunderstanding of the tissue anatomy. In laparoscopy, for example, misunderstanding of the anatomy of ducts between the liver and gall bladder can lead the surgeon to cut the wrong duct during gall bladder removal.

Morphological geometry, surface properties, and lighting conditions need to be modelled for all scene objects in order to attain a realistic appearance. This involves a number of different components such as an organ’s boundary representation and modelling, illumination, surface properties, shadows, anti-aliasing and colour/texture perception. In early simulation systems, organ surface appearance was mainly based on synthetic texture. For example, in the work presented by Cakmak et al. [38], synthetic texture and procedural turbulence functions are combined with grammar-based tree growing algorithms to give the impression of organ surfaces with fine arterial and venous trees. However, synthetic texture provides homogeneous surface with little or no more information than direct rendering of the 3D tomographic data. Recently, image-based rendering has been applied to surgical environments to increase the visual realism by accounting for global illumination and inter-object reflections [37 2005]. These techniques have achieved improved results, but issues related to visibility, reconstruction, as well as the huge data storage requirements have yet to be resolved. Since these methods do not explicitly represent geometry, the extension of these techniques for simulating physiological tissue response after cutting and dissecting is not trivial.

2.4.1.2 Tissue Deformation and Biomechanical Modelling

Tissue and human organs are elastic and they deform considerably during surgical operations. In any attempt of creating a meaningful learning experience, simulators must model this appearance and the behaviour of soft tissue in response to instrument interaction. Accurate modelling of biomechanical tissue properties is important for effective surgical training, planning and outcome evaluation [39]. Thus far, a number of physical and non-physical based deformable models have been proposed.

However, the quest for both photorealism and tissue deformation is not a trivial problem. Real-time simulation of soft tissue deformation is still the major obstacle in surgical simulation. Most applications sacrifice photorealism for achieving the interactive rendering rates required. Soft tissue is a non-linear viscoelastic anisotropic active material and in tissue modelling there are two fundamental problems that need to be addressed, i.e., how to measure tissue properties and how to create a realistic mathematical model with fast
convergence. The concept of strain, a measure of length change or displacement gradient, and stress, the force per unit area on an infinitesimally small plane surface within the material, are of essential importance in estimating biomechanical properties of the tissue and subsequent physiological modelling.

Physical-based models can incorporate material properties and they are indeed the most common methods used in surgical simulation. Mass-spring and Finite Element Models (FEM) fall into this category. Mass-spring model consists of a network of point masses connected by spring-dampers. They can be relatively large and complex without sacrificing real-time response due to user interaction. However, they generally have limited precision and realism, and can lead to numerical instability. In contrast, FEM can model tissue properties more accurately and it differs from the mass-spring models in that it permits a continuous formulation relating deformation to energy, and thus computes deformation over the entire volume instead of discrete points. The main disadvantage of the method is computational complexity.

In practice, measuring tissue properties is not trivial as different values can be obtained from one individual to another. Different forces, with a range similar to that of the real procedure, are exerted on selected tissue regions and the resulting displacements are recorded. Material parameters are then computed from the plotted force/displacement or stress/strain, relationships. In general, in vitro tissue characterisation is easier to achieve but the measured tissue properties can be significantly different to that of in vivo.

Alternatively, kinematic or non-physical models can be used to model tissue deformation. These approaches do not consider the actual physical characteristics but are based on abstract geometric manipulation under mathematical constraints, such as surface continuity and volume preservation. Examples of such techniques include parametric surface and Free-Form Deformations (FFD). Parametric models are usually based on splines and patches, such as Thin-Plate Splines, B-splines, and Non-Uniform Rational B-Splines. They are simple to manipulate and easy for collision detection. However, deforming the underlying mesh/volume based on these techniques usually involves a large amount of parameters and it is counter-intuitive for localised deformations. Furthermore, the inability to incorporate physical laws can result in a low level of deformation realism. Among these techniques, FFD provides a higher level of control by deforming the space where the objects positioned and it is a versatile tool for geometrical modelling of objects of different topologies.
Recently, Active Shape Models (ASMs) have been used to describe tissue deformation. They are parametric deformable models that deform in ways that are consistent with the class of motion they represent [40]. The intuitive benefit of the use of ASMs is that surface points of the underlying tissue are not moving independently, so the intrinsic dimension is significantly decreased than the number of control points. Although there is no need to measure tissue properties, they can accurately model both local and global deformation based on a training data set. ASMs are simple to build and manipulate, thus facilitating the development of patient-specific deformation models.

2.4.1.3 Physiological Response

Realistic physiological response due to pathology and surgical interaction plays a major role in surgical simulation. Usually these responses guide a surgeon’s decisions and instrument control. Presentation of accurate physiological cues and information on a consistent manner is crucial for MIS simulation. Most current simulation systems are of limited use for advanced skills training as they cannot reproduce accurately physiological responses and surgical interaction.

The challenges need to be addressed in order to produce behavioural realism are related to the physics of surgical manoeuvres such as cutting, tearing of soft tissue, and appearance of organic fluids and bleeding. Cutting and tearing of soft tissue is related to tissue biomechanical properties, whereas modelling bleeding involves computational fluid dynamics combined with stochastic processes. Bleeding is among the most difficult phenomena to simulate realistically. However, it is of great importance in MIS simulation since it affects the surgical environment by obscuring the image view and imposes risks to the patient. In simulation, bleeding has to be real-time and responsive to the action of the surgeon [41]. Instead of direct modelling, image-based approaches have already been used in simulation of bleeding for transurethral resection of the prostate [42]. These techniques use video sequences of blood flow as the source datasets that are positioned, oriented, and morphed to an existing virtual model of anatomy. Although these approaches are promising, further investigation is required in order for it to be applied to more complex procedures.
### 2.4.1.4 Haptics

Some of the essential cues used by surgeons for performing a surgical procedure are based on haptic feedback. In MIS, the sense of touch compared to open surgery is reduced. Surgeons must rely more on the feeling of the net forces resulting from tool-tissue interactions and they need more training to successfully operate on patients. In a clinical environment, force and visual feedback cannot be separated [43]. From open surgery to needle-based procedures, *force* and *tactile feedback* is important to permit the surgeon to apply appropriate tension to avoid damaging surrounding structures [21, 44]. Haptic feedback is important for developing certain motor skills. The integration of computer haptics in medical simulators, however, is challenging. It not only involves the task of developing software algorithms that synthesize computer generated forces to be displayed to the user for manipulating virtual objects through touch [45], but also involves the generic design of haptic hardware, tissue modelling, tool-tissue interactions, real-time graphical and haptic rendering, and measurement of tissue material properties [44].

Typically, the user interacts with the virtual environment through a haptic interface device. Information conveyed by the haptic interface includes position, velocity, acceleration and force. The force and motion involved are small but require the resolution to be fine and the device to be mechanically transparent. Therefore, realistic haptic reproduction can only be attained at high feedback frequencies around 1000\(\text{Hz}\). Furthermore, in modelling deformable objects, the device needs to estimate not only the direction and the amount of deformation at each node, but also the magnitude and direction of interaction forces that will be reflected to the user. Geometric contacts between haptic forces and objects in the virtual environment have to be detected. Contact forces determine how the object is perceived by the user in terms of stiffness and surface properties. Empirical investigations of *in vivo* tissue mechanics are critical to measure material parameters needed for realistic simulation. However, most current organ-force models are linear, homogeneous, and isotropic. They are inadequate to represent the inherent nonlinearities, anisotropy and rate dependence of the soft tissues [35, 44]. Haptics remain an active research area and yet many technological advances have to be achieved in order to provide users with a realistic sense of touch in complex scenes.
2.5 Simulation in Bronchoscopy

The tracheo-bronchial tree as part of the respiratory system is one of the most vital organs of the human body. The pulmonary-respiratory system is responsible for gas exchange between the external environment and the circulating blood supply in the body. This utilises a circulatory system to transport gases and energy to, and wastes from the tissues. Diseases such as cancer, emphysema, fibrosis, bronchitis, pneumonia, and asthma can disrupt normal circulation. For example, they alter the elastic properties of the lung tissue; block airways or blood vessels, changing the diffusing capacity of gas exchange surfaces; modify muscle function of the rib cage or diaphragm, altering the central nervous system processes that govern respiration. Bronchoscopy is the gold standard imaging modality in detecting and evaluating cancer in the tracheo-bronchial tree. Bronchoscopic images are used to identify stenoses and tumours, direct intra-airway interventions, and guide biopsy needles to suspected tumour sites. However, it is not a well tolerated procedure and training on the patient can result in prolonged operation and considerable discomfort. Virtual and augmented bronchoscopy has a number of advantages to offer in terms of operator skills training and assessment. In this section, the basic airway physiology and function are reviewed in order to provide an overview of the basic anatomical and mechanical properties of the airways and the potential complications related to operation of the fibreoptic bronchoscope.

2.5.1 Airways Anatomy and Physiology

The respiratory system is made up of the organs involved in the interchanges of gases and consists of the nose, pharynx, larynx, trachea, bronchi and lungs. Anatomically, it is divided into the upper and lower respiratory system. The upper respiratory system includes nose, nasal cavity, ethmoidal air cells, frontal sinuses, maxillary sinus, larynx and trachea. The lower respiratory tract includes the lungs, bronchi, and alveoli. The lungs are a pair of cone-shaped organs made up of spongy, pinkish-grey tissue. They take up most of the space in the chest, or the thorax, which is the part of the body between the base of the neck and diaphragm. The lungs are enveloped in a membrane called the pleura. The right lung has three sections, called lobes. The left lung has two lobes. When you breathe, the air enters the body through the nose or the mouth. It then travels down the throat through the larynx and trachea and enters into the lungs through tubes called main-stem bronchi. One main-stem bronchus leads to the right lung and one to the left lung. In the lungs, the main-stem
bronchi is divided into smaller bronchi and then into even smaller tubes called bronchioles. Bronchioles end in tiny air sacs called alveoli. The lungs take in oxygen, which cells need to live and carry out their normal functions and discard carbon dioxide, a waste product of the body's cells.

The human tracheo-bronchial tree consists of a set of connected, branching tubular structures that tend to decrease in diameter as the branching progresses as it is depicted in Figure 2.1. It is connected to the outside through the trachea, which splits into the main bronchi, right and left lung at the main bifurcation. The bronchial wall surrounds the air-filled lumen of the bronchi and its thickness is relative to the diameter of the airways. In humans, there are roughly 23 generations of bifurcations from the trachea down to the alveoli. An accurate and complete model of the tracheo-bronchial tree is difficult, since the structure of the airways differs between individuals. However, the most useful approach remains that of Weibel et al. [46], which numbers successive generations of air passages from the trachea (generation 0) down to the alveolar sacs (generation 23). Recent studies have shown that a regular dichotomy occurs for the first six generations, which means that each bronchus divides into two approximately equal-sized daughter bronchi [47]. The adult trachea has a mean diameter of 1.8cm and length of 11cm. The trachea and main bronchi are supported by U-shaped cartilages which are joined by smooth muscle bands. From the second to eleventh generation, the vessel wall appears irregularly shaped and helical. The small bronchi are extended between the fifth and eleventh generation and their diameter falls from 3.5 to 1mm.

The intra-thoracic airways of the lung are highly deformable. Unlike most other organs, they undergo large amplitude deformation even during normal breathing. On inspiration, active muscle-driven expansion of the rib-cage and diaphragm leads to expansion of the airways, which draws air into the lungs. On expiration, muscles relax and the lung typically contracts passively via its natural recoil, thus expelling the air. Air can also be actively pushed out of the lungs, for example during the respiratory manoeuvre called forced expiration [48]. During a cough, large airways such as the trachea and first several generations of airways tend to collapse dynamically, narrowing to a small fraction of their initial cross-sectional area.

Thus far, the basic mechanics of human airways are mainly studied to determine how their anatomy affects the function in normal and abnormal subjects. The structural properties and particularly bifurcation geometry of the airways are important for determining the stiffness of the lungs. Physiologists commonly use a simple physical analogue, called thin-walled elastic tube and a device, known as the Starling resistor, as a simplistic bench-top model of
a deformable airway to simulate flow dynamics [48, 49]. In the work of Kaye et al. [50] a simulation system that combines respiratory anatomy and respiratory mechanics has been developed based on a physical model. They consider lungs as a single-compartment, viscoelastic structure and the classical mechanical model is applied based on observable variables such as pleural pressure change, lung volume change, and airway pressure.

The lung is a highly elastic organ composed of fibres connecting the large airways, intricate vasculature and pulmonary interstice. Pathological processes that affect the lung typically alter the normal mechanical properties of lung tissue, and they are apparent as observable changes in lung morphology and function. The ability to quantify differences in pulmonary deformation is useful in the early detection of disease, evaluation of treatment efficacy, and improved assessment of disease staging and prognosis. Information about airways and how they function during inspiration and expiration can determine the causes of and suggest possible treatments for various pulmonary diseases [51]. The constriction of the intra-thoracic tree depends on a number of mechanisms related to the surrounding structures, as well as the biomechanical properties and anatomy of the elastic tissue of the airways. Radiological imaging has the potential to provide accurate and detailed description of the intra-thoracic tree during function. However, imaging of the human airways during respiration is not common because of technological limitations and patient safety regulations.

Figure 2.1: Main anatomy of the tracheobronchial tree.
2.5.2 Flexible Fibreoptic Bronchoscopy

Flexible Fibreoptic Bronchoscopy (FFB) is a commonly performed endoscopic procedure used for diagnosis and treatment of airway and pulmonary disorders. It allows the physician to directly visualize the interior passages of the lower respiratory tract through a bronchoscope. The basic reason for performing FFB is to detect anomalies in patient’s bronchus and to obtain a biopsy of the effected areas. Procedures are normally performed in fully awake patients or using light sedation. The bronchoscope is then inserted into the airways through either the nose or the mouth. Subsequently it passes through the vocal chords and reaches the main bifurcation, moving to the trachea. There are two types of bronchoscopes, flexible and rigid. The type of bronchoscope used determines the extent to which the bronchioles, smaller branches of the bronchi, can be visualized.

A rigid bronchoscope is a straight, metal, lighted tube capable of visualizing only the larger airways, thus limiting the diagnostic and therapeutic options available. However, certain conditions may warrant its use, such as aspiration of a large amount of secretions or blood, controlling significant bleeding, or removal of foreign objects and/or lesions (diseased tissue) within the bronchi. Generally, the rigid bronchoscope has been replaced by the flexible bronchoscope because it has less risk of traumatizing the tissue, thus improving patient tolerance and providing better access to smaller areas of the lung tissue. The design of the flexible bronchoscope is advantageous because it can be manoeuvred into the smaller bronchioles, yielding more information about their condition. In addition, the flexible fibreoptic bronchoscope has interior channels which increase the capabilities of treatment options, such as delivering oxygen, suctioning secretions, obtaining tissue samples during biopsy, instilling medications, and laser therapy. Recently, electronic video-scopes have been manufactured that use a video chip and strobe light mounted onto the tube for image capture.

Bronchoscopy may be performed for diagnostic or therapeutic reasons. Diagnostic indications include the detection of: (1) tumors or bronchial cancer; (2) airway obstructions or narrowed areas; (3) inflammation and infections such as tuberculosis, pneumonia, or fungal or parasitic lung infections; (4) interstitial pulmonary disease; (5) persistent cough or hemoptyisis; (6) abnormal chest x-rays; (7) biopsy of tissue or collection of other specimens; (8) vocal cord paralysis; and (9) bronchoalveolar lavage, or BAL, which implies the instillation of fluid through the bronchoscope to aid the diagnosis of certain lung disorders. Therapeutic uses of bronchoscopy, on the other hand, include: (1) removal of
secretions, blood, mucus plugs, or polyps to clear airways; (2) control bleeding in the bronchi; (3) removal of foreign objects or other obstructions; (4) laser therapy or radiation treatment for bronchial tumours; (5) stent placement; and (6) draining of an abscess.

During FFB, patients are awake although a sedative is generally given. Rigid bronchoscopy usually is performed in the operating room under general anaesthesia. Generally, a fibreoptic bronchoscopy procedure follows a standard process, which involves: (1) an intravenous (IV) line may be inserted in the arm or hand of the patient; (2) heart rate, blood pressure, respiratory rate, and oxygen level may be monitored during the procedure; (3) patient is positioned in a sitting position or lying on your back; (4) oxygen is received through a nasal cannula (tube) or face mask during the procedure; (5) slight sedative is given to make patient sleepy; (6) numbing medication is sprayed into the back of the throat of the patient to prevent gagging as the bronchoscope is passed down your trachea into the bronchi; (7) patients are not able to swallow the saliva that may collect in your mouth during the procedure due to the bronchoscope in your throat, the saliva is suctioned from their mouth regularly; (8) the physician advances the bronchoscope down the throat and into the airways, and as the bronchoscope is advanced, the tissues and structures are examined; (9) Patients experience some discomfort when the bronchoscope is advanced, but their airway is not blocked; and (10) the physician may obtain tissue samples for biopsy or other specimens for testing during the procedure.

As with any invasive procedure, complications may occur. Common complications related to bronchoscopy include bleeding, infection, bronchial perforation, bronchospasm or laryngospasm and pneumothorax, which is the state where air becomes trapped in the pleural space causing the lung to collapse. Major bleeding is the most common cause of death resulting from bronchoscopy and it accounts for the 3% of the total number of procedures [52]. Other procedures fail because the physician is unable to take a biopsy of the appropriate area. Needle and forceps biopsies are the most difficult tasks of the procedure. Needle biopsies are performed by inserting a needle into the bronchial wall to obtain tissue samples below the surface of the bronchus, while in forceps biopsies tissue sample are obtained from the surface using a small set of clamps. Biopsy presents a greater cognitive strain on the physician mainly because of the need to combine the mental mapping of the bronchial tree with the mental location of the particular site that changes from patient to patient. This is called biopsy location and usually it involves third generation navigation, locating the anomaly and determining the biopsy location. On the other hand, pre-biopsy navigation includes navigation through the first and second generations of the tracheo-bronchial tree and is typically the same for each patient.
Physicians operate the bronchoscope through the airways by controlling a lever that actuates the last few centimetres of the scope to bend the tip up and down. They also need to rotate the scope in order to gain a better control of the navigation and viewing angle. Mental image rotation imposes demands similar to the challenges of map orientation. It is a potential cause of physicians ‘getting lost’ in the patient’s bronchus, requiring more time to locate the area of interest within it.

Traditional educational schemes entail two to three years fellowship training after internal medicine residency. During fellowship training, novice bronchoscopists learn to manipulate the flexible fiberoptic or video bronchoscope by initially watching their attending faculty perform a procedure, and then by performing examinations themselves under supervision. Manual and technical skills, such as dexterity, speed and ability to thoroughly inspect tracheo-bronchial anatomy, are gradually acquired along with decision-making and interpretation skills through a combination of one-on-one interactions, patient-care experiences, and didactic lectures. Despite the widespread use of diagnostic flexible bronchoscopy, there are no firm guidelines that assure a uniform acquisition of basic skills and competency, nor are there guidelines to ensure uniform training and competency in advanced diagnostic flexible bronchoscopic techniques, such as transbronchial needle aspiration biopsy or fluorescence bronchoscopy [22]. More importantly, training using patients can result in prolonged bronchoscopy procedures, increased patient soreness and the risk for further complications and errors. More than 50% of the pulmonologists believe that their instructions in a newer diagnostic bronchoscopic procedure, such as transbronchial needle aspiration, had been insufficient during their training. Alternatively, animals and inanimate modes are used but they fail to simulate dynamic events such as respiratory movements, vocal cord closure, obstructing airway secretions, cough, or procedure-related hemodynamic compromise.

The incorporation of simulation technology for training bronchoscopists has transformed pulmonary procedural guidance and enhanced uniform quality of bronchoscopy training due to the following reasons [22]:

- It enables novices to attain a level of basic manual and technical skills when performing a thorough diagnostic bronchoscopic inspection of the tracheo-bronchial tree.
- It provides a short, focused course of instruction and unlimited, unsupervised practice.
• It has the potential of objective skill evaluation. Bronchoscopy simulations could distinguish levels of clinical experience between beginners and experienced bronchoscopists by assessing the duration of bronchoscopy, hands and tool motion and collisions with airway walls.

• Simulation devices provide equal opportunities of training in rare pathological cases, and decouple the dependency between patient flow and surgical training.

2.5.3 Augmented Bronchoscopy

The development of Virtual Bronchoscopy (VB) started over 15 years ago when the resolution of CT and associated radiation burden was amenable for routine examinations [53]. VB does not suffer from certain limitations of real bronchoscopic procedures such as the accessibility of distal airways. It has the unique ability of navigating through narrow airways and circumventing obstructing lesions. Applications of VB include screening airways for endo-luminal malignancy, evaluating airway stenoses, and pre/intra-operative surgical guidance. A detailed survey on the current systems can be found in the work by Bartz et al. [54], whereas Buthiau et al. [53] offers an extensive handbook of possible applications of VB as well as its advantages and limitations. Such an example is demonstrated in Figure 2.2 where a dental prosthesis is misidentified as a lesion due to the lack of texture information.

Traditional bronchoscopy displays only the inner surface of the airways and yields no information about the anatomy beyond the wall, which can be visualized in VB. The visualisation of the 3D relationship of the airways and the surrounding vessels has the potential benefit of facilitating needle aspiration biopsy procedures and general surgical guidance. The non-invasive characterisation of lesions as malignant or benign with Positron Emission Tomography has also brought new potential use of VB in non-invasive diagnosis of pulmonary abnormalities. Currently, the inability to provide detailed surface information is a major limitation of VB to be used for advanced skills training. To overcome this problem, VB should provide information that combines the information from both 3D CT scans and that of real bronchoscopy [55].
2.5.4 Technical Requirements of Augmented Bronchoscopy

The combination of 3D information derived from imaging with the bronchoscopic videos is the latest development in augmented MIS simulation environments. The basic steps involved in such a system are outlined in Figure 2.3. They include:

- High resolution imaging data of the airways
- Segmentation of the tracheo-bronchial tree
- Surface rendering of 3D geometry
- 2D/3D registration of the airways model to the in vivo bronchoscopic video
• Photorealistic rendering that involves the generation of novel views not previously acquired
• Navigation in real-time inside the tracheo-bronchial tree with haptic feedback.

Figure 2.3: Technical requirements of augmented bronchoscopy

2.5.4.1 High Resolution Image Acquisition of the Airways

Chest radiography is one of the oldest pulmonary imaging modalities. Radiography forms a 2D image that is the result of penetrating X-rays through the lung tissue. The outcome of the image is based on the fact that different tissues absorb different levels of radiation. For example, dense structures such as bones absorb more x-ray photons than air-filled lung tissue. Analysis of radiographic images identifies tissue discrepancies and abnormalities. Since the radiographic images are 2D projections of a 3D structure, there are problems with geometric ambiguities due to overlap. CT imaging eliminates these issues by constructing a high-resolution image representing the 3D volume of the body. It is formed as a number of cross-sectional images through the body by gathering many projections at different angles.

High Resolution CT currently dominates morphological lung imaging as it provides high spatial resolution and contrast of the airways. CT offers high quality images of sub-millimetre in-plane and through-plane resolution [56] and the entire airways can be scanned during a single breath hold, eliminating respiratory motion artefacts. The choice of CT reconstruction parameters, such as collimation width, smallest pitch and longitudinal resolution are important because they affect the size and the shape of the 3D model derived
HRCT allows the identification of airways that are 1-2 mm in size, suggesting the extent of the distal airways that the reconstructed VB can reach in practice [57].

CT entails a considerable radiation exposure, thus limiting its applications to static pulmonary images acquired during breath hold. In the work by Sera et al., the mouse lungs are visualised with micro-CT without dehydration and fixation in order to acquire high-resolution images during respiration. Pulmonary dynamics, such as air flow patterns and particle deposition, and the associated physiological implication were observed. Images with effective spatial resolution of $30\mu m$ were obtained with a total scan time of 15 minutes. In this study, not only small airways but also alveoli were visualised in detail. Such a technique offers valuable information for the respiratory system but the radiation burden is prohibitive for its use in human subjects.

In practice, there are several advantages of using MRI over CT, primarily due to the avoidance of ionising radiation and detailed high-resolution morphological and functional images that MRI can provide. Moreover, MRI has the potential to provide a higher sensitivity in detecting ventilation defects than other modalities [58]. Initially, MRI was not used for morphological lung imaging due to the lack of signal from the lung parenchyma. However, recent developments in ultra-short MR pulse sequences and the use of hyperpolarised gases have established the role of MRI in lung imaging [51]. Static pulmonary MRI involves imaging several sections of the lung during a single breath hold, which provides 3D information of the lungs. The depiction of airways by using static hyperpolarised gas is limited, because the lung periphery overshadows the signal from the airways. In contrast, dynamic pulmonary hyperpolarized He MRI consists of imaging the lungs during inspiration. The images are obtained as the gas flows through the airways to the lung periphery. There is limited time during the early inspiration phase when the gas travels solely through the airways. During this time, the signal will result only from the polarised magnetisation in the airways. After this point, the signal from the lung periphery can obscure the signal from the airways. With this technique, up to the seventh-generation of the airways can be visualised. Furthermore, this method has the potential to provide quantitative measurements such as airway diameters and volumes.

Experiments based on MRI to capture free-breathing airway function have been recently presented in the work by Sundaram et al. [59]. The images used in these experiments were acquired at a constant time interval while the volunteers breathed slowly and deeply, consciously controlling their respiration. Due to individual variations in respiration and the difficulties of real-time pulmonary imaging, the time window of each image acquisition bears little physiologic meaning. In this work, each subject’s expiratory image sequence
was re-parameterised with respect to normalised lung capacity. This was then combined with intra-subject pair-wise non-rigid registration to interpolate intermediate images in the normalised volume curve. Subsequently, an average representation of respiratory dynamics from free-breathing lung MRI of normal subjects was extracted.

2.5.4.2 Airway Tree Segmentation

For interactive rendering, segmentation of the intra-thoracic airways is necessary. Quantitative analysis of the extracted airway tree is also important for evaluating tree structure and function. Segmentation, which is used to identify the boundaries of an organ, is a common step in medical imaging. Although CT images are well calibrated, with air being at about -1000 Hounsfield Units (HU), bones at about +1000HU and soft tissue in the range of -100 HU to 200 HU, airway tree segmentation is a challenging problem. Image noise, partial volume effects, and movement artefacts in a complex branching structure make a fully automatic segmentation method difficult.

Partial volume effect occurs when a voxel represents more than one type of tissue. Due to the size of the voxel and image reconstruction artefacts, thin or stenosed airways can appear broken or discontinuous. Thus far, a number of strategies have been proposed [56, 60-62], but none of them can reconstruct accurately the lowest generations of the tracheo-bronchial tree. Most of them require the user to manually adjust segmentation parameters interactively during segmentation. The robustness of these methods is further restricted for low-dose CT images, where the signal-to-noise ratio is low. In general, current approaches to airway segmentation can be characterised into region-growing algorithms [63] and knowledge-based algorithms [64].

Region-growing algorithms build up a set of airway pixels by using intensity-based threshold and voxel connectivity from a ‘seed’. For airway analysis, the seed selection is achieved either manually or automatically. Voxels adjacent to the seed are added to the region if they satisfy the intensity criteria. Once a voxel has been added to the region, its neighbours are evaluated for selection and the procedure continues until no new luminal voxels can be added. Region-growing algorithms in general are not able to segment the tracheo-bronchial tree beyond the fourth generation because of the difficulties associated with connectivity and threshold selection. Furthermore, small airways oriented at an oblique angle to the imaging plane can also impose significant challenges. As the orientation of the branch tends to tilt in parallel to the imaging plane, the edge strength at the vessel wall is weakened gradually. This can result in the segmented tree ‘leaking’ out
into the surrounding lung parenchyma. As the diameter of the airways decreases, partial volume effects begin to increase, which greatly can affect the accuracy of threshold-based approaches.

Alternatively, knowledge based segmentation approaches have been developed, which use greyscale characteristics and shape information of the airways. In the work by Schlatholter et al. [62] a rule-based method for airway segmentation has been developed. The method uses the knowledge that airways and vessels often travel in parallel through the image dataset and a partial vessel wall is often visible around even the smallest detectable airways. While the sensitivity of the method is good, the associated specificity is low due to a large number of false airway detections. Fuzzy logic approaches have also been proposed [64-66]. These methods utilise fuzzy rules to describe image features such as region brightness, wall existence, and region connectivity. Fuzzy logic membership functions and fuzzy rules are constructed using real airway examples. The results show that fuzzy-logic methods can increase specificity and prevent leaking without sacrificing the underlying sensitivity of the method. Based on these approaches, a complete tracheo-bronchial tree up to the seventh generation has been reconstructed from low-dose CT data [65]. Given the tubular shape of the tracheo-bronchial tree, it is also natural to utilise the knowledge derived from central line extraction and thinning algorithms [67, 68] to enhance the robustness of the segmentation.

2.5.4.3 Volume and Surface Rendering

Advances in computer graphics, both in terms of algorithm design and hardware improvement, have led to significant progresses in volumetric reconstruction of medical images. In practice, two techniques are commonly used in visualisation. The first one is volume rendering which directly represents 3D objects as a collection of volume elements called voxels. Each voxel is a sample of the original volume and it is associated with one or more values quantifying the property of the object such as transparency, density, or velocity. The main advantage of this technique is in its ability to preserve the integrity of the original data. Therefore, visualisation of details beyond the tracheo-bronchial wall is possible with volume rendering. It is particularly useful in differentiating between benign and malignant structures. However, effective volume rendering is computationally expensive and it is difficult to achieve real-time response along with photorealistic rendering in most simulation environments without the use of dedicated hardware.
The second technique involves the construction of a set of polygons that represent the area of interest, such as the tracheo-bronchial wall. Polygonal models dominate interactive computer graphics because of their mathematical simplicity and that the rendering algorithms embed well in hardware. Polygonal surfaces that represent the outer surface of the 3D objects are usually calculated from a segmentation step. The marching cubes algorithm [69] or its variant can be used to extract polygonal surface. Marching cubes usually generate a large amount of polygons even for flat regions of the model. Subsequently, decimation algorithms are used to simplify the polygonal mesh to accelerate rendering without apparent deterioration of the image quality [70]. To this end, quadratic error metrics are often used with a manifold representation of triangle meshes. Quadratic error metrics provide a fast and simple way to guide the simplification process with relatively small storage cost. The visual fidelity of the resulting surfaces achieved tends to be high.

One interesting development in photorealistic rendering in recent years is the use of Image Based Modelling and Rendering (IBMR) techniques. IBMR is the intersection between computer graphics and computer vision. Computer graphics investigate the interaction of light with a set of objects where the geometrical and material properties are known. On the other hand, in computer vision often the inverse is presented, where a number of images are used to model the scene. With the use of IBMR techniques, it is possible to extend conventional texture mapping to support the representation of 3D surface details and motion parallax in addition to photorealism [71]. Existing research has shown that IBMR has the potential to create high fidelity rendering with manageable computational complexity by using the current graphics hardware. IBMR in MIS is not a trivial task since the lighting configuration is complex and the tissue deforms dynamically when the viewpoint is undergoing constant movement [72]. For patient-specific simulation, it is essential that the extraction of intrinsic surface texture and reflectance properties do not depend on the viewing conditions such that new views with different camera and lighting configurations can be generated [73]. In this way, free navigation of the acquired 3D model with enhanced photo-realism can be achieved.

2.5.4.4 Registration of the 3D Model with 2D Bronchoscope Video Sequences

As mentioned earlier, patient-specific bronchoscopy simulation requires the fusion of tomographic data and endoscope video in order to extract realistic surface properties from the real bronchoscope. This requires the tracking of the camera motion to determine the
pose of the camera inside the reconstructed 3D model that best matches with the bronchoscope video [74, 75][76, 77]. This is a classic 2D/3D registration problem in medical image computing.

Image registration is a well-studied topic in medical imaging which has a broad range of applications [78-82]. In bronchoscopy, however, the problem is complicated by the deformation of the airways. The pre-operative images generally do not accurately match to the geometry of the tracheo-bronchial tree during real bronchoscopy. This is because in tomographic imaging the patient is typically in a supine position, whereas in bronchoscopy examinations the patient usually adopts an upright posture. Extreme breathing patterns due to patient discomfort can occur, and they can cause severe deformation of the airways.

Thus far, most of the 2D/3D registration algorithms ignore the deformation of the tracheo-bronchial tree. Rigid registration techniques are used to match the CT data with bronchoscope video. For this reason, it is not guaranteed that these algorithms can converge to the solution since they depend on the initial pose and on the level of anatomical deformation involved. Registration failure occurs when the endoscope camera is close to the vessel wall or when image artefact is significant. In a typical examination, these effects account for 20-30% of the image sequence, which limit the current techniques being only applicable to short video sequences. A more detailed analysis of the current 2D/3D registration techniques will be provided in Chapter 3 and later in this thesis, we will focus on the development of a novel 2D/3D registration scheme that is immune to these problems.

### 2.5.4.5 Navigation and Interactivity

Navigation in virtual bronchoscopy involves the interaction of a user to control the camera movement in a way that is similar to real procedures. Even for a trained physician, navigation to the target can be difficult. The user should neither deal with too many parameters, creating a ‘lost-in-space’ feeling nor should be restricted to a heavily constrained navigation environment [83]. A near real-time frame rate is also an important goal for a effective navigation. Usually, visualization tools have three navigation modes: automated navigation, manual/free navigation, and guided navigation [84]. In automated navigation, the user provides the starting and ending positions, and the system automatically finds an appropriate path. Alternatively, the user can select key camera locations with the corresponding 3D representation. The system then calculates the in-
between camera poses. When the user is allowed to move freely along the calculated path, it is analogous to guided navigation. These features are essential in applications such as computer-assisted diagnosis and preoperative planning.

An indispensable part of realistic navigation is collision detection and haptic feedback. This determines the interaction between surgical tools, tissues and organs. Several collision detection techniques have been proposed. Most of them adopt a two-level approach, which includes collision bounding and collision refinement [21]. Collision bounding is a computationally inexpensive method. It is assumed that objects have simple geometry such as spheres or rectangles. When the objects interact, a more accurate representation is employed. Given that simulation devices require real-time interaction and visualization of the virtual anatomy, collision detection algorithms should be computationally efficient. A technique based on hardware implementation was proposed to accelerate collision detection [85]. It examines the volume around a simple shaped tool to check whether it intersects with other objects. This can be implemented easily on any available 3D graphics rendering software as a viewing volume with the camera at the tip of the tool. Although this technique cannot handle collision between complex objects, it is potentially useful in bronchoscopy simulation. The endoscope has a simple geometry and biopsy tools are the only tools used during navigation. Subsequently, a small viewing volume defined at the centre of the bronchoscopy lenses makes it possible for real-time collision detection on a conventional PC.

2.6 Discussion and Conclusions

With the introduction of MIS in medicine, surgical education and skills assessment have become an important part of the training procedure. Although the advantage of MIS over open surgery is widely recognised, a number of issues related to the complexity of instrument control and hand-eye coordination due to a loss of 3D vision must be considered. Extensive practice is required in order to automate the surgeons’ movements and familiarise them with the specific demands of MIS procedures. Medical simulators have the potential to provide an effective and complete surgical education scheme outside the operating room. Under this strategy, computer-based simulators can be used for both training and objective skill assessment. They reduce the time required from expert surgeons and provide the user with control over the learning process as well as the opportunity to
practice on rare pathological cases. However, current simulation systems are far from ideal due to the general lack of visual, physical, physiological, and tactile realism.

Patient specific models built from tomographic data and real endoscopic videos can overcome many of the limitations associated with photo-realism. This is because they combine the flexibility of scene geometry representation with photorealism based on image-based rendering techniques. The effectiveness of patient specific models is not only limited in visual reality but also extended to modelling tissue deformation and haptics.

In this chapter, we have reviewed the main requirements for effective surgical simulation and the current state-of-the-art. The work presented in this thesis is mainly to be focused on the development patient-specific models for bronchoscopy simulation. Bronchoscopy is the most reliable way for detecting cancer in the tracheo-bronchial tree. However, locating the lesion can be difficult and the branching structure can be disorientating. Patient specific models of the tracheo-bronchial tree can be created by fusing 3D CT images of the airways with real bronchoscopic video. This entails effective 2D/3D registration in the presence of airway deformation. Non-rigid 2D/3D registration is not a trivial problem. Although there is extensive literature on this topic, current techniques are mainly limited to rigid cases. This is due to the high dimensionality of the problem and the lack of an efficient way to capture and model tissue deformation. In practice, the tracheo-bronchial tree deforms due to respiration, cardiac contraction, coughing and pre-operative shifting caused by changes in the position of the patient. A non-rigid 2D/3D registration framework that recovers both the parameters of the camera motion and the deformation of the tracheo-bronchial tree is essential for the development of patient specific models.

Another major challenge of combining 2D video with 3D tomographic data for patient specific simulation is the extraction of intrinsic surface texture and reflectance properties that are independent of specific viewing conditions. This is required for generating new views with different camera and lighting configurations. For bronchoscope simulation, this permits the incorporation of tissue instrument interaction, and thus greatly enhances the overall realism of the simulation environment. The prerequisite of this approach is also in the accurate alignment of the 3D tomographic data with the bronchoscope video. In the following chapters, we will first review the current approaches to image registration and outline the major limitations of these techniques. A novel \( pq \)-space based 2D/3D registration technique is then proposed, which is further extended to incorporate statistical shape modelling for handling airway deformation.
Chapter 3

Non-Rigid 2D/3D Image Registration

In order to match video bronchoscopic images to the geometry extracted from 3D CT data, robust non-rigid 2D/3D registration techniques have to be developed. In non-rigid 2D/3D registration, the aim is both to localise the acquired image in relation to the 3D model and to extract the deformation parameters involved. This is an ill-posed problem and the inherent difficulty of the registration process is due to the differences in image formation, reduced image content and ambiguities in 2D images. Although image registration has been studied for many years, robust non-rigid registration is still a subject of on-going research. This is mainly due to the high degrees-of-freedom involved and it has been recognised that there is no single registration technique that can be applied to all image matching problems.

In general, a non-rigid image registration algorithm consists of four major components which include the target and source data sets, the transformation that relates the target and source, a similarity measure that estimates the similarity between the two, and an optimization technique that determines the optimal transformation parameters as a function of the similarity measure. The reference and target data can be intensity values, curves/surfaces, landmarks, or any combination of the above. The transformation model depends on whether the motion is rigid or deformable, and can be rigid, affine, piecewise linear or elastic. Finally, the similarity measure is based on either intensity values or geometrical features to determine how well the two data sets are aligned.

The main objective of this chapter is to introduce a mathematical framework of non-rigid 2D/3D registration and highlight the challenges involved. It also discusses the value of image-based registration in medical imaging and computer assisted surgical intervention.
Common methodologies and techniques are reviewed in order to address the weakness and limitations of the current approaches.

3.1 Image Registration in Medical Imaging

Medical imaging provides information about the morphology and function of the anatomical structures of individual patients as well as among a population. In clinical practice, it is common to image the same subject multiple times with the same image modality in order to monitor the progression of a disease, e.g., evolution of a malignant tumour, or assess the efficacy of the therapeutic process. Different image modalities can provide complementary information about a given symptom, and in surgery they can provide both pre-operative planning and intra-operative navigation. It is also desirable to compare images of a given subject to an atlas that reflects the average of a normal population. To this end, establishing the spatial correspondence of these data is the prerequisite of successful image interpretation and analysis.

Registration not only is a valuable tool for traditional medical imaging, but also opens up a range of new applications. These include serial imaging to monitor subtle changes due to disease progression or treatment. In these studies, the subject cannot be expected to remain in a fixed position. In image guided interventions, image data acquired prior to the procedure is registered to surgical devices so that the surgeons can use these data to guide the interventional procedure. Image registration has also become a valuable tool in basic sciences research, such as neuroscience, where serial and functional imaging studies are making substantial contributions to our understanding of the way the brain works.

Given the importance of image registration in medical imaging, there are many research surveys on the topic and registration techniques can be classified based on different criteria [78-80, 82, 86]. These include their dimensionality, registration basis, transformation methods, domain of transformation, interaction, optimization procedures, and modalities involved [80]. Dimensionality can be classified into spatial or temporal, where spatial dimensionality refers to the dimension of the images and thus can be 2D/2D, 2D/3D or 3D/3D. The nature of the registration basis refers to whether it is based on foreign objects introduced into the imaged space or it is purely based on image information. The transformation is related to the deformation involved, and therefore a registration can be
rigid, affine, or free-form. For existing registration algorithms, three levels of interaction are usually involved.

- **Automatic** - where the user only supplies the algorithm with the image data and possibly parameters related to the image acquisition;
- **Interactive** - where the user does the registration assisted by software supplying a visual or numerical impression of the current transformation;
- **Semi-automatic** - where the user initializes the registration procedure and guides the algorithm by rejecting or accepting suggested registration hypotheses.

The optimization procedure is based on analytical calculation or optimisation of the transformation parameters. A registration procedure is also dependent on the nature of the modalities involved, whether being multimodal or uni-modal. Alternatively, registration algorithms can also be classified according to their applications.

### 3.2 2D/3D Image Registration

Image registration becomes more difficult when the dimensionalities of the source and target data are different. In this case, 2D/3D registration is a typical example and as mentioned earlier, it is an ill-posed problem as the solution space is in 3D, whereas the available information is partially constrained to 2D. This is further complicated when deformation and image artefacts are present.

One approach to 2D/3D registration is to reconstruct the 3D volume from 2D images but this process itself is ill-posed and can be subject to significant errors. For this reason, most of the existing methods reduce the problem to 2D/2D registration between 2D ‘simulated’ images derived from the 3D volumetric dataset and the original 2D images. Therefore, a number of 2D volumetric images have to be compared to each of the 2D datasets and an optimisation procedure should be defined such that the best approximation is adopted as the solution. A schematic illustration of this method is provided in **Figure 3.1**. These approaches generally require a good initialisation of the pose parameters. To limit the solution space, application specific constraints can be used but local minima of the optimisation landscape are difficult to avoid.

2D/3D registration arises in a number of imaging applications, ranging from orthopaedics to neurosurgery. Typically, 2D/3D registration is employed in image-guided surgery and in
conventional X-ray radiography. The most common application of 2D/3D registration is in the alignment of intra-operative fluoroscopy images with pre-operative CT. In this case, each pixel in the fluoroscopic image corresponds to a combination of the X-ray attenuation along a projection line through the body. A common solution to this problem is to simulate X-ray projections from the CT data to produce a digitally reconstructed radiograph (DRR). Iterative pose estimation of the X-ray source of the fluoroscope in relation to the 3D CT data is then solved by optimising a 2D intensity similarity measure.

Figure 3.1: Diagrammatic illustration of 2D/3D registration showing images before registration (a) and after registration (b). The contour overlay represents the projection of the 3D model onto the endoscopic image.
3.3 Mathematical Formulation of the Registration Problem

Image registration is defined as the process of establishing a mapping between two or more images in order to achieve anatomical or functional correspondence. The computational process of registration results in a transformation between the coordinate systems of the source and target data. Once the correspondence is determined, one dataset can be readily mapped to the other by using the transformation parameters derived. When the images are of the same dimensionality then a transformation $T$ that transforms a position $x$ from one image $A$ to another $B$ is defined as [87]:

$$T : x_B \rightarrow x_A \iff T(x_B) = x_A \quad (3.1)$$

As the images $A$ and $B$ represent the same object $X$ imaged with the same or different modalities, there is a relationship between the spatial locations in $A$ and $B$. The registration process involves recovering the spatial transformation $T$ that maps $x_B$ to $x_A$ over the entire domain of interest. Practically, this may correspond to the field of view of the imaging device or the area of interest. It is useful to think of the two images themselves as being mappings of points in the patient to intensity values:

$$A : x_A \in \Omega_A \rightarrow A(x_A) \quad \text{and} \quad B : x_B \in \Omega_B \rightarrow B(x_B) \quad (3.2)$$

where $\Omega$ is defined as the domain of an image. The overlap domain between images $A$ and $B$ is defined as [87]:

$$\Omega_{A,B}^T = \{x_A \in \Omega_A \mid T^{-1}(x_A) \in \Omega_B\} \quad (3.3)$$

Therefore, the overlap domain depends on the domains of the original images $A$ and $B$, and also on the spatial transformation $T$. The overlapping between domains $\Omega_A$ and $\Omega_B$ plays an important role in formulating an accurate and reliable registration algorithm.

In 2D/3D registration, the spatial transformation $T$ involves a 3D-3D mapping followed by the projection of the 3D object onto a 2D plane. In endoscopic images, this involves a perspective projection with certain degrees of lens distortion. Endoscopic cameras have a large field of view to allow them the visualisation of a large part of the tracheobronchial structure even when the camera is close to the lumen’s wall. In this case the approximation of the perspective with an orthographic projection leads to significant errors. A perspective projection is described by four degrees-of-freedom, which include the focal length $(f_x, f_y)$.
and the principal point \((c_x, c_y)\). The focal length is the distance between the lenses and the image plane, whereas the principal point is defined as the point where the principal axis/camera axis intersects with the image plane. Subsequently, the transformation in 2D/3D registration takes the following form:

\[
T_{2D-3D} = \begin{bmatrix}
f_x & \vartheta \cdot f_x & c_x \\
0 & f_y & c_y \\
0 & 0 & 1
\end{bmatrix} \cdot T_{\text{distortion}} \cdot T
\tag{3.4}
\]

where \(\vartheta\) is a skewness coefficient describing the angle between x and y pixel axes. In the above equation, \(T_{\text{distortion}}\) is the transformation that describes barrel distortion due to imperfection of the camera lenses. Note that in endoscopic cameras, lens distortions are usually large due to the wide angle field-of-view. Distortion usually includes both radial and tangential [88] components, \(i.e.,\)

\[
T_{\text{distortion}} = T_{\text{radial}} + T_{\text{tangential}}
\tag{3.5}
\]

Generally, the intrinsic camera parameters and the distortion coefficients are constants and can be extracted prior to the image registration by appropriate calibration.

The nature of the transformation \(T\) defines whether the registration is rigid or not [89]. The most common transformations are shown in Table 3.1. The goal of rigid registration is to find the six-degrees-of-freedom of a transformation: \(T: (x, y, z) \rightarrow (x', y', z')\), which completely describe the rotation and translation that need to map each point in the source image \(I(x, y, z, t)\) at time \(t\) into the corresponding point of the target image, \(I(x', y', z', t_0)\).

Extending rigid-body registration, an affine transformation of 12 degrees-of-freedom allows for scaling and shearing. Affine transformations do not in general preserve angles or lengths, but parallel lines remain parallel. Therefore, there is no orthogonality constraint on the elements of the rotation matrix.

\[
T_{\text{global}}(x', y', z') = \begin{bmatrix}
\vartheta_{11} & \vartheta_{12} & \vartheta_{13} & \vartheta_{14} & x \\
\vartheta_{21} & \vartheta_{22} & \vartheta_{23} & \vartheta_{24} & y \\
\vartheta_{31} & \vartheta_{32} & \vartheta_{33} & \vartheta_{34} & z \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\tag{3.6}
\]
<table>
<thead>
<tr>
<th>Rigid Transformation (6DoF)</th>
<th>Affine Transformation (12DoF)</th>
<th>Projective Transformation (15DoF)</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}
\] | \[
\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}
\] | \[
\begin{bmatrix} A & t \\ v^T & 1 \end{bmatrix}
\] |

Table 3.1: Common geometric transformations and the degrees-of-freedom (DoF) involved. a) Original Shape, b) rigid transformation expressed by rotation \( R \) and translation \( t \), c) affine transformation which preserves parallel lines (\( A \) is not an orthogonal matrix), and d) projective transformation, where the projective part is expressed as a three-dimensional vector \( v^T \).

Linear transformation should not be confused with affine transformation. A linear transformation satisfies the condition [87]:

\[
T(\alpha x_A + \beta x_B) = \alpha \cdot T(x_A) + \beta \cdot T(x_B) \quad \forall x_A, x_B \in \mathbb{R}^D
\]  (3.7)

Affine transformations are described as the combination of linear transformations with translations.

In the case that deformation must be accounted for, the simplest method is to generalise the rigid body transformation. This can be represented as global polynomial function, or piecewise polynomial relations, or splines. For example, a quadratic transformation model can be defined by second-order polynomials:

\[
T(x, y, z) = \begin{bmatrix}
    a_{00} & \cdots & a_{09} & a_{09} \\
    a_{10} & \cdots & a_{18} & a_{19} \\
    a_{20} & \cdots & a_{28} & a_{29} \\
    0 & \cdots & 0 & 1
\end{bmatrix} \begin{bmatrix}
    x^2 \\
    y^2 \\
    \vdots \\
    1
\end{bmatrix}
\]  (3.8)

These transformations can be extended to higher order polynomials, but they are generally constrained only to global shape changes. Global polynomial transformations are only useful to account for low-frequency distortions because of their unpredictable behaviour when the order of the polynomial is high. Global mapping functions do not always
adequately capture anatomical deformation that is intrinsically local. Therefore, it is difficult to use a parameterized transformation to describe local anatomical shape variability.

3.4 Non-Rigid 2D/3D Registration

A non-rigid transformation can be described as a combination of a global and a local component [89]:

\[ T(x, y, z) = T_{\text{global}}(x, y, z) + T_{\text{local}}(x, y, z) \]  

(3.9)

The global deformation component describes the overall deformation of the object. It is usually either rigid or affine. The local transformation describes local deformation that cannot be modelled as linear. The level of non-rigid deformation depends on the resolution of the control points and is also directly related with the computational complexity. The transformation model plays a major role in non-rigid image registration. Since the local component encodes variability, it is inherently related to the notion of deformable models. However, it should be noted that deformable models describe both shape and shape variability, whereas a transformation is only a mapping mechanism from one image to the other.

There are a number of transformation models that have been proposed in the literature for non-rigid registration. Their formulation is also dependent on whether an intensity or feature-based similarity measure is employed. Intensity based techniques iteratively optimise the similarity between the corresponding voxels or pixels of the source and target images. They transform the image at each iteration by using the current estimate of \( T \) to evaluate the similarity measure, and the type of interpolation used can alter the solution obtained.

In computer guided interventions, surface based registration is also common as surface rendering requires less computational resources (CPU, RAM) than volume rendering. In general, surfaces provide more redundancy than landmarks and this redundancy can be useful for characterizing non-rigid motion. Furthermore, surface-based approaches are more useful for aligning partially overlapped objects.
3.4.1 Transformation Models for Non-Rigid Registration

Non-rigid (also called non-linear) transformations are often used to describe transformations with higher degrees-of-freedom than that of an affine transformation. These transformations are meant to model local changes in the shape of an object. Thus far, a number of models have been proposed which include those based on geometric primitives, such as Thin-Plate Splines, B-splines, Multiquadric, Piecewise Linear and Weighted Mean representations, and those based on physical models such as elastic, fluid, finite element models and mechanical models.

Most non-rigid registration algorithms based on a non-physical transformation rely on the use of a set of control points between the two image domains. These control points can either correspond to particular features extracted in images or be regularly spaced within the spatial domain. The transformation either interpolates or approximates the displacement required to map each control point of the source image to the corresponding control point in the target image. An interpolation condition arises when the number of control points is equal to the number of unknown terms in the transformation function. Subsequently, the system of equations that describes the transformation is fully determined. When the number of control points is greater than the number of unknown parameters, only approximate transformation coefficients can be derived. This implies that residual errors may exist between the corresponding control points.

In physically-based transformations, model-based techniques are used to model the shape and the dynamic behaviour of the surface. Such methods generally compute curve and surface motion not by explicit matching, but by implicit consideration of image motion in the form of virtual forces.

3.4.1.1 Spline Based Transformations

A spline-based transformation provides a smoothly varying displacement field, which, for example, can be formulated as three thin-plate spline functions [87]:

\[
T = (t_1, t_2, t_3)^T
\]  

(3.10)

Thin-plate splines are defined as a combination of \( n \) radial basis functions \( \vartheta \).
\[ t(x, y, z) = a_1 + a_2 x + a_3 y + a_4 z + \sum_{j=1}^{n} b_j \cdot \vartheta \left( \| \varphi_j - (x, y, z) \| \right) \]  

(3.11)

The coefficients \( a \) characterise the affine part of the transformation, whereas the coefficients \( b \) characterise the non-affine part of the transformation. \( \varphi \) is the location of the control points.

Radial basis functions have infinite support, which implies that each control point has a global influence on the entire transformation. This is undesirable when local deformation needs to be modelled. Alternatively, B-splines are locally controlled, which makes them computationally efficient for a large number of control points. The most commonly used B-spline is the cubic B-spline [90], \( i.e. \),

\[ T(x, y, z) = \sum_{l=0}^{3} \sum_{m=0}^{3} \sum_{n=0}^{3} B_i(u) B_m(v) B_n(w) \phi_{j+l+m+k+n} \]  

(3.12)

When the local transformation model is based on cubic B-splines, the resulting deformation produces a smooth and \( C^2 \) continuous transformation.

### 3.4.1.2 Free-Form Deformation

Free-Form Deformation (FFD) provides a higher level of control of the deformation during registration. FFD changes the shape of an object by deforming the space where the object lies. In other words, the object is embedded in a lattice of grid points using either cubic or cylindrical coordinates. Manipulation of the nodes of the grid induces deformation on the space inside the grid, which then transforms the graphics primitives describing the object. If \( \{U_i\} \) is the set of three-dimensional cells defined by the grid, a free-form deformation can be thought of as a collection of mappings of the form: \( f_i : U_i \rightarrow \mathbb{R}^3 \). Subsequently, direct movement of the surface or object points is achieved by converting the desired movements of these points to equivalent grid point movements. This under-constrained problem is typically solved by choosing the grid point movement with the minimum least-squares energy that produces the desired object manipulation.

FFD mathematically can be defined in terms of a tensor product trivariate Bernstein polynomial [89]. First, a local coordinate system on a parallelepiped region is specified such that every point \( X = (s, t, u) \) in the 3D space can be described as:
\[ X = X_0 + sS + tT + uU \] (3.13)

Every point inside the parallelepiped conforms to the following constraints: \(0 < s < 1\), \(0 < t < 1\), \(0 < u < 1\). Subsequently, the location of the control points is defined as:

\[ P_{i,j,k} = X_0 + \frac{i}{l} S + \frac{j}{m} T + \frac{k}{n} U \] (3.14)

where \(l + 1\), \(m + 1\) and \(n + 1\) are the number of the grid planes in the \(S\), \(T\) and \(U\) directions, respectively. The deformation is specified by moving the control points away from their original positions. The deformed position \(X_d\) of an arbitrary point \(X\) is derived by evaluating the vector valued trivariate Bernstein polynomial:

\[
X_d = \sum_{i=0}^{l} \binom{l}{i} (1-s)^{l-i} s^i \left[ \sum_{j=0}^{m} \binom{m}{j} (1-t)^{m-j} t^j \left[ \sum_{k=0}^{n} \binom{n}{k} (1-u)^{n-k} u^k \cdot P_{i,j,k} \right] \right] 
\] (3.15)

This deformation can also be formulated in terms of other polynomial bases, such as tensor product B-splines.

FFD is a versatile tool for representing different geometric deformations. For example, a parametric surface remains parametric after deformation. Furthermore, it is possible to apply two or more FFDs in a piecewise manner so as to maintain the cross-boundary derivative continuity. Another reason that FFD is so useful in solid modelling is that it provides control over the volume change of the deformable object. In general, the volume change is given by the Jacobian of the FFD. Therefore, by imposing a bound on the Jacobian over the region of deformation also results in a bound of the volume change. Such a bound is easily provided when the Jacobian is expressed as a trivariate Bernstein polynomial. Subsequently, the two extremes of the coefficients provide upper and lower bounds on the volume change. Thus far, FFD has been successfully used for 3D-3D image registration for a number of applications [91]. Unlike radial basis functions, FFD requires a regular mesh of control points with uniform spacing. Subsequently, its applications are limited to volumetric images and solid object representations.
3.4.1.3 Physical Transformation Models

Physically-based techniques simulate natural motions and shape deformations based on mathematical models that specify the physical behaviour of the object. Usually, they are characterised by a high accuracy at the cost of a higher complexity. Elastic, viscous fluid and biomechanical models belong to this category. The idea behind elastic transformations is that the underlying deformation is a physical process that resembles the stretching of an elastic material. This process is governed by both internal and external forces. The internal force is caused by the deformation of elastic material and is opposite to the force that deforms the elastic body from its equilibrium. The deformation of the object stops when all the forces acting on the object form an equilibrium solution, which is described by the Navier linear elastic Partial Differential Equation (PDE) [87]:

$$\mu \nabla^2 u(x, y, z) + (\lambda + \mu) \nabla (\nabla u(x, y, z)) + f(x, y, z) = 0$$

(3.16)

where $u$ describes the displacement field, $f$ is the external force acting on the elastic object, $\nabla$ denotes the gradient operator, and $\nabla^2$ is the Laplacian operator. The parameters $\mu$ and $\lambda$ are Lamé’s elasticity constants, which describe the behaviour of the elastic body. The external force drives the registration process and it can be defined as the gradient of an intensity-based similarity measure or the distance between curves, surfaces and anatomical structures.

Fluid registration enables the modelling of highly localised deformations including corners. Fluid registration, similarly to elastic registration, is described by the Navier-Stokes partial differential equation:

$$\mu \nabla^2 v(x, y, z) + (\lambda + \mu) \nabla (\nabla v(x, y, z)) + f(x, y, z) = 0$$

(3.17)

where $v$ describes the velocity field. The relationship between the Eulerian velocity $v$ and the displacement field $u$ is:

$$v(x, y, z, t) = \frac{\partial u(x, y, z, t)}{\partial t} + v(x, y, z, t) \cdot \nabla u(x, y, z, t)$$

(3.18)

One way to solve these partial differential equations is the Finite Element Methods (FEM). These are numerical techniques that discretise the continuum into a number of elements in order to approximate the continuous body equations over each element. FEM is computationally intensive, and thus realistic deformation cannot be achieved in real-time. Mass-spring-damper systems, Boundary Element Method and Finite Difference Method
(FDM) are also used to model physical deformable objects [92]. In Boundary Element Method only the object boundary is considered. Although the resulting system is one dimension less than FEM, it is still very dense and can not model internal behaviour of non-homogeneous materials. In FDM, the differential operator is approximated by algebraic difference operators. FDM is efficient and accurate when the geometry of the problem is regular, but discretisation becomes extremely dense when it is used to approximate the boundary of an irregular object.

Generally, FEM is superior when handling geometric approximations, because it uses the exact differential operators that apply only in subspaces of the solution fields. Many different approaches have been proposed in order to improve the time efficiency of FEM [93]. The key to the efficient solution of this problem is the placement of nodal points and elements that constitute the 3D mesh. For example, regions with large gradients need a higher mesh density. In order to obtain an optimal mesh, every element should contain the same predefined allowable error. Adaptation in FEM is essential to the performance of the algorithm while maintaining local deformation details.

The main problem of physically-based transformations is that \( \mu \) and \( \lambda \) are characteristic parameters of the biomechanical tissue properties and they need to be measured prior to the registration process. These parameters differ from subject to subject and they have a high degree of intra-subject variability. These measurements also differ for \textit{in vitro} and \textit{in vivo} specimens. Another problem associated with physically-based methods is the large number of degrees-of-freedom involved, which makes the optimisation process difficult to converge, thus leading to mis-registration errors.

### 3.4.2 Surface Representation and Deformable Models

Surface representation is an important element of non-rigid 2D/3D registration. In medical image segmentation, a cloud of points is often derived for the anatomical structure and it is common to use triangulation to form a 3D mesh. One way to construct a surface deformable model is to use scattered data interpolation techniques. The problem of scattered data interpolation uses a continuous function that interpolates values that are known only at some scattered points. Approaches to the interpolation of scattered data can be classified as global and local methods. In global methods, the interpolated values are influenced by all of the data and usually they are limited to small datasets because of the computational complexity involved. In local methods, the interpolated values are only
influenced by the values at neighbouring points, and thus these methods are suited for modelling larger datasets. While interpolation techniques construct functions that pass through the given data points, data fitting schemes construct functions that maintain the overall trend of the input data and they do not necessarily pass through all the input points. Scattered data interpolation functions can be formed based on splines and patches. They include, for example, Bezier curves, double quadratic curves, B-splines, rational B-splines and non-uniform rational B-splines (NURBS). These methods can represent both planar and 3D curves and relate 2D patches for specifying surfaces. In these representations, the curve or surface is represented by a set of control points. Therefore, the shape of the object can be adjusted by moving the control points to new positions. This parameter-based object representation is computationally efficient and can support interactive modifications. However, precise alterations of the curves or surfaces can be laborious and a perceptually simple modification may require the adjustment of many control points [94].

A NURBS surface is a bidirectional parametric representation of an object. Points on a NURBS surface are defined by two variables, $u$ and $v$, representing the longitude and latitude, respectively. Points defined in Cartesian coordinates ($x$, $y$, $z$) can easily be converted into surface coordinates ($u$ and $v$) and vice versa. The extraction of a NURBS surface can be based on patient image data and it permits the representation of a surface at any resolution. Furthermore, NURBS can be altered easily via affine or other transformations for dynamic modelling, as well as capturing anatomical variations among patients. The main disadvantages of the NURBS are the inherent mathematic complexity and the relatively large number of parameters that need to describe a 3D surface. The added complexity, however, is beneficial for modelling biological shapes [95].

### 3.5 Similarity Measures for Image-Based Registration

A key issue of non-Rigid 2D/3D registration is how to define the distance measure between the 3D data and the 2D projection. Similarity measures are based on either intensity values or geometrical features. Both techniques involve optimising a similarity measure, which evaluates how close the 3D model is to the corresponding 2D video frame. Intensity based techniques can be based on cross-correlation or mutual information, whereas feature based techniques rely on the extraction of features from the model and their alignment to the correspondent images. Feature based techniques generally involve the following steps: (1)
extraction of landmarks, (2) pair-wise correspondence, and (3) estimation of the transform model.

3.5.1 **Intensity-Based Similarity Measures**

Intensity based techniques, such as normalised cross correlation and mutual information are popular in medical imaging. They have been applied successfully to a number of applications as well as modalities such as microscopy, MRI, CT, X-rays, and ultrasound. They have also been applied to multimodal registration, such as MR-CT, MR-PET, MR-SPECT, MR-US, CT-PET, and CT-SPECT. Intensity based techniques tend to be computationally intensive but they have the advantage of not requiring pre-segmentation or feature extraction, thus facilitating fully automatic image registration.

3.5.1.1 **Sum of Square Differences**

Sum of Square Differences (SSD), also called Euclidean Distance, is the simplest way to evaluate the similarity between two images. The aim of SSD is to find the transformation \( T \) that minimises the sum of the square differences between the pixels intensity \( I \) of the two images.

\[
\arg \min \sum_{x_A, x_B \in \Omega} (I(x_A) - I(T(x_B)))^2
\]  

(3.19)

This measure is optimum when the images differ only by Gaussian noise. Similar to SSD is the Sum of Absolute Differences, which uses the absolute intensity differences instead of the squared differences, and the Mean of Square Differences. In order to eliminate the effect of contrast and brightness changes, the normalised SSD is normally used in practice, i.e., the cost function is designed to seek

\[
\arg \min \left\{ \frac{\sum_{x_A, x_B \in \Omega} (I(x_A) - I(T(x_B)))^2}{\sqrt{\sum_{x_A \in \Omega} (I(x_A))^2} \sqrt{\sum_{x_B \in \Omega} (I(T(x_B)))^2}} \right\}
\]  

(3.20)
3.5.1.2 Normalised Cross-Correlation

The cross-correlation measure between two images is defined as:

\[
\text{arg max} \sum_{x_A, x_B \in \Omega} I(x_A) I(T(x_B))
\]  

(3.21)

Similarly to SSD, the normalised cross-correlation (NCC) is preferred in order to make this measure immune to linear intensity transformations, such as image contrast and brightness. The NCC cost function is defined as:

\[
\text{arg max} \left\{ \frac{\sum_{x_A, x_B \in \Omega} (I(x_A) - \bar{I}_A)(I(T(x_B)) - \bar{I}_{TB})}{\sqrt{\sum_{x_A \in \Omega} (I(x_A) - \bar{I}_A)^2} \sqrt{\sum_{x_B \in \Omega} (I(T(x_B)) - \bar{I}_{TB})^2}} \right\}
\]  

(3.22)

where \( \bar{I}_A \) is the average intensity of the reference image \( A \) and \( \bar{I}_{TB} \) is the average intensity of the transformed image \( B \). For correlation measures, a linear relationship of the corresponding voxel/pixel intensities is assumed. However, even in images of the same modality, this requirement can not always be met. For example, this can be due to the bias field in MR images caused by RF inhomogeneity, and motion artefacts [96]. In practice, the sensitivity of correlation-based similarity measures to non-linear intensity transformations can be improved by performing correlations on the magnitude of the intensity gradient. This can be useful for images with discontinuous surface properties.

3.5.1.3 Fourier Methods

Fourier Methods are preferred over correlation-based methods when images are large and computational speed is important. Fourier-based registration is originated from the Fourier Shift Theorem, which states that a shift in spatial domain leads to a linear phase change in the frequency domain. Originally, the technique has been introduced for the registration of translated 2D images but it has been extended to correct for rotation and scaling in 2D as well as 3D images. Fourier-based registration techniques have been used to achieve sub-pixel accuracy, avoiding image distortion introduced by interpolation. However, their application is limited mainly to rigid registration.
3.5.1.4 Registration based on Geometrical Moment Invariants

Moment invariants were initially developed for positional invariant pattern recognition. This is a convenient way of characterizing the geometric properties of objects in 2D or 3D \[97\]. Other techniques, such as Gabor filters, wavelets and low frequency representations can also be used to extract the geometric properties of the objects. For a 3D continuous function \( f(x_1, x_2, x_3) \), the 3D regular moments of order \((p + q + r)\) are defined by the Riemann integrals:

\[
M_{p,q,r} = \int \int \int_{R^3} x_1^p x_2^q x_3^r f(x_1, x_2, x_3) \, dx_1 \, dx_2 \, dx_3
\]  

Although geometric moments operate directly on the image intensity values, they reduce their content to a representative set of scalars and orientations \[80\]. Based on these methods, the image centre of gravity and its principal orientation/principal axes are computed from the first order moments. Subsequently, registration is performed by aligning the centre of gravity and its principal orientations, also called invariant attribute vectors.

The notion of ‘invariant attribute vectors’ has been also used in \[98\] in the context of surface registration. The goal is to distinguish different parts of a boundary according to the shape properties. With this method, an attribute vector is attached to each vertex of the model in order to reflect the geometric structure of the model from a local to a global level. The attribute vector is an extension to curvature, which is invariant to rotation and translation.

3.5.1.5 Mutual Information

Mutual Information (MI) is an entropy-based measure that was introduced in \[99\] to solve the so called ‘sensor fusion’ problem. It uses a new information-theory approach for finding the pose of an object based on a formulation of the joint entropy between the model and the image. This is called Empirical Entropy Manipulation and Analysis. MI is an effective method for image registration, particularly for multi-model applications. MI intuitively encodes the statistical relationship of the voxel intensities, thus yielding more consistent registration results.

Mathematically, MI of \(A\) and \(B\) is defined as the amount of information gained about \(A\) when \(B\) is learned and vice versa. It is zero if and only if \(A\) and \(B\) are independent.
The basic idea is that when images $A$ and $B$ are aligned, there will be a consistent relationship between image $A$ intensity and image $B$ intensity. A transformation is considered to be consistent if points that have similar intensities in image $A$ are projected to similar values in image $B$. A formal definition of consistency can be formulated based on the principle of maximum likelihood. However, a similarity measure based on neighbour likelihood alone suffers from certain drawbacks as it assumes that image $A$ is a function of image $B$, which may not be true in multi-modal images [100]. Furthermore, it may be susceptible to outliers caused by occlusion and specularities.

Alternatively, Viola et al. [99] suggested using predictability and subsequently information entropy based on the assumption that image $A$ is predictable from image $B$ but not necessarily a function of the later. Information entropy, also called Shannon’s entropy or marginal entropy describes how much uncertainty or randomness is encoded by a signal. The more random a signal is, the higher the entropy. The entropy of a random variable $x$ is defined as:

$$h(x) \equiv -\int p(x) \ln p(x) \, dx$$ \hspace{1cm} (3.24)

The joint entropy of two random variables $x$ and $y$ is defined as:

$$h(x, y) \equiv -\int p(x, y) \ln p(x, y) \, dx\, dy$$ \hspace{1cm} (3.25)

The concept behind image alignment using the joint entropy is that the predictability of one image based on the other is maximised when the two images are aligned correctly and this results in minimising the corresponding joint entropy. However, joint entropy is also minimised when image intensities are constant, and thus the image by itself is easily predictable. To overcome this problem, MI is expressed as the sum of the entropy of the individual signals less their joint entropy:
\[ M(x, y) = h(x) + h(y) - h(x, y) \] (3.26)

The relation between entropy and mutual information can also be depicted using Venn diagrams as shown in Figure 3.2, where MI is zero when two images are uncorrelated and reaches maximum when they are registered to each other. The fact that MI is not minimised by image constancy can be better shown by considering a transformation \( T \) that seeks to align transform image \( B \) to \( A \). The MI of the intensity values of the source image \( I(x_A) \) and the transformed image \( I(T(x_B)) \) that takes the form:

\[
M(I(x_A), I(T(x_B))) = h(I(x_A)) + h(I(T(x_B))) - h(I(x_A), I(T(x_B)))
\] (3.27)

Registration algorithms based on MI of the images \( A \) and \( B \) seek to find a transformation \( T' \) that maximises their mutual information over the domain of possible transformations \( T \).

\[ T' = \arg \max_T M(I(x_A), I(T(x_B))) \] (3.28)

In Equation (3.27), the first term of the MI equation expresses the entropy of the source image \( A \) and it does not depend on the transformation \( T \), hence it is constant for a given set of images. The second term is maximised when the transformation \( T \) projects \( B \) into the complex parts of image \( A \) and it is minimised when the transformation leads to constant intensity. The last term of (3.27) encourages a transformation that maximises predictability. Therefore, both the second and third terms strives to minimise entropy, while avoiding a transformation that leads to constant image intensities.

Studhome et al [101] pointed out that MI is sensitive to the changes of image overlap. This implies that a change in the proportion of the image overlap can result in changes of MI that do not reflect the quality of alignment between the two images. For this reason, they formulated the Normalised Mutual Information, which provides a measure that is independent to the field of view.

\[
MN(I(x_A), I(T(x_B))) = \frac{h(I(x_A)) + h(I(T(x_B)))}{h(I(x_A), I(T(x_B)))}
\] (3.29)

MI has been evaluated from a number of different groups [86, 96, 102]. Although it is widely accepted as one of the most reliable similarity measures in multi-modal image
registration, it also has certain drawbacks. Problems usually arise due to poor image resolution and small overlapping areas between the two images. MI can fail if the 2D histogram is relatively sparsely populated, which is the case when image resolution is low. This problem is also apparent in 2D/3D registration, since the number of pixels reduces considerably when compared to 3D/3D registration [86]. Problems also arise when the function contains many local minima [103]. Furthermore, MI decouples intensity values and location information. As a result, the dependency of the intensity values on neighbouring voxels/pixels is ignored [102]. In practice, incorporation of spatial information can improve the robustness of the registration process. However, as MI only relies on an analysis of the global intensity information, it is not well suited to tasks where the statistical aspects of the intensity values vary within the image, which may be the case for endoscopic images because of shadows and intensity gradients due to directional illumination [104].

3.5.1.6 Beyond Mutual Information

Recently, several approaches have been introduced to complement or replace MI. For example, in the work by Likar et al. [105] noted that searching for the global registration transformation with a MI-based similarity measure is not practical for highly non-rigid deformations. Typically, this approach leads to a complex similarity function that may have many local maxima. They proposed to subdivide the image into blocks that can be registered locally by a rigid or affine transformation. The registered images at a higher level are then partitioned into progressively smaller sub-images. Once all the sub-images are registered, a globally consistent registration is achieved by using the centres of the registered sub-images as point pairs and an elastic thin-plate-spline interpolation. However, the reduction of the number of samples due to the partition of the images into smaller sub-images can reduce the statistical power of the joint intensity histogram. In order to improve the probability estimation, random re-sampling is introduced, which prevents the alignment of a too large number of grid points to minimise the interpolation artefacts.

Dinggang et al [98] also noted that minimising an intensity-based similarity measure does not necessarily imply good registration of the underlying anatomy. In order to address this problem, they proposed a new image registration approach based on a Hierarchical Attribute Matching Mechanism (HAMMER). Their work has two main features. Firstly, they use attribute vectors calculated from the tissue maps across different scales for each voxel of the image. Secondly, in order to avoid being trapped by local minima, HAMMER
uses a successive approximation of the energy function optimized by lower dimensional smooth functions. By hierarchically selecting the features that have distinct attribute vectors, the ambiguity in establishing correspondence is significantly reduced.

In HAMMER, the transformation between two images is defined as a mapping of one coordinate system to the other (rigid-body transformation) followed by a shape transformation. Let \( I_A(x) \) be the intensity of voxel \( x \) in a template brain image, and \( I_B(y) \) be the intensity of voxel \( y \) in another brain image. Here, \( x \) and \( y \) are the 3D coordinates of the voxels volumes \( V_A \) and \( V_B \), respectively. The displacement field \( u(x) \) defines the mapping from the coordinate system of image \( A \) to image \( B \), while transformation \( h(x) = x + u(x) \) defines the mapping that transforms image \( A \) into the shape of image \( B \). The inverse transformation is defined as \( h^{-1}(x) \).

The hierarchical attribute matching mechanism for non-rigid registration uses a sequence of lower dimensional energy functions to approximate the following multivariate energy function:

\[
E = \sum_{x \in V_A} \omega_A(x) \left\{ \sum_{z \in n(x)} \varepsilon(z) \left( 1 - m\left( \alpha_A(z), \alpha_B\left( h(z) \right) \right) \right\} \sum_{z \in n(x)} \varepsilon(z) + \sum_{y \in V_B} \omega_B(y) \left( 1 - m\left( \alpha_A\left( h^{-1}(z) \right), \alpha_B(z) \right) \right) \sum_{z \in n(x)} \varepsilon(z) + \beta \sum_{x \in V_A} \left\| \nabla^2 u(x) \right\|
\]  

In the first term, the similarity of the attribute vectors for each template voxel \( x \) in its spherical neighbourhood \( n(x) \) is calculated. It is worth noting that the similarity is estimated over a neighbourhood area, rather than on individual voxels. In (3.30), \( \alpha_A(z) \) and \( \alpha_B(z) \) are the attribute vectors of the voxel \( z \) in image \( A \) and \( B \), respectively. Function \( m(\ldots, \ldots) \) measures the similarity of two attribute vectors within the range of 0 to 1.

In (3.30), the denominator \( \sum_{z \in n(x)} \varepsilon(z) \) is used as a normalization factor. The parameter \( \omega_A(x) \) is the relative weight for each voxel \( x \) in image \( A \). The voxels with the distinct attribute vectors are assigned with larger weights, because they can be identified more reliably than others. In a hierarchical registration scheme, these voxels drive the deformation of the image \( A \) by initially focusing on the first term of the energy function.
The second energy term is similar to the first, but it is defined in the domain of image $B$ and it is used to constrain the inverse transformation $h^{-1}(\cdot)$, from image $B$ to $A$. Compared to other techniques, the robustness of HAMMER is achieved through a combination of the first two terms of the energy function, i.e., the first term is focused on deforming image $A$ to $B$, whereas the second is focused on deforming image $B$ back to $A$. The third term is a smoothness constraint on the displacement fields, where $\nabla^2$ denotes the Laplacian operator and the parameter $\beta$ controls the smoothness of the deformation field.

One important factor of HAMMER is the way that the attribute vector is defined. If it contains rich enough information then it is possible to distinguish between different parts of the anatomy, even when this cannot be inferred from the intensity values alone. Typically, the attribute vector is defined for each voxel $x$ in a volumetric image and it reflects the anatomical structure at different scales. Therefore, an attribute vector $\alpha(x)$ consists of three vectors:

$$\alpha(x) = [\alpha_1(x) \; \alpha_2(x) \; \alpha_3(x)]$$

(3.31)

$\alpha_1(x)$ represents the edge type of the voxel $x$ in the image and it is designed based on the classification of tissue between the voxel $x$ and its neighbouring voxel $z$. In this way, prior knowledge about the image features and the type of the deformation is also incorporated into the registration scheme. In (3.31), $\alpha_2(x)$ represents the intensity of voxel $x$ with a range of grey levels. Finally, vector $\alpha_3(x)$ represents the Geometrical Moment Invariants (GMI)s of each tissue at different scales.

An intuitive way of using an intensity based approach to deal with 2D/3D registration has been suggested in [106] for the alignment of 2D optical images with 3D surface data of the human faces. In this work, registration is performed by optimizing a similarity measure based on photo-consistency with respect to the transformation parameters. The concept of photo-consistency can be summarised as follows. When the scene radiance is assumed to follow a locally computable lighting model, which means that shadows, antireflections and transparencies are not allowed, the observed intensity depends on the cosine of the angle between the surface normal and the vector to the light source. Note that it does not depend on the angle between the surface normal and the direction to each optical camera. Therefore, a surface point projected into different image planes should yield identical image intensities in all views. Given $N$ images of the 3D model, a similarity measure is
defined in order to decide whether a point in one view corresponds to a point in the other. The arithmetic mean of the pixel values associated with a given point is calculated over all optical images. Subsequently, the mean squared image intensity differences are calculated and a similarity measure based on a threshold value is introduced.

3.5.2 Feature-Based Techniques

Although the accuracy of intensity-based registration is usually better than feature-based registration, in some cases intensity-based techniques can fail due to anatomical or scene structure changes. Another important advantage of feature-based techniques is that they converge faster because they use a reduced representation of the image. For this reason, they are preferred when real-time response is necessary. These techniques have been successfully applied to AR and surgical applications involving implanted fiducial markers.

3.5.2.1 Feature Detection

Feature detection depends on the nature of the application and plays an important role in the accuracy of the registration. In general, features should be distinctive and efficiently detectable in both images. They should also be fixed during the whole experiment and sufficiently populated in the image plane. Possible features include landmarks and their surrounding regions, curves, segmented sets, or points. In 2D/3D registration, the notion of edges, regions or points needs to be generalised to 3D surfaces, such that a meaningful match between 2D edges and 3D ridges is possible. To this end, a number of definitions have been utilised to accomplish this, which also reveals that a mapping between the 2D and 3D feature space is not trivial.

In these approaches, regional features are mainly selected by segmentation and they should ideally be invariant to scale, rotation, and illumination changes. Curve and line selection is also popular. Standard edge detection methods, such as Canny detector, are often employed in 2D/2D registration [78]. However, they are usually not appropriate for 2D/3D registration as salient curves on the 3D model surface may not have an accurate match in the 2D image data. In the work by Pennec et al. [107], features of the 3D volume data are identified through differential geometry where ridges are extracted from the medial axis transform. It is worth noting that this definition does not represent salient curves on a surface.
Crest and extremal lines describe the loci of the surface where one of the principal curvatures reaches a local minimum/maximum along the principal curvature. Directional curvatures at a surface point \( P \) are defined by the intersection of the plane \( (P, t_1, \overline{n}) \) with the surface at point \( P \), where \( \overline{n} \) is the normal to the surface point \( P \) and \( t_1 \) presents the principal direction, which is the extremum of the surface curvature at point \( P \). Subsequently, a crest line can be extracted as the intersection of two implicit surfaces [107]:

\[
f = I \quad \text{and} \quad e_1 = 0
\]  

(3.32)

where \( f \) represents the intensity value of the image and \( I \) is an iso-intensity value. In (3.32), \( e_1 = \nabla k_1 \cdot t_1 \) and it is called the extremality function.

Extremal points are close to the notion of corner points. They can also be identified along the same line as extremal lines. Extremal points are defined as the intersection of three implicit surfaces [107]:

\[
f = I \quad \text{and} \quad e_1 = 0 \quad \text{and} \quad e_2 = 0
\]  

(3.33)

Similarly to \( e_1 \), \( e_2 \) is defined as \( e_2 = \nabla k_2 \cdot t_2 \) where \( k_2 \) is the associated principal curvature. In 2D images, the point features are typically extracted as the intersection of salient lines. In general, feature based methods are sensitive to feature extraction and selection. Although various techniques have been developed for automatic feature selection, a certain amount of user interaction is often required.

### 3.5.2.2 Feature Correspondence and Pose Determination

In feature-based registration, both correspondence between extracted features and pose should be determined. Usually they are tackled simultaneously in an iterative manner. Features are treated as points either by extracting the geometric centroids or by assuming that curves and lines are comprised of a set of points. The problem of establishing correspondence between two sets of points is not trivial especially when noise and localised deformation are present. In general, the set of points may be unequal in size and can differ due to non-rigid transformations. Point correspondence usually is formulated as a discrete linear assignment problem and the pose estimation is considered as a continuous least-squares problem. The combination of the two makes the non-rigid point registration
problem difficult. Thus far, a number of approaches have been developed to tackle these issues. The Iterative Closest Point (ICP) algorithm is one of the popular techniques to register 3D point clouds, curves and surfaces.

ICP is an iterative approach that identifies the optimal transformation between 3D shapes [besl92]. During each iteration, the algorithm establishes point correspondence by identifying for each point on the reference surface the closest point in the target surface. Typically, ICP uses a mean square distance metric over all six degrees-of-freedom and the algorithm can converge monotonically to the nearest local minimum. However, it requires an initial pose that is close to the solution in order for the algorithm to converge. In the case of large non-rigid deformation, the ICP algorithm does not always converge to the global minimum. This is due to the fact that ICP uses a crude way to assign correspondence, which can generate local minima and cannot guarantee one-to-one correspondence. Subsequently, its performance can deteriorate with the presence of outliers.

As an alternative, Chui et al. suggested the use of an energy function to solve both the correspondence and the transformation problem for non-rigid registration [108]. They point out that solving for binary, one-to-one correspondences and outliers at each step is not meaningful when the transformation is far away from the optimal solution. To cater for the uncertainty of whether a point matches with another, one allows fuzzy, partial matches between the two sets of points. This is the basic idea of softassign, which has also been used in [109]. In other words, the discrete problem is now turned into a continuous one in order to avoid getting trapped in a local minimum. Softassign has emerged from neural networks and it is concerned with how to assign fuzzy values in a set of variables in order to form a doubly stochastic matrix. Doubly stochastic matrix is a square matrix with all positive entries with rows and columns summed to one. It has been proved that a doubly stochastic matrix is obtained from any square matrix with all positive entries by the iterative process of alternating row and column normalisations. This is known as the assignment problem, which is a classical problem in combinatorial optimisation. However, non-rigid point-matching problem is much harder than the linear assignment problem, which can be solved in polynomial time.

Another useful optimisation technique, called deterministic annealing, can also be used to directly control the fuzziness by adding an entropy term to the energy function, called temperature [108]. Higher temperature values correspond to a higher degree of fuzziness in correspondence. The minima obtained at each temperature are used as initial conditions for the next stage as the temperature is lowered. The resulted energy function expresses both the correspondence and the pose problems at once and it is formulated as:
In the above energy function, two point sets $U$ and $V$ are defined in $\mathbb{R}^n$, $n = 2, 3$, so they consist of points $U \equiv \{u_a, a = 1, 2, \ldots, N\}$ and $V \equiv \{v_b, b = 1, 2, \ldots, K\}$. The non-rigid transformation is represented by function $f$, where $\|L^f\|^2$ is a smoothness term that imposes constraints on the mapping function. In (3.34), $m_{ba} \in [0, 1]$ and it is the fuzzy correspondence matrix which satisfies:

$$
\sum_{a=1}^{N+1} m_{ba} = 1 \text{ for } b \in \{1, \ldots, K\} \text{ and } \sum_{b=1}^{K+1} m_{ba} = 1 \text{ for } a \in \{1, \ldots, N\}
$$

(3.35)

where $T$ is the temperature and it is followed by the entropy term. In (3.34), the last term is the robustness control term that prevents the rejection of too many points as outliers. Although this formulation is limited to the alignment of 2D or 3D point sets, it can also be extended to registering data from different dimensions and modalities.

When it comes to 2D/3D registration, the perspective projection should also be taken into account. Existing approaches have addressed the problem of registering a 3D model and its perspective projection using a combination of the softassign technique for the problem of correspondence [110, 111]. It incorporates an iterative technique called Pose from Orthography and Scaling with Iterations (POSIT) for pose estimation. The basic principle of POSIT is based on the fact that the pose problem for perspective projection can be solved by iteratively refining the pose computed for scaled orthographic projection. According to the idea of ‘softassign’, the correspondence between image features and model features in the global objective function is expressed as a correspondence matrix in which each term is a weight for the match between an image point and a model point. This weight is a function of the distance between the scaled orthographic projection of the model point and the scaled orthographic projection after that model point is displaced onto the line of sight of the image point (for a camera pose that is refined during the iteration process). For the pose problem, scaled orthographic projection is used as an approximation of perspective projection, because it allows one to solve the pose problem with linear algebra and with fewer unknowns than with projective geometry.
3.6 Optimisation Techniques

Most of the image registration approaches involve an optimization step. Optimisation techniques are iterative processes for identifying the optimal transformation that maximizes the similarity between the two images/data sets. This is not an easy task especially when the transformation has large degrees-of-freedom and the similarity measure exhibits many local minima. Subsequently, the selection of an appropriate search space is crucial. In practice, the search space is affected by the similarity measure, the type of the transformation, and the image data involved.

Based on the similarity measure and the nature of transformation, a cost function can be constructed. Subsequently, global or local optimization techniques are used to minimize the cost function. Global optimization techniques are used to find the best set of parameters to optimize an objective function. One such method is the simulated annealing algorithm, which is a generalization of a Monte Carlo method applied to combinatorial optimization problems. Local optimization methods, on the other hand, usually converge faster but they cannot guarantee a global optimum solution. These methods include Powell’s method, Simplex, and Gradient Descent Algorithms. In these approaches, only information about the neighbourhood of the current approximation is used and subsequently, the algorithm is expected to converge to a local minimum/maximum close to the initial position.

3.7 Evaluation of 2D/3D Registration Algorithms

In image guided surgery, registration errors can have major consequences in patient safety. The quantification of the registration quality is also an important issue. For example, registration errors can have a strong influence on the quantification of the lesion progression and mis-alignment of important anatomical structures. Accurate evaluation of the performance of 2D/3D registration algorithms is of crucial importance in surgical guidance. Currently, there is a lack of a consistent, common scheme that would permit an objective, direct, and data-invariant comparison between different registration methods. Thus far, most techniques have been developed under the strict constraints of a particular application framework with lack of general applicability. Kraats et al [112] investigated this issue and introduced a standard validation method based on the 3D Rotational X-ray imaging system. However, this system is designed to address only the
problem of registering 3D CT with X-ray images. Otherwise, the ground truth data cannot be evaluated with this method and the target-registration error cannot be estimated.

The most fruitful part of this work is the criteria of validation, which can be applied to other registration problems. These are the ground truth data and the determination of accuracy. Although the availability of common datasets is necessary in order to establish a standardized evaluation framework, ground truth data are not always available. Usually, the ground truth data can be created under a controlled environment with additional tracking devices. The determination of accuracy involves the estimation of validation statistics under a range of different starting positions and experimental conditions.

In the case of bronchoscopy registration, 2D/3D algorithms have been investigated by a number of research groups [74, 75, 77, 113, 114]. However, a common framework for validating these algorithms has yet to be created. For example, Mori et al. [75] used visual inspection to decide whether a frame was successfully registered. In this way, there is no quantitative measurement of the error involved and an indication of the sensitivity of the algorithm to different registration parameters. In the work presented by Helferty et al., a rubber mould from a human airway tree was used to assess a biopsy procedure and average positional error between a target metal bead and the needle. However, there is no systematic assessment of the angular accuracy of the technique. The exploration of the influence of different initialisations on the convergence of the algorithm is presented in [77], although the notion of the ‘ideal registered position’ and ‘registration error’ has not been fully clarified.

3.8 Discussion and Conclusions

Image-based registration is a valuable tool for monitoring the progression of diseases, surgical planning, and intra-operative guidance. In general, image registration consists of four major components. These include the target and source datasets, the transformation model, a similarity measure, and an optimisation technique. The choice of the transformation model and the similarity measure plays a key role in the success of registration techniques and this will be the focus of the work presented in this thesis.

For free-form deformation, transformation can be described by a local and global component. The local component encodes the variability of the model and a number of
geometrical- and physical-based models have been proposed. Geometrical-based models offer a large number of unrestricted control points but it may be difficult to adjust in practice. Physical-based models provide an effective way of modelling the physical behaviour of complex structures. However, their application requires accurate measurement of the tissue properties, which in practice can be difficult.

For image registration, similarity measures can be classified as intensity-based and feature-based approaches. Intensity-based techniques compare image intensities without structural analysis, whereas feature-based techniques are dependent on the extraction and matching of features of the reference and target images. Intensity based techniques have a number of limitations. For example, cross-correlation based methods are sensitive to intensity changes introduced by varying illumination and changing modality. When applied to endoscopic image registration, careful adjustment of illumination condition is important to the accuracy of the registration result. Mutual information is based on the statistical dependency between datasets and thus is more suitable for registering multi-modal images. However, it can fail when the histogram is sparsely populated, which is typical for 2D/3D registration. A major disadvantage of intensity-based techniques is that image similarity does not necessarily imply correct registration. In practice, feature based approaches can augment the registration performance but the robust extraction of salient features used for registration can be a difficult task.

Despite extensive research in registration, non-rigid 2D/3D registration remains a challenging issue. This is partly due to the ill-posed nature of the problem as well as the difficulty in handling both global and local deformation. For this reason, most existing techniques reduce the problem to a 2D/2D registration scheme by using the 2D projections of the 3D data. However, this procedure introduces further ambiguity when deformation is present. In the next chapter, we will demonstrate that for bronchoscopy navigation, it is possible to exploit the unique camera-lighting configuration of the system to provide a robust surface geometry based 2D/3D registration algorithm. We will demonstrate why the proposed method is superior to existing techniques and how it handles significant lighting condition changes during bronchoscope examination.
Chapter 4

2D/3D Registration based on \( pq \)-based Representation

Registration of 2D bronchoscope video to 3D tomographic data is a popular research topic in medical imaging due to improved imaging hardware in recent years. For example, the work by Helferty et al. [113] analysed preoperative CT data of the airways to compute a segmentation of the tracheo-bronchial tree and the central lines through the major airways. In this way they used the central lines as a path to guide the registration process. In this method, the similarity measure is based on Normalised Mutual Information, which is calculated from weighted entropy terms, estimated from the normalised image histograms of the video and CT reconstructed images. Their hypothesis is that the darker pixel values have less noise and are more likely to form a match in the two images. In other words, dark areas give more useful information, as they correspond to distal lumen and airway bifurcations.

In the work by Bricault et al. [77], a multilevel strategy was adopted by considering the fact that the bronchial lumen presents no specific texture and grey values vary with the camera position and lighting conditions. In order to enhance the robustness of the algorithm, a model-based shape-from-shading algorithm is used to reconstruct the surface of the lumen and the subsequent 2D/3D registration is transformed into a 3D/3D registration problem.

Recently, Mori et al [74] presented a registration method that does not depend on the airway structure. The objective of the work was to develop a general framework for tracking the viewpoint and view direction of the fiberscope in relation to the 3D model. The method searches for the best viewpoint and view direction by calculating a matching ratio
between a reconstructed virtual bronchoscopic image and a real one. Camera motion is also estimated by extracting optical flows. Optical flows of a video sequences are the velocity fields that indicate the motion of objects from one frame to the other. For this method, since a cross-correlation based similarity measure was used, the lighting properties of the 3D model played an important role in the registration process. If the lighting is not carefully adjusted to match to that of the bronchoscope video, cross-correlation based similarity measures can fail. In real endoscopy, the illumination power decreases with the square of the distance from the light source. To simulate this effect, the parameters are adjusted manually such that similar visual appearance can be achieved. Throughout the registration process, the specular and reflection properties of the tissue are ignored.

Since image registration often depends on the inherent characteristics of the image data, it is necessary to exploit the unique constraints of the image formation process to facilitate faster and more accurate convergence of the registration algorithm. These special conditions, although may not be generally true, can reduce the ambiguity of the otherwise ill-posed problem. In this chapter, a hybrid approach is proposed. It is based on the extraction of surface normals from the bronchoscope video images by using the camera-lighting constraint of the bronchoscope. The components of these normals can be represented in the \((p,q)\) gradient space that expresses the slopes of the surface in the \(x\) and \(y\) directions as shown in Figure 4.1. This permits the development of a linear shape-from-shading technique to derive the 3D geometry of the 2D bronchoscope data such that registration can be performed based on the alignment of the surface normals. In this way, there is no need to adjust the lighting of the 3D model, and hence significantly increases the robustness of the registration algorithm.

The basic process of the proposed technique involves the following steps. Firstly, the surface normal for each pixel of the 2D video bronchoscope image is extracted by using a linear local shape-from-shading algorithm based on the camera/lighting constraint of the bronchoscope. The corresponding \(pq\) vectors of the 3D tomographic data are calculated from direct \(z\)-buffer differentiation. Subsequently, a similarity measure based on angular deviations of the \(pq\) vectors derived from 2D and 3D data sets is used to find the optimal pose of the camera in relation to the 3D model, thus establishing the correspondence between the 2D video and the 3D tomographic data.
4.1 pq-based 2D/3D Registration

In this study, our main interest is focused on the estimating pq-components of the exposed airway surface from monocular bronchoscopic images without the reconstruction of the whole surface. To this end, a linear shape-from-shading technique is adapted [115] and in this section detailed processing steps will be provided. The direct use of surface pq-values for 2D/3D registration avoids error prone processes of depth integration. First, we will introduce the basic concept of shape-from-shading and image formation process. We will then demonstrate how to incorporate the special camera-lighting configuration of the bronchoscope to reduce the problem to a linear form. Finally, a pq-space based similarity measure is constructed for the alignment of the 2D/3D data. Detailed phantom and in vivo experimental results are demonstrated at the end of this chapter.

![Figure 4.1](image.png)

**Figure 4.1:** A schematic illustration of the basic configuration of the image formation process for bronchoscope. In this figure, the camera coordinate system is used with the Z-axis towards the image plane. \( \hat{N} \) is the unit normal of a surface point. Variables \( p, q \) are called the surface gradients at that point, which are the slopes of the surface along the \( x \) and \( y \) direction, respectively. In the case of endoscope images, the viewing vector (the vector from the camera to the surface point) coincides with the lighting vector (the vector from the light to the surface point). However, the camera is close to the object and the angle between the viewing vector and the optical axis, \( \alpha \), is not negligible.
4.2 Shape-from-shading

Shape-from-shading is a classic problem in computer vision that has been well investigated since the pioneering work of Horn [116]. It addresses the problem of extracting both surface and relative depth information from a single image. Before we move on to the special case of extracting surface information from bronchoscopic images, we need to define a few basic concepts of how image intensity is related to scene geometry. To extract the 3D structure from a monocular image, it is also important to study how the brightness pattern of an image depends on the shape of the object being imaged. Most of the notations used in this chapter follow the convention adopted by Horn on his original work on Shape-from-shading and photometric stereo [116-118].

4.2.1 Human Vision and Depth Perception

How the human visual cortex infers depth information from 2D images is an issue that has intrigued scientists and philosophers for centuries. There are many cues that help the vision system to recover the third dimension. Well known cues for depth recovery include motion parallax and binocular disparity. Motion parallax is the apparent shift of an object against the background. Binocular disparity, on the other hand, is the difference between the relative positions of corresponding features resulting from the spatial separation of the two eyes.

When an image is projected onto the screen, the apparent depth information is lost. However, humans can still infer shape from monocular images with a remarkable degree of details [119]. It is believed that the extraction of shape-from-shading is a visual process, developed prior to higher-level cognitive tasks, such as perceptual grouping and motion perception [120]. The leading cue in a monocular image is the variations in brightness, also called shading. Shading should not be confused with shadowing, which is the locus of points where the rays graze the surface (self-shadow), or a region of darkness where light is blocked (cast-shadow). Shading arises partly from the fact that the fraction of the light source that appears at a particular point on the surface depends on surface orientation. Subsequently, shading provides cues all over the surface and not just along special contours. However, shading is related to shadowing in the sense that shadowing of parts of the microstructure of the surface may contribute to the shading effects apparent on the macro scale.
The recovery of shape-from-shading is by no means trivial, since the problem is ill-posed. Simply speaking, the orientation of an object located at a given distance from the camera depends on two parameters while there is only one measurement of brightness for each pixel. Therefore, to solve the shape-from-shading problem mathematically, additional constraints must be adopted. One common way of providing these constraints is to assume a smooth and continuous surface, implying that the surface orientations of the neighbouring surface patches are not independent. It has been shown that for the human vision system, object boundaries and prior knowledge plays an important role in resolving depth ambiguities. We often simplify the task of interpreting shading by assuming a single light source emitting light from above in relation to the retinal coordinates [119]. Furthermore, in our visual environment, we mainly deal with opaque objects, where the light rays do not penetrate through the material. Subsequently, we only need to focus on the surface of the object. It is with these simplifications, that the 3D shape of an object can be recovered given only one monocular image. To understand how to use the shape information encoded in the intensity values of an image, it is important to study the relationship between the light falling onto a surface and that radiated from the same point according to the reflectance properties and orientation of the surface. It is also important to study image formation process itself.

4.2.2 Understanding Image Intensities

The image of a 3D object depends on its shape, its reflectance properties, and the location of the light sources. It is also a function of the position of the object relative to the camera and the object’s orientation in space. Radiometry is the field that studies the measurement of electromagnetic radiation, including visible light. It is central to the determination of how the brightness at a particular point of the image is formulated. In this field, the amount of light falling onto a surface is called the irradiance and the amount of light radiated from a surface is called the radiance. Light transports energy through space and subsequently brightness is determined by the amount of energy the imaging system receives per unit area. The fact that a surface can anisotropically radiate into a whole hemisphere of possible directions is what complicates the notion of radiance, and subsequently the process of image formation. This is modelled with the Bidirectional Reflectance Distribution Function (BRDF), which is a measure of how much of the incident light is reflected in a particular direction. More accurately BRDF is the ratio of radiance to irradiance. Analytical
estimation of the reflectance function is difficult to construct, and hence it is usually determined empirically.

Assuming a single point light source and a unique value of BRDF, which can be assumed for most surfaces, the geometry of reflection is governed by three angles.

- The incident angle, which is the angle between the incident ray and the local normal;
- the emittance angle, which is the angle between the ray to the viewer and the local normal;
- the phase angle, which is the angle between the incident and the emitted ray.

In this way, object geometry, expressed as the normal to each surface point, is related to the light reflected in the direction of the viewer, and hence it is related to image intensity. A surface normal is the vector perpendicular to the tangent plane of a surface point and it can be described by two parameters only as it defines only the orientation of the surface. Moreover, it is unique as long as the surface is smooth. Here, it is practical to describe surface orientation based on viewer-centred coordinate system, such that one axis is aligned to the optical axis of the imaging system. By following this convention, it has been shown that in the image coordinate system the unit surface normal is described from the equation below [116].

\[
\hat{n} \cdot \mathbf{n} = \frac{(-p,-q,1)^T}{\sqrt{1 + p^2 + q^2}} \tag{4.1}
\]

where \((p,q)\) is called the gradient space, since its components represent the slopes of the surface in the \(x\) and \(y\) directions, respectively. These components can also be seen as the \(x\) and \(y\) projections of the normal to the camera coordinate system. The sign depends on whether the normal is pointing inside or outside of the 3D-object.

Gradient space is a map that provides insight into the geometrical properties of a surface. For a given type of surface and distribution of light sources, there is a fixed value of BRDF for every point in gradient space. Subsequently, image intensity is a single-valued function of \((p,q)\). The plot of the intensity values against the gradient space is also called reflectance map and it only depends on the surface properties and the light source distribution. In other words, a reflectance map encodes the relation between brightness and surface orientation, and the pattern it follows is characteristic of the surface and lighting properties. Subsequently, this map holds properties that make it attractive to image analysis.
because it establishes an explicit relationship between surface orientation and brightness. For example, the angular position of a point in gradient space corresponds to the direction of steepest descent on the original surface. Furthermore, a line that appears on the image and corresponds to the intersection of two surfaces would be mutually perpendicular to the gradient-line, which is the line that connects the points corresponding to each plane in the gradient-space.

The dependence of brightness on surface orientation can also be described as image irradiance equation, which illustrates that image irradiance $E(x, y)$ is proportional to the radiance $R(p, q)$ at the corresponding point on the surface.

$$E(x, y) = c \cdot R(p, q)$$ (4.2)

Note that $R(p, q)$ is described from the reflectance map and $c$ is a constant. Image irradiance equation is an important tool in recovering surface shape from a monocular image. However, the image irradiance equation only restricts the orientation of a surface point and cannot uniquely define the surface itself. Although there is a unique mapping from surface orientation to radiance, the inverse mapping is not unique. An infinite number of surface orientations give rise to the same brightness. To recover surface orientation locally, additional constraints must be introduced. For example, many surfaces are smooth without discontinuous in depth and the same is true for their partial derivatives. The assumption of smoothness is a strong constraint, since adjacent patches of the surface cannot have arbitrary orientations.

### 4.2.2.1 Special Cases of Reflectance Map

In this section, some special cases of illumination are further described as they are common in bronchoscopy and related to the problem of reconstructing 3D shape from endoscopic images. In practice, a number of factors can complicate the problem. For finding an analytical solution, certain assumptions must be taken into consideration. For example, it is common to assume a Lambertian model with an ideal point light source as well as a pinhole camera setup. However, it is also important to investigate the errors that these assumptions introduce.

The situation where a point source is located at the same place as the viewer leads to a Rotational Symmetric Reflectance Map (RSRM), which generally holds when the light
source is distributed in a rotationally symmetric fashion about the viewer. This is described from the equation bellow, where $g$ is assumed to be a strictly monotonic and differentiable function.

$$R(p, q) = g(p^2 + q^2) \quad (4.3)$$

Based on the image irradiance equation a system of five differential equations is constructed.

$$\begin{align*}
\dot{x} &= p, \quad \dot{y} = q, \quad \dot{z} = p^2 + q^2 \\
\dot{p} &= \frac{E_x}{2g}, \quad \dot{q} = \frac{E_y}{2g}
\end{align*} \quad (4.4)$$

where $(E_x, E_y)$ is the image brightness gradient, and the dots denote differentiation with respect to a step $\delta \xi$ in the direction of the steepest ascent. Giving starting values, this set of differential equations can be solved numerically.

It is common for shape-from-shading algorithms to assume an ideal Lambertian surface because of its mathematical simplicity. Lambertian surface under point source illumination is an idealised surface material that satisfies two conditions: (1) it appears equally bright from all viewing directions, and (2) it reflects all incident light. This surface when illuminated from a single light source obeys Lambert’s cosine law, which states that the amount of reflected light is proportional to the cosine of angle of incidence. In this case, the RSRM takes the form:

$$R(p, q) = \frac{1}{\sqrt{1 + p^2 + q^2}} \quad (4.5)$$

This special case can be used also when brightness decreases monotonically with the surface slope. It has been proved that for a surface that is illuminated symmetrically around the viewer, i.e., RSRM, with a Lambertian reflectivity model (not self-occluded and wholly contained in the field of view) a shaded image uniquely determines the imaged surface [121].

In most cases of recovering 3D shape from image intensities, mutual illumination is ignored. Mutual illumination is created from the interaction of objects, so that light is reflected from one to another. If the intensity of an image is also subject to mutual illumination, then the reflectance map cannot be uniquely determined. This implies that the
assumption of a unique value of image intensity with every surface orientation does not hold any more. The effects of mutual illumination include a reduction in contrast between faces and the appearance of gradation of light on images of planar surfaces. In the absence of this effect, one would expect planar surfaces to form images of uniform intensity, since all points on a plane surface have the same orientation. However, when the light source is close to the scene, significant gradients can appear.

Since mutual illumination is due to global lighting interaction, it is difficult to account for this effect in shape-from-shading schemes. In these cases, it is better to rely on the discontinuities of radiance rather than its actual values, since the former are related to the scene geometry [122]. It has also been noted that in the presence of inter-reflections, the extracted shape appears shallower than that of the real one and a refinement of the solution is required [123]. If the surface has small albedo, which implies that the surface material reflects only a small portion of the incident light, the inter-reflections can be ignored and the solution is very close to that suggested by the image irradiance equation.

4.2.3 Image Formation

Image formation is a transformation between two worlds: (1) a 2D surface and the depth values, and (2) a 2D image and its intensity values. The relationship between object coordinates and image coordinates is given by the well-known perspective projection equations where $f$ is the focal length.

$$x_0 = \frac{x}{z} f, \quad y_0 = \frac{y}{z} f$$

(4.6)

where $f$ is the focal length. Horn [118] also proved that image irradiance is proportional to scene radiance by taking in consideration an ideal pinhole imaging system. This is achieved by considering a patch on the surface of the object that makes an angle $\alpha$ between the optical axis and the light ray going through the centre of the solid angle subtended by the object’s surface patch. Note that the solid angle subtended by a surface patch is defined as the surface area of a unit sphere covered by the surface’s projection onto the sphere. This area is proportional to the area of the patch and the cosine of the angle of inclination, but it is inversely proportional to the square of its distance from the origin. The solid angle defined by the origin of the lens and the surface patch must be equal to the solid angle defined by the lens and the corresponding image patch. Assuming that no light from other areas reaches this image patch, it follows that:
\[ E(x, y) = L \left( \frac{d}{f} \right)^2 \cos^4 \alpha, \text{ where } \tan \alpha = \frac{1}{f} \sqrt{x^2 + y^2} \] (4.7)

where \( E(x, y) \) is the irradiance of the image, \( L \) is the reflected radiance of the surface in the direction towards the lens, \( d \) is the diameter of the lens, and \( f \) is its focal length. This is a basic relationship for recovering information about the shape of an object and it is also related to the image irradiance equation defined above.

4.3 Shape-from-shading in Bronchoscopy

The essence of shape-from-shading problem is to recover surface \( z(x, y) \) given image \( E(x, y) \) and a reflectance map \( R(p, q) \). The image irradiance equation can be viewed as a non-linear first-order partial differential equation. Since the original development of shape-from-shading in computer vision [116], many different techniques have been developed. Zhang et al. presents a survey of most well-known algorithms in the field. In general, each algorithm works well for a certain type of images, but can perform poorly for others. An early solution was based on singular points that have maximum brightness. The orientation of the surface can be determined directly and uniquely at these points because their normals are parallel to the principal axis of the camera. Subsequently, the solution can be extended from these points to the rest of the surface by using method of characteristic strips [118].

The method of characteristic strips is classified as a propagation approach, because it propagates the shape information from a set of surface points to the whole image. The image irradiance equation provides a system of five differential equations, three related to the spatial coordinates of the system and two related to image gradients. A solution of these differential equations produces a curve on the surface of the object. These equations are integrated numerically along these curves of the object, and therefore called characteristic strips. The projections of characteristic strips on the image are called base characteristics. The direction of characteristic strips is identified as the direction of intensity gradients. To obtain the depth distribution of the whole surface, characteristics strips must be joined together. Each of them requires an initial starting position, possibly provided as a singular point of a boundary constrain. The method of characteristic strips has been refined from a number of research groups, among them Kimmel et al. [124], who suggested an Eulerian
formulation of the problem. Nevertheless, propagation approaches suffer from a number of problems, such as sensitivity to measurement errors and their accumulation by the numerical integration of the differential equations, as well as numerical instabilities.

Alternative solutions can be found by forming a minimisation problem of the difference between the observed brightness and that predicted from the estimated shape, thus leading to relaxation/minimisation techniques [118]. In other words, an energy function, which consists of the brightness and the smoothness constraints, is minimised. Additional constraints based on the integrability of the surface and regularisation terms are also introduced. These techniques are also called variational approaches because they use calculus of variations to find the appropriate function to minimise. In general, minimisation approaches are more robust than propagation methods but they require a number of additional parameters to be estimated related to the constraints of the minimisation function. An important disadvantage of this method is that the image irradiance equation is not satisfied as a hard constraint. As a result, the reconstructed image can be different from the initial image.

Most of the shape-from-shading algorithms assume that both the viewer and the lighting source are relatively far from the object’s surface, which suggests an orthographic projection and parallel illumination. Under orthographic projection, the position of an image projection is independent of the depth of the surface point. Note that parallel illumination light leads to uniform illumination intensity for the entire object surface. Therefore, the main analysis is based on the assumption that the angle between the viewing vector \( \hat{V} \) and the Z-axis, \( \alpha \), is negligible. However, this is only correct when the object size is small compared to its distance from the camera, and these assumptions are violated in endoscopic images.

Recovering shape-from-shading from an endoscope images is a special case. In endoscope, both the camera and light source are close to each other and positioned in close proximity to vessel walls. Furthermore, the direction of the illuminating light coincides with the axis of the camera. Indeed, endoscopes usually have two light sources that they are symmetrically placed on either side of the lens. This light distribution can be approximated as a light source at the tip, which coincides with the centre of the lens [125]. In this case, the image intensity depends not only on the 3D shape of the object, but also on the distance between the light source and the surface. This is also apparent by studying the image formation equation. Even when the illumination is uniform, the term \( \cos^4 \alpha \) implies non-uniform image brightness. The wide-angle lenses of the endoscope also enlarge the error
assumed by orthographic projection. In this case, no assumption can be made on the angle between the viewing vector and the principal axis, $\alpha$, being negligible and lighting being uniform. Conventional algorithms, such as of Bichsel et al. [126], Oliensis et al. [127], Tsai et al. [128], and Worthington et al. [129] are not applicable. Although the light source distribution in endoscope images is not the same as in conventional shape-from-shading formulations, it offers an advantage to solve the problem as the direction, as well as the distance, from an object’s surface point to the light source and the projection centre is the same. Furthermore, the light is distributed in a rotationally symmetric way around the viewer that leads to a RSRM discussed earlier.

Due to the fact that shape-from-shading is an ill-posed problem in a general context, the practical uptake of the technique is slow. However, recently it has been shown that for the special lighting-camera configuration in endoscopy, the problem can be considerably simplified and constrained. There are a number of studies focused on 3D shape from endoscopic images [115, 130-132]. Their main objective of these studies is to reconstruct the 3D shape of an endoscopic view of the inner vessel wall, with applications to tumour assessment and surgical planning.

In the work by Deguchi et al. [133] and Okatani et al. [130] is suggested an extension to the propagation technique based on the work of Kimmel et al. [124], which exploits the endoscopic configuration setup of the light source with relation to the camera. They introduce the notion of equal distance contours, which are space curves on the endoscopic surface with points at equal distance from the light source. In other words, they are the intersection of the object surface with a sphere whose centre is the light source. Based on the constraints introduced from the camera-lighting configuration, the surface normal at each point on the contour is completely defined. If an initial equal distance contour is given, the total surfaces can be recovered by propagating this contour with respect to the distance from the lighting source. Finally, a numerical algorithm is constructed based on finite difference approximation. To reconstruct the shape of an object with this method, it is necessary to determine at least one initial contour. This method suffers from the same problems as the propagation methods presented earlier. For example, error due to image noise can accumulate by the numerical integration involved.

Henrique et al. extends the work of Okatani et al. [130] to account for the radial distortion of endoscopic images for non-Lambertian surfaces. Henrique et al. observed the strong radial image distortion in the endoscopic images. This is a result of the small focal length and the wide-angle lenses used. To correct for radial distortion, the work by Henrique et al. uses a model based on a spherical projection followed by an orthographic projection.
Concerning the Lambertian surface assumption, this work uses a dichromatic model to remove the specular reflection component present in the image to obtain a Lambertian surface equivalent. A dichromatic model suggests that light is reflected in one of two ways: surface reflection, which encompasses mirror surfaces and body reflection, which refers to matte surfaces. However, this work did not contribute to any substantial algorithmic change in the main algorithm of propagating equal distance contours introduced in the work by Okatani et al. [130]. In both studies by Okatani et al. and Henrique et al. the image formation model was based on the fact that the light source and the camera projection centres are at the same position and close to the surface. This can be described by the equation below:

$$E(x, y) = \rho(x, y) \frac{G(\cos \vartheta)}{r^2}$$  \hfill (4.8)

where $E(x, y)$ is the image irradiance, $\rho$ is the surface albedo, $\vartheta$ is the angle between the surface normal and the observed direction, coincident with the light source direction. In Equation (4.8), $r$ is the distance between the projection centre and the corresponding point on the surface, whereas function $G(\cos \vartheta)$ models the BRDF under the assumption that it is monotonically decreasing with respect to $\vartheta$. Note that this assumption is more general than the Lambertian model.

Rashid et al. [115] also investigated the problem of reconstructing 3D shape from endoscopic images. The main objective of his study was to recover the gradient space ($pq$-components) for each pixel of the image so that the surface can be reconstructed later by using an integration step. He also exploited the special camera-light configuration of endoscope and by considering a Lambertian surface property of the vessel lumen. To model the dependency between the distance of the 3D object to the light source, and the image intensity of the corresponding image patch he added one more factor to the image irradiance equation. This was a monotonically decreasing function $f(r)$ between the surface point and the light source. Therefore, the image irradiance function $E$ is formulated as:

$$E(x, y) = s_0 \cdot \rho(x, y) \cdot \cos \vartheta \cdot f(r)$$  \hfill (4.9)

where $s_0$ is a constant related to the camera, $\rho$ is the surface albedo, and $\vartheta$ is the angle between the incident light ray and the surface normal. The main advantage of his technique is in its mathematical simplicity, which leads to a system of linear equations. In this way,
the \( pq \)-vectors can be calculated from a closed-form solution as it is shown in the following section.

Recently, Prados et al. [132] studied the Lambertian shape-from-shading problem for a pinhole camera with a point light source located at the optical centre. Under these assumptions, the problem is well-posed and the solution can be found by solving the partial differential equation without additional constraints. Their main contribution of this work is that it proves the existence and uniqueness of the solution in the case of the light source locating at the same place as the camera. They also provide an iterative algorithm that is robust to image noise and errors on the estimation of intrinsic camera parameters and lighting configuration.

\subsection{A closed-form solution for extracting \( pq \)-components}

The method for \( pq \)-value extraction for 2D/3D registration in this thesis follows the work of Rashid \textit{et al} [115]. It is a linear method of determining local surface orientation of a smooth, Lambertian surface and it does not require prior knowledge of the albedo or the illumination strength. Instead, it assumes that the light source coincides with the camera lenses, which is true for endoscopic procedures. The method accepts surfaces with a slowly varying (or locally constant) albedo and perspective image projection was used for the image irradiance equation, \textit{i.e.},

\begin{equation}
E(x, y) = k \cdot \rho \cdot I_0 \frac{\cos \vartheta}{r^2}
\end{equation}

where \( k \) is a camera constant and \( I_0 \) the intensity strength of the light source, which is also assumed to be constant. In the above equation, \( \vartheta \) is the angle between the incident light ray and the surface normal defined as \( \mathbf{n} = [p, q, -1] \), whereas \( r \) is the distance between the light source \( S = (X_S, Y_S, Z_S) \) and a surface point \( P = (X, Y, Z) \). The surface albedo \( \rho \) is defined as a function of \( (x, y, z) \). Subsequently, the cosine of \( \vartheta \) can be expressed by the normalised dot product of the surface normal and the vector that connects point \( P \) with the light source \( S \):
\[
\cos \vartheta = \frac{\left( X_s - X \right) p + \left( Y_s - Y \right) q - \left( Z_s - Z \right)}{\sqrt{\left( p^2 + q^2 + 1 \right) \left[ \left( X_s - X \right)^2 + \left( Y_s - Y \right)^2 + \left( Z_s - Z \right)^2 \right]}}
\]  

(4.11)

The distance from the light source is:

\[
 r^2 = \left( X_s - X \right)^2 + \left( Y_s - Y \right)^2 + \left( Z_s - Z \right)^2
\]

(4.12)

Based on these formulations, image irradiance at an image point \((x,y)\) that corresponds to the brightness of the surface point \(P\) can be written as:

\[
E (x, y) = k_0 \rho \frac{(X_s - X) p + (Y_s - Y) q - (Z_s - Z)}{(p^2 + q^2 + 1)^{1/2} \left[ (X_s - X)^2 + (Y_s - Y)^2 + (Z_s - Z)^2 \right]^{3/2}}
\]

(4.13)

Note that all distance measures have been normalised with respect to the focal length so that:

\[
k_{0} = k \frac{I_0}{f^2}
\]

(4.14)

By placing the point light source at the origin of the camera coordinate system simplifies these equations. As mentioned earlier, although physically the light source cannot be placed exactly at the same point as the camera, endoscopes have two light sources placed symmetrically on each side of the lens. This configuration can be approximated well with a system where both the camera and the light source are at the origin of the coordinate system and have the same direction pointing towards the -Z axis (principal axis) [115].

\[
X_s = Y_s = Z_s = 0
\]

(4.15)

Furthermore, the well known relationship between \(P\) and its image position \((x,y)\) under perspective projection can be used to replace the 3D coordinates with the corresponding image coordinates. Subsequently, the image irradiance equation can be written as

\[
E (x, y) = k_0 \rho \frac{1 - p - q}{Z^2 \left( p^2 + q^2 + 1 \right)^{1/2} \left( 1 + x^2 + y^2 \right)^{3/2}}
\]

(4.16)

By assuming a smooth surface and continuous differentiable function \(Z = Z(x, y)\), local Taylor series expansion at \(P_0 = (X_0, Y_0, Z_0)\) can be used to derive \(Z\), i.e.,
\[ Z = Z_0 + (X - X_0) \frac{\partial Z}{\partial X} + (Y - Y_0) \frac{\partial Z}{\partial Y} + \cdots \]  
(4.17)

By omitting the higher order terms, we have:

\[ Z = Z_0 + (X - X_0) p_0 + (Y - Y_0) q_0 \]  
(4.18)

where the components of the normal vector \([p_0, q_0, -1]\) at the surface point \(P_0\) are given by:

\[ p_0 = \frac{\partial Z}{\partial X} \big|_{X=x_0, Y=y_0} \text{ and } q_0 = \frac{\partial Z}{\partial Y} \big|_{X=x_0, Y=y_0} \]  
(4.19)

Function \(Z\) can also be represented in terms of the image coordinates by taking into account the perspective projection, which yields

\[ Z = \frac{Z_0 (1 - p_0 x_0 - q_0 y_0)}{1 - p_0 x - q_0 y} \]  
(4.20)

The substitution of the above expression into the image irradiance equation results in the following formulation:

\[ E(x, y) = k_0 \rho_{av} \frac{(1 - p_0 x - q_0 y)^3}{Z_0^2 (1 - p_0 x_0 - q_0 y_0)^2 (1 + p_0^2 + q_0^2)^{1/2} (1 + x^2 + y^2)^{3/2}} \]  
(4.21)

This equation represents image irradiance corresponding to a planar patch approximation of the surface around \(P_0 = [x_0, y_0, Z_0, Z_0]\). In Equation (4.56), \(\rho_{av}\) represents the average albedo in the neighbourhood of \(P_0\), and it is evident that the above image irradiance equation is a non-linear function of \(p_0, q_0, Z_0\) and \(\rho_{av}\).

Further information about the local analysis of image irradiance is obtained from the directional derivatives. The variation of image irradiance along the \(x\) and \(y\) directions can be obtained by using the partial derivatives of the image intensity, \(i.e.,\)

\[ dE = \frac{\partial E}{\partial x} dx + \frac{\partial E}{\partial y} dy \]  
(4.22)

By differentiating the image irradiance equation with respect to \(x\) and \(y\), we have:
\[
\frac{dE}{E} = -3 \left[ p_0 dx + q_0 dy + \frac{x_0 dx + y_0 dy}{1 + x_0^2 + y_0^2} \right]
\]

There are two important outcomes from this formulation. Firstly, the derivative of the image irradiance equation is proportional to the image intensity itself. Secondly, the average albedo \( \rho_{av} \) and the camera constant \( k_0 \) have been cancelled out. Subsequently, the surface information encoded in the normal components \( (p_0, q_0) \) depends only on image intensities and the image coordinate position of the correspondent pixel. Since \( dx \) and \( dy \) are in two independent directions, the \( x \) and \( y \) components of the normalised derivatives can be separated. Therefore, after some algebraic manipulations, a system of two linear equations with the two unknowns can be derived:

\[
\begin{align*}
A_1 \cdot p_0 + B_1 \cdot q_0 + C_1 &= 0 \\
A_2 \cdot p_0 + B_2 \cdot q_0 + C_2 &= 0
\end{align*}
\]

where

\[
\begin{align*}
A_1 &= (-x_0 \cdot R_x + 3) \cdot (1 + x_0^2 + y_0^2) - 3 \cdot x_0^2 \\
B_1 &= -R_x \cdot (1 + x_0^2 + y_0^2) \cdot y_0 - 3 \cdot x_0 \cdot y_0 \\
C_1 &= R_x \cdot (1 + x_0^2 + y_0^2) + 3 \cdot x_0 \\
A_2 &= -R_y \cdot (1 + x_0^2 + y_0^2) \cdot x_0 - 3 \cdot x_0 \cdot y_0 \\
B_2 &= (-y_0 \cdot R_y + 3) \cdot (1 + x_0^2 + y_0^2) - 3 \cdot y_0^2 \\
C_2 &= R_y \cdot (1 + x_0^2 + y_0^2) + 3 \cdot y_0
\end{align*}
\]

In the above equation, \( R_x = E_x / E \) and \( R_y = E_y / E \) are the normalized partial derivatives of the image intensities, \( E \) is the intensity of the pixel under consideration and \( x_0 \) and \( y_0 \) are the normalized image plane coordinates. In practice, the partial derivatives involved in the above equations can be estimated by convolving the image with a 3×3 Sobel kernel.

To demonstrate how the above algorithm works, Figure 4.2 demonstrates the derived \( pq \)-distribution from \textit{in vivo} bronchoscopic images. These values can be integrated to form a 3D surface representation of the airways, as shown in Figure 4.2, where the reconstruction was based on the algorithm proposed by Agrawal \textit{et al} [134]. This technique uses graph
theory to impose constraints and enforce integrability. The depth map shown in Figure 4.2 also illustrates the potential problems associated with surface reconstruction from $pq$-distribution. Usually, these reconstruction algorithms assume integrability of the vector field, which is not true in endoscopic images due to large surface discontinuities. Furthermore, the integration process exacerbates inherent inaccuracies of the shape-from-shading algorithm due to specularities and deformation. For this reason, we have therefore used the $pq$-distribution directly in deriving the registration algorithm.

Figure 4.2: a) Example endoscopic video images. b) The derived $pq$-vectors based on the linear shape-from-shading algorithm of Rashid et al. c) The depth map derived from the integration of the $pq$-vectors. d-e) The reconstructed surface from two different viewing angles.
4.3.2 Extraction of pq-components from the 3D Model

![Figure 4.3](image.png)

Figure 4.3: a) A rendered pose of the 3D model and b-c) the corresponding $p$, $q$ components, respectively. The $p$, $q$ components have been derived by differentiating the z-buffer and the colormap encodes the value of $p$ and $q$ for each pixel.

As for tomographic images, the extraction of the $pq$ components from the 3D model is relatively straightforward as the exact surface representation is known. Differentiation of the z-buffer for the rendered 3D surface will result in the required $pq$ distribution, which also elegantly avoids the tasks of occlusion detection. The effect of perspective projection has been taken into account during the rendering stage. An example of the extraction of the $pq$-components from the z-buffer is shown in Figure 4.3.

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}$$  \hspace{1cm} (4.26)

4.3.3 Similarity Measure

At a first glance, one would expect to use the angle $\vartheta$ between the surface normals extracted from shape-from-shading and those from the 3D model for constructing a minimization problem for 2D/3D registration, i.e.,

$$\vartheta = \frac{p_{img}p_{3D} + q_{img}q_{3D} + 1}{\sqrt{(p_{img}^2 + q_{img}^2 + 1)(p_{3D}^2 + q_{3D}^2 + 1)}}$$  \hspace{1cm} (4.27)

This, however, was proved ineffective because of the scaling problems and image noise involved. We used instead the $pq$ components directly to formulate the similarity measure.
Analytically, for each pixel of the video frame, a $pq$-vector corresponding to the normalised surface normal, $\mathbf{n}_{img}(i,j) = [p_{i,j}, q_{i,j}]^T$, was calculated by using the linear shape-from-shading algorithm described above. Similarly, for the current pose of the rendered 3D model, the corresponding $pq$-vectors $\mathbf{n}_{3D}(i,j) = [p'_{i,j}, q'_{i,j}]^T$ for all rendered pixels also extracted by differentiating the z-buffer. The similarity of the two images was determined by evaluating the dot product of corresponding $pq$-vectors as

$$\varphi\left(\mathbf{n}_{3D}(i,j), \mathbf{n}_{img}(i,j)\right) = \frac{\mathbf{n}_{3D}(i,j) \cdot \mathbf{n}_{img}(i,j)}{\|\mathbf{n}_{3D}(i,j)\| \cdot \|\mathbf{n}_{img}(i,j)\|}$$

(4.28)

By introducing a weighting factor that is proportional to the norm of $\mathbf{n}_{3D}$, the above equation can be reduced to:

$$\varphi_w\left(\mathbf{n}_{3D}(i,j), \mathbf{n}_{img}(i,j)\right) = \frac{\mathbf{n}_{3D}(i,j) \cdot \mathbf{n}_{img}(i,j)}{\|\mathbf{n}_{img}(i,j)\|}$$

(4.29)

In practice, the weight needs to be normalised to reduce the effect of spurious data due to noise amplified by the differentiation of the z-buffer. This is also necessary to avoid the degeneration of the similarity measure and the introduction of local minima. This problem is caused by low values of the $pq$ components, such as when the camera directly faces the tracheo-bronchial wall. The normalised similarity measure can be represented as:

$$\varphi'_w\left(\mathbf{n}_{3D}(i,j), \mathbf{n}_{img}(i,j)\right) = \frac{\mathbf{n}_{3D}(i,j) \cdot \mathbf{n}_{img}(i,j)}{\|\mathbf{n}_{3D}(i,j)\| \cdot \|\mathbf{n}_{img}(i,j)\|} N_{3D}$$

(4.30)

where $N_{3D}$ is the average length of the $pq$ vectors derived from the 3D model. In this formulation, the $pq$ vectors are also normalised, $\|\mathbf{n}_{3D}(i,j)\|_{\text{norm}}$, based on an empirically chosen upper limit $K$.

$$N_{3D} = \frac{\sum_{i,j} \|\mathbf{n}_{3D}(i,j)\|_{\text{norm}}}{K}$$

(4.31)

By incorporating the mean angular differences and the associated standard deviations $\sigma$, the following similarity function can be derived:
To formulate a minimisation problem, the above similarity measure takes the form as shown below:

\[
S = \sum \left( \varphi \right) \cdot \sum \left( \sigma \left( \varphi \right) \cdot N_{3D} \right) \quad (4.32)
\]

By minimising the above equation, the optimal pose of the camera for the video image can be derived. The reason for introducing a weighting factor is due to the fact that \(pq\)-estimation from the 3D model is more accurate than that of the shape-from-shading algorithm. This is because it is not affected by surface textures, illumination conditions or surface reflective properties. The weighting factor therefore reduces the potential impact of erroneous \(pq\)-values from the shape-from-shading algorithm and improves the overall robustness of the registration process. The standard deviation has been introduced to increase the specificity of the algorithm based on the premise that a systematic error may not always indicate pose misalignment. On the contrary, it may be due to inherent inaccuracies of the \(pq\) estimation technique. Based on the standard deviation, poses that introduce mis-alignment between the image and model \(pq\)-vectors are penalised.

**Figure 4.4** demonstrates the effect of the normalised weighting factor on the optimisation landscape. In this figure, the origin corresponds to the optimal pose based on visual inspection and manual refinement. By evaluating the similarity measure in a square area around this pose, the shape of the function to be optimised is revealed. The white square indicates where the minimum is located. The weighting factor bounds the minimum in an area that is close to the optimal position. Without the use of this factor, the similarity measure can easily slide to a minimum introduced by texture discrepancies.

Ideally, the optimisation landscape should be smooth and free of local minima. For the \(pq\)-based registration approach, this landscape depends also on the distinctive features of the endoscope image. Bifurcations tend to be the best example of optimisation landscape due to the uniqueness of the geometrical feature, whereas frames acquired when the camera faces the lumen exhibit a poor global minimum.
Figure 4.4: Examples demonstrating the effectiveness of the normalised weighting factor. On the left column is the similarity measure with the normalised weighting factor. On the right column is the similarity measure based on the angle. The position at (0,0) coordinates corresponds to the optimal pose based on visual inspection and manual refinement. By evaluating the similarity measure in a square area around this pose, the shape of the function to be optimised is revealed. The white square indicates where the minimum lie.
4.3.4 Tissue Deformation

Figure 4.5: a) A video frame from a deformed airway phantom, b) the associated p-q space deformation map where bright intensity signifies the amount of deformation detected. (c-d) The superimposed 3D rendered image with pose estimated from intensity-adjusted registration and pq space registration with deformation weightings, respectively.

By the use of the proposed pq-space representation, the angle between the normal vectors before and after rigid body transformation will remain the same for every surface point. Local deformation can therefore be identified at surface points where the angle diverts from the mean angle of the 3D model. Given the fact that the 3D model is more or less registered to the corresponded video frame, localized inter-frame deformation can be isolated and subsequently excluded from the pose estimation process. In this study, we used the pq deformation map as a weighting factor during the registration process such that the weighting provided was inversely proportional to the amount of deformation detected.

The effect of localised deformation on the pq-space representation is illustrated in Figure 4.5, where (a) is the original video bronchoscope image and (b) is the derived pq-space deformation map. Figure 4.5 (c) and (d) demonstrate the accuracy of the pose estimation with the traditional intensity based technique and the proposed pq-space registration with deformation weighting, respectively. It is evident that in the presence of tissue deformation, the intensity-based technique has introduced significant error, despite careful adjustment of illumination conditions.
4.3.5 Video Preprocessing

![Figure 4.6:](image)

The pre-processing steps applied to the bronchoscope videos before 2D/3D registration. a) Original video frame acquired from the prototype bronchoscope, b) de-interlaced video frame, c) after lens distortion correction, and d) final smoothed image by using an anisotropic filter that preserves local geometrical features.

Pre-processing of the videos was necessary in order to alleviate the effects of interlacing, lens distortion and filter out redundant texture information. In Figure 4.6 the main steps involved in video pre-processing are illustrated, which include de-interlacing, distortion correction, and anisotropic filtering.

Digital camcorders and VCRs that use either NTSC or PAL format interlace two consecutive frames into one such that one video frame contains the even and the next the odd lines. We used VirtualDub filter (http://biphome.spray.se/gunnart/video/) to remove interlacing. Furthermore, the wide angle lens of the bronchoscope camera can cause severe barrel distortions. In general, methods that correct for ‘barrel’ distortion must calculate the centre of distortion and correct for both radial and tangential components. Radial distortion is the most commonly seen distortion and it causes the actual image plane to be displaced radially in the image plane. Tangential distortion is due to ‘decentering’ or imperfect centring of the lens or other manufacturing defects. For camera calibration, we used the
OpenCV matlab toolbox, which includes both radial and tangential distortion correction steps.

The initialization of the calibration parameters is similar to Zhang et al. [135] based on a closed-form solution after distortion correction. The intrinsic parameters of the camera have been estimated along with the first five coefficients that characterise the distortion of the lens. Subsequently, the distortion parameters are used to restore each endoscopic frame, and the intrinsic camera parameters are used to derive the perspective projection. The mathematical details involved in this step are provided in Appendix A. Finally, a structure adaptive anisotropic filter was applied to the video image to remove image noise and artefacts [136]. The method uses a local orientation and an anisotropic measure of level contours to control the shape and extent of the filter kernel, and thus ensures that corners and edges are well preserved throughout the filtering process.

4.4 Experiments and Results

The proposed method was implemented in Microsoft Visual C++ on a conventional PC machine (2 GHz Intel Pentium 4 processor, 512MByte main memory, nVidia GeForce 4 MX 440 graphics card, with Microsoft Windows 2000 operating system). Surface rendering was implemented using OpenGL. The interface was based on FLTK (www.fltk.org). The technique has been validated both with phantom data and in vivo data. For the phantom validation ground truth data are acquired from a 6DoF EM tracker attached to the camera. For the in vivo study ground truth data has been collected by manually registering selected poses and interpolating the intermediate points.

4.4.1 Phantom Study

In order to assess the accuracy of the proposed algorithm, an airway phantom made of silicone rubber and painted with acrylics, as shown in Figure 4.7 (a), was constructed. The phantom has a cross-sectional diameter of 12cm at the opening and narrows down to 5cm at the far end. The inside face was coated with silicone-rubber mixed with acrylic for surface texturing and left to cure in the open air. This gives the surface a specular finish that looks similar to the surface of the lumen. A real-time, six degrees-of-freedom Electro-Magnetic
(EM) motion tracker (FASTRAK, Polhemus), shown in Figure 4.7 (b), was used to validate the 3D camera position and orientation. The EM-tracker has an accuracy of 0.762mm RMS. The tomographic model of the phantom was scanned with a Siemens Somaton Volume Zoom four-channel multi-detector CT scanner with a slice thickness of 3mm and in-plane resolution of 1mm. A CMOS camera and NTSC standard with frame rate of 29.97fps was used.

![Figure 4.7: a) An airway phantom made of silicon rubber and painted with acrylics was constructed in order to assess the accuracy of the pq-based registration. b) A real-time six DOF EM tracker motion tracker (FASTRAK, Polhemus) was used to validate the 3D camera position and orientation.](image-url)
4.4.2 In vivo Validation

Figure 4.8: Assessment of manual alignment error compared to the EM tracker as assessed by using the 3D bronchial model. a) Euclidean distance between the first and subsequent camera positions as measured by the EM tracker and after manual alignment with step equal to 10. b) Inter-frame angular difference between manual alignment and readings from the EM tracker.

For preliminary in vivo validation, bronchoscopy examination was performed in one patient according to a conventional clinical protocol. During the bronchoscope procedure a prototype videoscope (Olympus BF Type; with field of view 120°) was used. Video images from the bronchoscopic examination were transferred to digital videotapes in PAL format at 25fps. Since the original endoscopic video frames contain both the endoscopic image and redundant black background, only the endoscopic view was digitised and cropped to images of 454×487 pixels. All images were converted to greyscale before the pq-space
analysis. Similar to the phantom study, the CT images were acquired from the Siemens Somaton Volume Zoom four-channel multi-detector CT scanner with a slice width of 3mm and collimation of 1mm, and the acquisition volume covered from the aortic arch to the dome of hemi-diaphragm.

The technique has been evaluated both with qualitative and quantitative analyses. Quantitative results were obtained with gold standard data derived from manual alignment of the 3D model to the video frames. Manual alignment always entails an error. To assess this, the distance from the initial reference position and the angle from the initial vector are displayed in **Figure 4.8** for both the EM tracking data and the manual alignment. The measurements have been done by using the airway phantom and the EM tracker. Analogous to phantom validation, a video sequence has been tracked for more than 400 frames using the EM tracker. For the same sequence we used manual alignment for every 10 frames to get the position of the virtual camera that matches best the corresponding video view. The average positional error is equal to 3mm with STD 2.26mm and the angular error is 0.0381rad with STD 0.0285rad. This indicates that the error is consistent throughout the video sequence and relatively small. We expect that this error will be smaller in real bronchoscope images as the phantom’s scale is greater than the real size of the airways.

### 4.5 Results

#### 4.5.1 Phantom study

To assess the accuracy of the proposed algorithm in tracking camera poses in 3D, **Figure 4.9** and **Figure 4.10** compare the relative performance of the traditional intensity based technique and EM tracked poses against those from the new method. Since the tracked pose has six degrees-of-freedom, we used the distance travelled and inter-frame angular difference as a means of error assessment. The video acquired has 1.73 minutes duration (25fps). A continuous part of 40 sec (1000 frames) has been used for tracking. Traditional cross-correlation with the illumination conditions manually adjusted failed when the bronchoscope gets through the main bifurcation to the left bronchi and faces directly the far end of the tubular phantom airway, (394 frame). Three different variations of the pq-space technique were tested and compared against the gold standard EM tracker data (black line) and the intensity based technique (blue line). As expected the intensity-based technique is highly sensitive to lighting condition changes, and with manual intensity adjustments, the
convergence of this method is improved. However, the proposed $pq$-space registration has significantly consistent results, which were close to those measured by the EM tracker. It is also evident that the weighting factor affects greatly the performance of the method and can enhance significantly its accuracy. By choosing a weight related to the z-buffer variations, errors caused by variations in texture and surface reflection can be minimised.

Figure 4.9: Euclidean distance between the first and subsequent camera positions as measured by four different tracking techniques corresponding to the conventional intensity based 2D/3D registration with or without manual lighting adjustment, the EM tracker and the proposed $pq$ space registration technique.

Figure 4.10: Inter-frame angular difference at different time of the video sequence, as measured by the four techniques described in the above figure.
4.5.2 In vivo validation

Qualitative results from the in vivo validation are demonstrated in Figure 4.11, where sample frames from the video sequence are displayed. The proposed $pq$-space based registration has been applied to a video sequence of 31sec (797 frames) of the one-patient study. The bronchoscope video sequence starts from the main bifurcation and continues through the left bronchi. Visual inspection of the real and the virtual endoscope images proves that the $pq$-based registration technique can track the tip of the bronchoscope relatively accurately and is stable under sudden movements or large rotation angles.
However, similar to Mori, when mis-tracking occurs in one frame, tracking of subsequent frames almost always fails, as the initial starting position deviates too far away from the correct result. The reason that the tracking sequence is limited to only 31sec (797 frames) is that bubbles and deformation occlude and distort the anatomical features, respectively and thus $pq$-based registration fails to work under these conditions.

**Figure 4.12** and **Figure 4.13** demonstrate the quantitative comparison between $pq$-based registration and manual alignment, similar to that of phantom validation (Figure 4.9 and **Figure 4.10**). The experimental results suggest that the proposed method can track the bronchoscope tip satisfactorily. The average positional error is equal to 3.5mm with STD 2.42mm and the angular error is 0.059 rad with STD 0.058 rad.

We also investigated the limitations of the proposed technique in a clinical environment. Fluids such as blood and mucus dynamically change the appearance of the lumen, as shown in **Figure 4.14**. Appearance of bubbles is common and usually covers the whole video frame resulting in a failure of the registration algorithm. Respiratory motion and extreme breathing patterns deform the airways significantly and distort severely the anatomical features that are essential in 2D/3D registration. An example of large tissue deformation is also shown in **Figure 4.14**. A process of identifying these phenomena combined with temporal information would facilitate tracking of the bronchoscope during the whole procedure.
Figure 4.12: *In vivo* validation: Euclidean distance between the first and subsequent camera positions as measured by the \( pq \)-based 2D/3D registration and from manual alignment for ten random frames of the sequence.

Figure 4.13: *In vivo* validation: Inter-frame angular difference at different time of the video sequence, as measured by the two techniques described in Figure 4.12.

Figure 4.14: Common image artefact that can affect image-based 2D/3D registration techniques: a) excessive bleeding due to pathology, b) appearance of bubbles when patient coughs, and c-d) large tissue deformation between successive image frames.
4.6 Discussions and Conclusions

In this chapter, we have proposed a new $pq$-space based 2D/3D registration method for recovering camera poses of bronchoscope videos. The results indicate that based on the $pq$-space and the 3D model, reliable bronchoscope tracking can be achieved. One major advantage of the method is that it is not affected by illumination conditions. This is a condition that the traditional intensity-based techniques fail due to dynamic changes in the overall illumination parameters when the bronchoscope passes through the main bifurcation to the bronchi. The $pq$-space based technique is less sensitive to these changes and it does not require the extraction and matching of explicit feature vectors.

However, $pq$-based registration can be influenced by the surface texture of the endoscopic images. For this reason a weighting factor has been incorporated into the similarity measure. This factor is estimated from the salient features that are present in both the projected images of the 3D model as well as those that appear in the endoscopic video. In this way, erroneous surface normals due to noise and texture discrepancies are ignored. Phantom as well as in vivo validation has shown that the similarity measure is robust and resilient to variations of surface texture details.

Another important implication of the incorporation of surface normals in the registration scheme is the ability to identify airway deformation between successive image frames. Preliminary results indicate that deformation of the airways can be identified and excluded from the registration process. However, in doing so we have assumed that the bronchoscope moves smoothly through the airways and the deformation is only limited to a small part of the video frame. Further investigation of the accuracy of the method for large tissue deformation is required. To this end, explicit airway deformation modelling may become necessary.

It should also be noted that there are a number of factors that can affect the accuracy of the $pq$-space algorithm. The 3D reconstruction of the tracheo-bronchial tree can involve artefacts due to respiratory motion and partial volume effects. Since the respiratory status during imaging and that during video bronchoscope examination are not matched, the 3D airway anatomy can differ significantly from the dynamic appearance of the tracheo-bronchial tree during the examination. Furthermore, bronchoscope cameras typically have a wide angle coverage, which can also cause adverse effects on the accuracy of $pq$-space estimation despite the use of distortion correction.
With the proposed method, the intrinsic robustness of the technique is dependent on the performance of the shape-from-shading method used. The use of camera/lighting constraints of the bronchoscope greatly simplifies the 3D pose estimation of the camera. Nevertheless, it is important to note that shape-from-shading is also affected by specularities and inter-reflectance caused by mucus and the lumen surface. A number of improvements need to be introduced for further enhancement of the accuracy of the proposed framework by explicit incorporation of the effect of mutual illumination, inter-reflectance and the specular components.

Overall, the most severe artefacts encountered in bronchoscopy are due to bleeding, mucosa, and large deformation of the tracheo-bronchial tree. In these cases, it is necessary to introduce temporal constraints to stabilise the registration results. In the following chapter, we will show that temporal information can be utilised through a probabilistic framework. Another means of enhancing the robustness of the 2D/3D registration is by incorporating EM tracking devices, which are capable of real-time tracking of the bronchoscope tip. As will be discussed later, EM tracking facilitates registration even in the presence of large tissue deformation and allow for a robust non-rigid registration scheme.
Since 2D bronchoscope video only provides localised views of the inner surface of the lumen, the exact 3D location of the structure it represents is ambiguous. Different segments of the airways may well have a similar local structure. An example is illustrated in Figure 5.1, which shows bronchoscope frames from different parts of the tracheobronchial tree. It is evident that the bronchus is a relatively symmetrical structure. Therefore, visual tracking should not depend only on a frame-by-frame analysis. It should also consider the path followed by the camera and its motion characteristics. The use of temporal information to derive the trajectory of the bronchoscope in 3D space is important. Another advantage of the use of temporal correspondence is that the estimates of the camera’s orientation and position can be used to accelerate the registration and tracking of consecutive video frames.

For video bronchoscope navigation, Naga et al [137] used Kalman filtering to exploit temporal information in order to increase the speed and accuracy of the registration process. Kalman Filtering was introduced in 1960 by Rudolph Kalman as an optimal linear estimator. It is a recursive data processing algorithm which makes optimal use of imprecise data on a linear (or nearly linear) system with Gaussian errors to continuously update the best estimate of the system's current state. Kalman filter is statistically optimal in that it minimizes the mean-square error and incorporates all available information to estimate the current value of the variables of interest. The information used includes (1) knowledge of the system and measurement device dynamics; (2) the statistical description of the system noises, measurements errors and uncertainty in the dynamics models; and (3) information about initial conditions of the variables of interest. In other words, Kalman filter is, in fact, a learning method. It uses a model of the estimation problem that distinguishes between
what is observed, what is really going on, and the state of knowledge about the real status of the system and the one that can be deduced from the observations. That state of cognition is represented by probability distributions that correspond to knowledge of the real world.

Kalman filtering is a viable framework for tracking the bronchoscope camera, since it provides a means of inferring the missing information from indirect and noisy measurements. However, practical application of Kalman filtering requires careful consideration of underlying statistical modelling and numerical precision. Typically, Kalman filtering is based on the assumptions of linearity, white noise and Gaussian noise \[138\]. These assumptions cannot always be fulfilled in a real situation. The robustness of Kalman filtering in practice depends on the extent of these assumptions being violated. There are methods for extending Kalman filtering to more general cases but usually they compromise the optimal behaviour of the filter and lead to considerably more complex systems. Furthermore, these approaches do not take into account all prominent statistical characteristics of the process under consideration, and hence they often yield poor results.

In bronchoscopy, tissue deformation, inter-reflection, multi-texture background, and view dependent specularity due to mucosa can limit the accuracy of image-based algorithms. Kalman filtering does not provide a mechanism to overcome this problem and the effectiveness of the method has so far been limited. Dis-occlusion is common in bronchoscopy navigation, since bubbles and luminal fluids can cover partially or completely a video segment. Although dis-occlusion often lasts only for a few seconds, during this time, the position of the bronchoscope camera can be moved significantly. Without temporal tracking, the current techniques can only work for relatively short segments. Furthermore, the potential artefacts described above can result in a multi-modal distribution of the probability density function of the state vector. Therefore, the observation probabilistic model cannot be approximated as a Gaussian distribution. It is also important to note that the assumptions of linearity and whiteness may be also violated in bronchoscope tracking.

To circumvent the above problems, we aim to develop in this thesis a predictive tracking-algorithm to propagate the probability distribution of the state space based on the Conditional Density Propagation (Condensation) algorithm \[139\]. The method is designed to cater for the general situation when several competing observations forming a non-Gaussian state-density are encountered. It uses a stochastic approach that has no restriction on the system/measurement models used and the distribution of error sources. Therefore, none of the assumptions about linearity, gaussianity and whiteness needs to hold.
For the tracking method proposed, an autoregressive algorithm is used as a predictive model [139]. This is based on the fact that during bronchoscope navigation, the motion is restricted within a bounded area of the lumen, and a rapidly moving camera is more likely to slow down or change direction, rather than accelerate further. The proposed method provides a systematic learning procedure with modular training from the ground truth data such that information from different subjects can be integrated for creating a dynamical model that accommodates the learnt behaviour [140, 141]. Experimental results from both phantom and patient data demonstrate a marked improvement in tracking accuracy especially in cases where there is airway deformation and image artefacts.

**Figure 5.1:** Image frames demonstrating the local ambiguities during endoscope navigation due to the relatively symmetrical branching structure of the tracheobronchial tree. For example, frames a) and c) show the left and right bronchi after passing through the main bifurcation (b). It is difficult to locate the position of the camera by inspecting individual image frames.
5.1 A Probabilistic Framework in Visual Tracking

Most of 2D/3D registration applications use either image-based techniques and/or incorporate positional information from tracking devices. Both of these approaches have their limitations in terms of accuracy and computational complexity. Image-based registration techniques are derived from mathematical models that are not perfect. They depict only those characteristics of direct interest and under essential simplifications in terms of lighting conditions and camera modelling. As described in Chapter 4, Lambertian surface properties and a pinhole camera model are usually assumed, and inter-reflections, dis-occlusion due to bleeding and mucosa fluids, and deformation are typically ignored. Image-based registration is also prone to local minima due to ambiguities of the local structure, such as when the endoscope is very close to the tracheo-bronchial wall. Once the tracking fails, the registration process cannot be re-initialised automatically and manual intervention is necessary.

The use of physical tracking devices can potentially solve the problems stated above. The technique, however, also has certain limitations. For example, accurate mapping of the EM tracking world coordinate system to the CT coordinate system under deformation is not trivial, as will be shown in the following chapters. Nevertheless, a probabilistic framework that combines the measurements of the tracking device and the estimations of the image-based technique in an optimal fashion is desirable. To provide current estimates of the system variables a statistical model should be designed to properly weight each new measurement relative to past information.

The basic concept of a probabilistic tracking framework is summarised in Figure 5.2. The camera pose is estimated using one or more measurement models. These measurement models are based either on image features or data provided from tracking devices. These measurements, also called observations, are inputs to the probabilistic model that incorporates information about the error both in the prediction and the observations so as to derive a better estimation of the pose for the next image frame. The prediction model is the combination of two components: (1) a mathematical model that approximates the camera motion based on previous measurements; and (2) an uncertainty term that accounts for the inherent incapacity of such a model to describe, for example, the unknown and the inherent inaccuracy of the model. The parameters related to the prediction/state model and the observations uncertainty are estimated or approximated prior to the actual tracking process, either with a systematic training approach or performed empirically when the motion is simple to describe.
Observation uncertainty factors can also be updated on-line, according to how close the predicted pose is to the actual measurements. System observability also plays an important role in system analysis and parameter estimation [142]. It relates to the amount of information contained in the measurement about the state of the system, and decides whether the system state is recoverable from the measurements. In other words, if the system is not observable, its state cannot be completely determined from the measurements, and additional or alternative measurements should be considered. This can be used as an evaluation metric of the 2D/3D registration performance.

In summary, the role of the suggested stochastic framework is to estimate the state of dynamic systems. For the case of bronchoscopy tracking, this state would most probably be the position and orientation of the camera during navigation. This state can also encode other information, such as the speed and acceleration of the camera tip. In general, the dynamic aspects of a system cannot be estimated precisely. This stochastic framework allows expression of this partial ignorance using probabilities. Therefore, the state of dynamic systems with certain types of random behaviour can be approximated using statistical information. Furthermore, this scheme also has the ability to analyse and evaluate the performance of its components. This analytical capability allows a system designer to assign an uncertainty factor to subsystems of an estimation system, and to trade off between computational complexity and real-time processing to optimize the overall system performance.

Figure 5.2: A schematic diagram of the proposed framework.
5.2 Endoscope Tracking Based on Particle Filtering

In visual tracking, it is generally recognised that designing effective tracking algorithms is difficult, requiring mechanisms to deal with issues such as weak image features, background clutter, erratic and discontinuous motion, multiple occluding and deforming objects. Some of these effects are prominent in endoscope tracking and as a result the typical assumptions of linearity, gaussianity and whiteness do not hold. Filters based on Bayesian theory, known as the Condensation algorithm or particle filtering, utilise a more general approach, without over-simplifying the underlying hypothesis. This method can be seen as a generalisation of Hidden Markov Models in that it allows a discrete set of states with probabilistic transitions between states. These approaches are flexible, easy to implement and parallelisable.

The Condensation algorithm is the extension of a sequential Monte Carlo algorithm for visual tracking of outlines and features of foreground objects, modelled as curves. It has been introduced by Isard et al. [143] for developing agile trackers that are immune to distraction from background clutter. The Condensation algorithm is designed to accommodate complex posterior distributions, non-linear dynamics and ambiguous image data [144]. Therefore, it is capable of addressing the more general situation of a non-linear system with non-Gaussian state distributions. Nevertheless, it is considerably simpler than the Kalman filter. Despite the use of random sampling which is often thought to be computationally inefficient, the algorithm can be executed in near real-time. This is because tracking over time maintains relatively tight distributions for the position of the camera at successive time-steps, which is the case, given the availability of accurate, learned models of the camera motion.

The algorithm uses factored sampling to stochastically propagate a non-Gaussian, multi-modal estimate of the target posterior with a stochastic differential equation model for camera motion. To deal with non-Gaussian observation distributions, it maintains multiple hypotheses, particle sample, and automatically focuses more on the most likely ones. The weight of each particle is represented by the likelihood of that particular state variable. This likelihood value is determined by comparing the observed data with the measurement data. Each hypothesis/particle is propagated through a dynamical model of the camera motion. The random component of the dynamical model leads to increasing uncertainty while the deterministic component causes the density function to drift bodily.
The main stochastic mechanism that constitutes the probabilistic endoscope tracking framework developed in this thesis can be summarised into the following three steps: (1) sampling, where \( N \) particles from the original set are chosen, with the probability of selecting each item being proportional to its weight; (2) evolving, where the dynamic model is applied to the selected particles; and (3) measuring the coefficients based on the observations, so the weights of the new density are estimated. Since this includes multimodal distributions, particle filters can cope effectively with temporarily ambiguous image support.

The main idea of the statistical framework is to maintain a time-evolving probability distribution \( p(x_t | Z_t) \) of the tracker state \( x_t \) over time \( t \). The \textit{a posteriori} density \( p(x_t | Z_t) \) represents knowledge about \( x_t \) deducible from the observation history \( Z_t = \{ z_1, \ldots, z_t \} \). The effective prior distribution \( p(x_t | Z_{t-1}) \) in the dynamic case is unknown and therefore cannot be represented with a closed-form solution. The \textit{Condensation} algorithm does not explicitly represent the density function. It uses instead statistical factor sampling, which provides a way of approximating \( p(x_t | Z_{t-1}) \) by using a random number generator for sampling \( p(x_t) \). The sampling technique is based on the Bayesian theory which states [145]:

\[
p(x | z) \propto p(z | x) p(x)
\]

Bayesian theory allows a prediction model to incorporate inherent knowledge, while improving its ability to make decisions through the seamless integration of auxiliary information.

Factored sampling needs to be extended to be applied iteratively to the image sequence. The process at each time-step is a self-constrained iteration of factored sampling [146]. In order for the algorithm to run under a constant computational complexity, the sample sets should be retained at a fixed size \( N \). During statistical factored sampling, a sample set \( \{ s^{(1)}, \ldots, s^{(N)} \} \) is generated from the prior density \( p(x_t) \), where \( N \) is the number of sample sets (particles). A weight is subsequently assigned to each particle according to the observation density

\[
\pi^{(n)} = \frac{p(Z | x = s^{(n)})}{\sum_{j=1}^{N} p(Z | x = s^{(j)})}
\]
The weighted point set is then used as a representation of the posterior density, which is becoming more accurate as $N$ increases.

Therefore, the procedure is to sample with replacement $N$ times from the set $\{s_{t-1}^{(n)}\}$, choosing an element with probability $\pi_{t-1}^{(n)}$. Elements with high weights are more likely to be chosen several times, leading to identical copies of elements in the new set, while others with relatively low weights may not be chosen at all. Subsequently, each element chosen from the new set is subjected to a predictive step, which corresponds to sampling from the distribution $p(x_t | x_{t-1})$. In this way, the sample set $\{s_t^{(n)}\}$ for the new time-step is generated but without its weights. Finally, the observation step from factored sampling is applied, generating weights from the observation density $p(z_t | x_t)$ to obtain the sample-set representation $\{s_t^{(n)}, \pi_t^{(n)}\}$ of the state density for the new time-step. By evaluating the moments of the state density, it is possible to extract the current state. As an observation/measurement model, the $pq$-space based 2D/3D registration technique described in the previous chapter has been used.

5.3 Prediction Model for the Bronchoscopic Camera

In the Condensation algorithm, if the prediction model is too simplistic or its parameters are not tuned properly then it might not be able to represent the dynamics of the system adequately. In that case, the estimated posterior gradually drifts away from the observation value, which can result in a zero weighting factor that completely spoils the outcome of the algorithm. Therefore, it is important to develop a prediction model and a training system that can model the motion of the endoscopic camera adequately.

In bronchoscope tracking, the estimated pose in the $(t-1)^{th}$ frame of a sequence is the initial pose for the $t^{th}$ frame. In a probabilistic context, this is equivalent to applying a Gaussian prior distribution to each frame with fixed covariance but whose mean is simply the estimated pose from the previous frame. When temporal tracking is employed, we could take the posterior from the $(t-1)^{th}$ frame as the prior for the $t^{th}$ frame rather than fixing the prior via one constant covariance for all image frames. However, this idea is not effective for inferencing the motion between successive time-steps, which is reasonable only when the camera moves slowly and the registration process is accurate. Furthermore,
as the posterior at one time-step is regarded as the prior of the next, statistical information from measurements steadily accumulates. In other words, the \( pq \)-space registration results will have a time-decreasing influence on the estimated pose. An adequate statistical framework for motion tracking must provide not only a prior for the first frame, but also a prior for possible motions, in the broad sense of rigid motion. This dynamical prior distribution should apply between all successive frames. It needs to have a deterministic part, giving the expected displacement between successive image frames, as well as a stochastic component to counteract the otherwise unlimited accumulation of information.

For building a dynamical model for tracking the pose of the bronchoscope camera, it is important that successive positions are tightly constrained. Rather than using only the global prior distribution \( p(x) \). A Markov chain model assumption is therefore more meaningful. A Markovian assumption presumes that the present state of a system can always be predicted given the previous \( n \) states. A Markov analysis looks at a sequence of events, and analyses the tendency of one event to follow another. In this way, a new sequence of random but related events can be produced to have properties similar to the original. A first-order Markov chain model correlates the distribution of the vector state at a given time with the immediate predecessor only:

\[
p(x_t | x_1 \ldots x_{t-1}) = p(x_t | x_{t-1}) \tag{5.3}
\]

where \( p(x_t | x_{t-1}) \) is the conditional distribution, which indicates the probability of \( x_t \) given \( x_{t-1} \). This is an assumption that can greatly simplify the probabilistic model. Note that a second-order Markov model taking two predecessors into account can also apply.

A simple, isotropic, first-order Gaussian Markov process is defined as:

\[
p(x_t | x_{t-1}) \propto \exp \left( -\frac{1}{2b^2} \| x_t - x_{t-1} \|^2 \right) \tag{5.4}
\]

where \( b \) is the average root-mean-square step length and ‘isotropic’ has the meaning of steps equally likely to occur in all directions. This type of motion is known as Brownian motion. Such a distribution is too random to be useful for modelling of real object motion. Instead, a constrained Brownian process is adopted. Although, it initially appears to behave randomly, in the long-term it is limited within a restricted area of a Gaussian envelope.

All these Markov processes belong to a broader family that is called Auto-Regressive (ARG) processes and they meet some of the requirements of dynamic modelling by
simulating the entire family of motions. The first-order ARG processes deal with changes in position and can impose strong incremental constraints when global constraints are weak. They allow a global distribution to be chosen in addition to, and independently of, the local process. Although, they appear to be the simplest available models that can achieve these properties, they do have certain limitations. A first-order ARG process seems to model noisy directional motion at a constant average velocity. However, it has an average direction that is fixed over time and must be known in advance.

Substantial changes in velocity components over time and oscillations are beyond the scope of the first-order model. This can limit the applicability of the system in a bronchoscopy application where the respiratory motion results in a periodic movement of the bronchoscope tip. A second-order dynamical model is a natural extension of a first-order ARG process and it can give a much improved result, while it has the potential to specify an ARG process representing a second-order Markov chain. In other words, the state vector at a given time depends on two previous time-steps, rather than just one as in the first-order process.

In order to construct a motion model for the endoscope camera that moves freely in the 3D tracheo-bronchial tree, a second order auto-regressive model is used [139]. This is in contrast to the ‘constant acceleration’ model used in [137], which effectively implies that the camera acceleration is expected to be constant during bronchoscope tracking. In this study, the auto-regressive model takes into account that during bronchoscope navigation, motion occurs within a bounded area, and a rapidly moving camera is expected to slow down or change in direction rather than accelerate further [147]. This ensures smooth navigation of the camera, and with modular training [140], multiple training sets can be used to obtain a more general model of the motion behavior. For a 2\textsuperscript{nd}-order auto-regressive model, the following equation can be used:

\[
x_t = \sum_{k=1}^{2} A_k x_{t-k} + B w_t
\]

where \( A_k \) represents the collection of damped harmonic oscillators associated with vibrational modes and \( w \) the white noise with covariance coefficient \( B \).

In Figure 5.3, an example of the application of a first, as well as a second order ARG model is demonstrated. The first order ARG model fails to follow the position of the camera even during the phase of training.
5.4 Learning Dynamics of Bronchoscope Navigation

In practice, it is possible to build a tracking model by approximating its general behaviour to intuitive expectations of the observed motion. Hand-built models are often used when the prediction model is described by a first-order autoregressive process. However, for the purpose of bronchoscopy navigation it is difficult to use this approach due to the high-dimensionality and complex motion involved. Instead, a training sequence (gold standard data) can be used. Mathematically, learning motion characteristics from a training sequence is effectively to estimate the coefficients $A_k$, the mean value $\bar{X}$, and the random component $B$ of an autoregressive process that best model the motion in a training sequence involving camera poses of $x_1, \ldots, x_M$. The problem can be expressed in terms of maximising the log likelihood function $L$ over all coefficients [140]. For a second order autoregressive model this takes the form:

![Figure 5.3: a-b) Training results based on a first order ARG model. c-d) Training results based on a second order Autoregressive Model.](image)
\[
L \left( x_1, \ldots, x_M \mid A_1, A_2, C, \bar{X} \right) = \\
-\frac{1}{2} \sum_{t=3}^{M} \left[ B^{-1} \left( x'_t - A_2 x'_{t-2} - A_1 x'_{t-1} \right) \right]^2 - (M - 2) \log \det B
\]

where

\[
x'_t = x_t - \bar{X}
\]

and

\[
C = BB^T
\]

In general, this is a non-linear problem, because \( L \) is quartic in the unknowns, due to the fact that \( \bar{X} \) is explicitly estimated together with \( A_1, A_2 \) and \( B \). The non-linearity can be removed by defining the constant parameter \( D \) [140]:

\[
D = (I - A_1 - A_2) \bar{X}
\]

The values of \( A_1, A_2, B \) and \( D \) that maximise the likelihood function \( L \) can be found by solving the linear system of equations below [140]:

\[
\begin{align*}
R_2 - A_1 R_{0} - A_2 R_{1} - D (M - 2) &= 0 \\
R'_{20} - A_1 R'_{00} - A_2 R'_{10} &= 0 \\
R'_{21} - A_1 R'_{01} - A_2 R'_{11} &= 0
\end{align*}
\]

The auto-correlation coefficients \( R_{i,j} \) and \( R'_{i,j} \) can be computed for \( i, j = 0, 1, 2 \) as:

\[
R_i = \sum_{t=3}^{M} x_{i-t} \quad R_{i,j} = \sum_{t=3}^{M} x_{i-t} x'_{i-j} \quad R'_i = R_{i,j} - \frac{1}{M - 2} R_i R'_j
\]

Subsequently, \( A_1, A_2 \) and \( D \) are given by:

\[
\begin{align*}
A_2 &= \left( R'_{02} - R'_{00} R'_{11} R'_{12} \right) \left( R'_{22} - R'_{20} R'_{11} R'_{12} \right)^{-1} \\
A_1 &= \left( R'_{01} - A_2 R'_{21} \right) R'_{11}^{-1} \\
D &= \frac{1}{M - 2} \left( R_0 - A_2 R_2 - A_1 R_1 \right)
\end{align*}
\]
The mean of the ARG process, on the other hand, is calculated as below, given that \((I - A_i - A_j)\) is not singular.

\[
\bar{X} = (I - A_i - A_j)^{-1} D
\]  

(5.13)

Finally, the covariance coefficient \(B\) is estimated as a matrix square root of \(C\):

\[
C = \frac{1}{M - 2} \left( R_{00} - A_2R_{20} - A_1R_{10} - DR_{0}^T \right)
\]  

(5.14)

Note that parameter \(B\), which expresses the stochastic part of the ARG process, cannot be determined uniquely, but any solution that satisfies \(C = BB^T\) is acceptable.

Another important issue to consider is the dimensionality of the data. For a system with just one degree-of-freedom, the situation is relatively straightforward, while multi-dimensional motion is more difficult to describe. A 6-DoF pose estimation problem is easier to handle if it is described by a two independent 3-DoF tracking problems that model the position and the orientation of the camera separately. This implies that for each of these systems the second order equation can be described from different parameters, thus being more flexible and easier to train. This is illustrated in Figure 5.4, where for three in-vivo datasets the Euclidean distance is estimated for both a 6DoF ARG model and a 3DoF-3DoF model.

### 5.4.1 Modular Training

Usually, it is desirable to broaden the dynamical model to include more than a specific motion. For the purpose of bronchoscope simulation, it is more meaningful to collect several training sets from the same as well as different operators in order to construct a more representative dynamic model. In this case, the problem becomes how to combine multiple training sets in order to gain knowledge of a more general behaviour. Modular learning cannot be addressed by simply concatenating the different training sets and then treating them as one because the abrupt transition between different data sets would be interpreted as an actual behaviour of the system [139]. To this end, the auto-correlation coefficients of each training set can be calculated individually and then combined in a linear fashion. This is similar to the multi-variate algorithm that Reynard et al. [148] and Wildenberg et al. [140] have used for motion tracking over classes of objects with variable mean shapes and positions.
Figure 5.4: (a,c,e) Training results based on a 6DoF, second order Autoregressive Model. (b,d,f) Training results based on two simultaneously second order Autoregressive Models of tracking the position and the orientation of the camera separately.

In the work presented by Wildenberg et al. [140], modular learning is also based on the maximum likelihood estimation to find the best model that fits all the training sequences. The extension of the training algorithm for a single to multiple training sets is presented in Blake et al. [139] and it is straightforward when the mean of the state vector, $\bar{X}$, is
simultaneously estimated with the covariance matrix, $B$, and the coefficients of the deterministic part, $A_1$ and $A_2$. Therefore, the learning algorithm for a set of training sequences is formed as follows. Given training sequences numbered as: $\gamma = 1, \ldots, \Gamma$, with the $m^{th}$ data point of the $\gamma^{th}$ sequence represented by $X_m^\gamma$ and the $\gamma^{th}$ sequence having $M_\gamma$ data points, the $S$ moments from the $\gamma^{th}$ data sequence are defined to be:

$$S_i^\gamma = \sum_{m=1}^{M_\gamma} X_{m+i}^\gamma \quad \text{and} \quad S_{i,j}^\gamma = \sum_{m=1}^{M_\gamma} (X_{m+i}^\gamma)^T \quad \text{where} \ i, j = 0, 1, 2 \quad (5.15)$$

Assuming that all of the training sequences have exactly the same dynamical model, including the same $\overline{X}$, the moments describing the cumulative system can be written as the sum of the moments of each individual data sequence. A correction for the effect of $\overline{X}$ on $S_{i,j}$ moments should also be taken in consideration. That is:

$$S_i = \sum_{\gamma=1}^{\Gamma} S_i^\gamma \quad \text{and} \quad S_{i,j} = -S_i S_j^T + \sum_{\gamma=1}^{\Gamma} S_{i,j}^\gamma \quad \text{where} \ i, j = 0, 1, 2 \quad (5.16)$$

Finally, the unknown parameters are estimated by the following linear equations:

$$D = (I - A_1 - A_0) \overline{X} \quad (5.17)$$

The covariance matrix $B_0$ is determined as a solution of $BB^T = C$ and it is not unique, where

$$C = \frac{1}{M-2} \sum_{m=1}^{M-2} R_m R_m^T \quad (5.18)$$

In the above equation, $R_m$ is an instantaneous measure of error of fit of the autoregressive process, which can be written in a linear form:

$$R_m \equiv x_{m+2} - A_1 x_{m+1} - A_0 x_m - D \quad (5.19)$$

Finally, to calculate $A_1, A_2, B$ and $\overline{X}$ the following equation system needs to be solved.
\[
\begin{align*}
S_2 - A_0 S_0 - A_1 S_1 - D(N - 2) &= 0 \\
S_{2,0} - A_0 S_{0,0} - A_1 S_{1,0} &= 0 \\
S_{2,1} - A_0 S_{0,1} - A_1 S_{1,1} &= 0
\end{align*}
\] (5.20)

Figure 5.5: Four sequences of the camera position during in vivo navigation have been used for modular training. These have been extracted from four different subjects. This figure presents the Euclidean distance from an initial reference point of each of these training sequences.

### 5.4.2 Model Dynamic Variability

In the work presented by Wildenberg et al., it is also noted that the mean of the sequence is not the mean of the system. However, the mean of the sequence are often used as a good estimation when the training data are stable over a long time period:

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} X_i = \bar{X}
\] (5.21)
Nonetheless, if the training sequence is short and weakly damped there is a systematic error in $\mathbf{X}$. It can cause the rest of the terms to be falsely estimated. In practice, the numerical error can be large enough to change the overall characteristics of the system, causing erroneous damped behaviour and shifts of the phase and the frequency. This is profound when several training sequences are employed. Since each of these dynamic systems may have a different mean value, the use of pre-estimated mean value of the system can result in a strongly biased prediction. Alternatively, $\mathbf{X}$ can be estimated on-line as part of the state vector. In this way, the state vector is extended by regarding $\mathbf{X}$ as a variable, rather than a constant [140, 148]. Hence the prediction model can be written as:

$$
\begin{bmatrix}
X_{n+1} \\
X_{n+2} \\
\bar{X}_{n+2}
\end{bmatrix} =
\begin{bmatrix}
0 & I & 0 \\
A_0 & A_1 & I - A_0 - A_1 \\
0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
X_n \\
X_{n+1} \\
\bar{X}_{n+1}
\end{bmatrix} +
\begin{bmatrix}
0 \\
B_0 w_n \\
0
\end{bmatrix}
$$

(5.22)

Here $\mathbf{X}$ is not known as a priori but it is updated during tracking, and $\bar{X}$ is initialized with its estimated mean value $\bar{X}_0$ and its associated variance and covariance.

The ground truth data of the camera pose from four different patients have been used to train the auto-regressive model. This data are displayed in Figure 5.5. Figure 5.6 illustrates the effectiveness of the training process involved in this study. The performance of the trained model was evaluated on a different subject. The Euclidean distance between the first and subsequent camera positions predicted from the Condensation algorithm was used for error analysis. Similar analysis for the error in orientation was also performed. In Figure 5.6 (a), the system mean has been predefined according to the training sequences, whereas for (b), it was included in the state vector of the predictive model such that it was updated in real-time. It is evident that without continuous updating of $\bar{X}$, the derived tracking value is heavily biased.
Figure 5.6: Assessment of the accuracy of the training model and the effect of excluding a), and including b), mean value estimation of $\bar{X}$ as part of the state vector.
5.5 Experimental Design

In order to assess the accuracy of the proposed algorithm, an airway phantom made of silicone rubber and painted with acrylics was constructed. The phantom has a cross sectional diameter of 12cm at the opening and narrows down to 5cm at the far end. The inside face was created such to give the surface a specular finish that looks similar to the surface of the lumen. This phantom has been described in details in Chapter 4. A real-time, six degrees-of-freedom Electro-Magnetic (EM) motion tracker (FASTRAK, Polhemus) was used to validate the 3D camera position and orientation. The EM-tracker has an accuracy of 0.762mm RMS. The tomographic model of the phantom was scanned with a Siemens Somaton Volume Zoom four-channel multi-detector CT scanner with a slice thickness of 3mm and in-plane resolution of 1mm. A CMOS camera and NTSC standard with frame rate of 30fps was used.

For *in vivo* validation, bronchoscopy examination was performed in five patients according to a standard clinical protocol. During the bronchoscope procedure two similar type videoscopes (Olympus BF Type; with field of view 120°) were used. Video images from the bronchoscope examination were transferred to digital videotapes in PAL format at 25fps. Since the original endoscopic video frames contain both the endoscopic image and redundant black background, only the endoscopic view was digitized and cropped to images of 454×487 pixels. All images were converted to grayscale before the *pq*-space analysis. Similar to the phantom study, the CT images were acquired from the Siemens Somaton Volume Zoom four-channel multi-detector CT scanner with a slice width of 3mm and collimation of 1mm, and the acquisition volume covered from the aortic arch to the dome of hemi-diaphragm. Pre-processing of the video images was necessary in order to alleviate the effects of interlacing, lens distortion and unnecessary texture information. To remove noise and image artifacts, anisotropic filtering was applied to each image.

5.5.1 Obtaining the Training Sequences

Training sequences were produced by the manual alignment of selected key frames. Subsequently, both the position and orientation of intermediate frames can be estimated by interpolating the measurements. For positional interpolation, cubic spline-based, piecewise cubic Hermite and linear interpolation have been tested. Spline-based interpolation can cause some artefacts that do not reflect the actual dynamic motion behaviour of the camera. This
is apparent in Figure 5.7. The piecewise cubic Hermite interpolation was preferred, as it is also smoother than linear interpolation and it does not introduce erroneous oscillations. In order to interpolate the orientation of the camera, the Euler angles were transformed into quaternions and subsequently, spherical linear interpolation was applied. Both Euler angles and quaternions can represent an orientation. However, quaternions provide a unique and intuitive representation of an angle and thus they are preferred in interpolation problems. Note that spherical interpolation may result in discontinuities. It is also feasible to obtain \textit{in vivo} ground truth data with the incorporation of EM trackers as it will be shown in Chapter 7.
5.6 Results

The detailed assessment results for the phantom and patient studies are summarized in Table 5.1 and Figure 5.8. It is evident that in both cases the 2D/3D registration accuracy has been increased significantly by the use of the proposed predictive tracking algorithm. More notably, the method permits more stable tracking results in patients where image artefacts (e.g. partial occlusion of the images due to mucosa or bleeding) and sudden airway deformation due to coughing can introduce large propagation errors to the original pq-space registration technique. In Figure 5.9, we demonstrate the extent of this effect on the actual bronchoscope views. The left column shows the original frames from the bronchoscope video, whereas the middle and right columns are the virtual views of the 3D model by using pq-space registration without and with predictive camera pose tracking. It is worth noting that pre-processing, including radial distortion correction, de-interlacing and anisotropic filtering, has been applied to the real bronchoscope images before the registration step.

<table>
<thead>
<tr>
<th>Case</th>
<th>Frames</th>
<th>Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position (mm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pq Mean ±Std</td>
</tr>
<tr>
<td>Phatom</td>
<td>300</td>
<td>36.5 ±22.7</td>
</tr>
<tr>
<td>Pat1</td>
<td>100</td>
<td>7.0 ±2.0</td>
</tr>
<tr>
<td>Pat2</td>
<td>120</td>
<td>13.3 ±5.8</td>
</tr>
<tr>
<td>Pat3</td>
<td>243</td>
<td>26.98 ±13.0</td>
</tr>
<tr>
<td>Pat4</td>
<td>144</td>
<td>5.87 ±5.5</td>
</tr>
<tr>
<td>Pat5</td>
<td>100</td>
<td>14.4 ±10.12</td>
</tr>
</tbody>
</table>

Table 5.1: Quantitative assessment of the pq-space based registration with and without the Condensation algorithm (pq-CD), respectively.

Figure 5.7: The effect of the interpolation on the dynamics of the prediction model. (a,c) correspond to B-spline interpolation and (b,d) correspond to Piecewise Hermite interpolation.
Figure 5.8: Bar chart showing the quantitative assessment of the pq-space based registration with and without the condensation algorithm.
5.7 Discussions and Conclusions

In this chapter, we have described the use of predictive camera tracking for increasing the general accuracy and robustness of 2D/3D registration involved in virtual bronchoscope modelling. A stochastic filter is used to resolve the inherent global ambiguity in tracking by exploiting the temporal coherence of the camera tip. Particle filters, particularly the Condensation algorithm, permits the use of multi-modal probability distributions and it is not restricted to linear systems and white noise. Instead, it uses statistical factor sampling to approximate any form of probability distribution without any assumptions. The results from both phantom and patient data demonstrate a significant improvement in tracking accuracy especially in cases where there is slight airway deformation and image artefacts. The method effectively avoids the registration algorithm being trapped in local minima and increases the overall accuracy. It is also able to recover when the image-based registration has failed, because it does not use the previous camera pose to initialise the following registration step.

It has been shown that the use of a second order ARG model is particularly effective for bronchoscope camera motion prediction that it is highly restricted inside the tracheobronchial tree. Another important feature of this model is the use of two previous camera poses to estimate the subsequent camera positions. This contributes to the robustness of the proposed framework against failures of the image-based algorithm. This is because the system has a memory encoded in the form of particles. Particles tend to concentrate around the areas indicated by the measurements. Subsequently, erroneous measurements that are completely outside of the underlying neighbourhood of possible positions are rejected as outliers.

The estimation of the parameters of both the auto-regressive motion model and the noise distribution plays an important role in the algorithm proposed. In this work, the parameters are estimated through a systematic procedure based on the principles of the maximum likelihood learning. The main advantage of this approach is that it extends modular learning. Modular training facilitates the incorporation of multiple sequences from different patients during training. In bronchoscopy, ground truth data are difficult to be obtained. Therefore, it is desirable to create a prediction model that is not patient specific. Our results indicate that modular training can be successfully implemented when the mean of the system is also a part of the state space. In practice, the mean typically varies from system to
system and it cannot be estimated by simply averaging the individual mean values. If the mean value is not calculated properly, it can cause the rest of the terms to be falsely estimated. In this case, the numerical error can be large enough to change the overall characteristics of the system.

It should be noted that the proposed method can be further extended to multi-class motion description such that the dynamic behaviour of camera navigation in different parts of the tracheo-bronchial tree can be incorporated [141]. In this way, several auto-regressive models can be utilised and trained to allow for more accurate predictions. It is also necessary in practice to take into consideration of the technical skills of the operators and separate them into different groups. It is also worth noting that temporal tracking has also its limitations. Although it improves 2D/3D registration under airways deformation and occlusion, it may also fail when prolonged image artefacts are encountered. The actual quality of modular training also reduces with the number of degrees-of-freedom. Therefore, it is more efficient to propagate separately the positional information from the angular state space. Nevertheless, there is a relationship between changes in the camera position and changes in the orientation. Further investigation is required to clarify how this information can be incorporated in an efficient way.
Figure 5.9: The effect of airway deformation and partial occlusion of the image due to mucosa and blood on the accuracy of the 2D/3D registration technique without (mid-column) and with (right-column) predictive camera tracking.
In the last two chapters, we have developed a $pq$-space based framework for 2D/3D image registration. We have also demonstrated how temporal tracking can be used to improve the overall system accuracy and robustness when severe artefacts are encountered. From the *in vivo* experiments, it has become clear that image-based techniques have certain limitations. Although the proposed $pq$-space based similarity measure is immune to lighting parameters, airway deformation has so far only been considered qualitatively in an *ad hoc* manner. In Chapter 4, local deformation of the airways is detected from temporal variations of the $pq$ distribution and the registration algorithm assumes that this deformation is relatively small. For patient-specific bronchoscope tracking and modelling, however, respiration and patient motion can introduce significant distortions to the tracheo-bronchial tree. For the proposed algorithm to be clinically useful, further enhancement in effective motion handling is necessary.

To this end, EM tracking devices offer an attractive solution. The latest generation of miniaturised catheter tip EM tracking devices allow the insertion of the EM sensor into the biopsy channel of the endoscope. This offers the potential for significantly improving the robustness and accuracy of current registration techniques in the presence of large tissue deformation. In this way, the camera position can be tracked during the whole procedure with sub-millimetre accuracy and real-time response. The major advantage of these systems is their immunity to the quality of the video frames, as well as the imaging artefacts involved.
However, there are several challenges need to be addressed before these systems can be safely applied to clinical simulation. The choice of the tracking system is influenced by a number of parameters that depend both on the nature of the application and the required level of accuracy. Ferromagnetic distortion, tissue deformation and patient movements reduce the accuracy of these systems in clinical environments. Particularly in bronchoscopy, respiratory motion and global shifting of the position of the patient can interfere with the measurements of the EM tracker and subsequently reduces its effectiveness. In this chapter, we will address the technical issues related to the use of EM tracking devices for bronchoscope tracking, and emphasise on the issue of how to decouple global and respiratory motion of the EM tracking data such that its reading can be effectively used to guide the 2D/3D registration algorithm.

6.1 Positional Trackers in Medical Applications

Tracking devices are well known in image-guided surgery because they provide real-time measurement and have stable performance compared to image-based fusion techniques [149, 150]. In many surgical applications, it is important to correlate in real-time the intra-operative field with the imaging space. For example, in neurosurgery tumours identified in CT or MR images may be visually difficult to distinguish from the surrounding normal tissue during surgery. In neurosurgery, optical tracking devices were developed in the early 1990’s and they gradually replaced neuronavigator arms, which have been used since the end of 1980’s as passive intraoperative position systems. The main advantage of optical tracking is that they are easy to handle and have fewer restrictions to the surgeon’s movements.

Optical tracking systems provide the most reliable and precise position sensors available in medical applications [151]. Objects are tracked by rigidly attaching retro-reflective spheres or active infrared LED’s. The spheres can be detected by the camera system and used to determine the location and orientation of the object. Infrared light is used for maximal accuracy and decreased sensitivity to ambient light. Optical tracking provides an attractive solution for tracking rigid tools due to its linearity, stability and accuracy. Compared to ultrasound and EM trackers it offers a great potential in terms of accuracy and speed of acquisition. Subsequently, it has been popular in surgical operations related to brain, spine and general orthopaedics, in which it has been established as part of the standard clinical practice. However, the method is constrained to the line-of-sight applications and thus it is inappropriate for tracking flexible instruments inserted into patients.
6.2 Miniaturised Electro-magnetic Trackers

Recently, Electro-magnetic Tracking (EM) has made significant advances both in terms of its accuracy and miniaturisation. Examples of such systems that can be used for MIS include Aurora® (NDI), MiniBird® (Ascension) and Magellan® (Biosense Webster). They are equipped with cylindrically shaped sensor coils to the size of the tip of a biopsy tool so that they can be embedded in catheters and thin needles. The advent of miniaturised trackers has a variety of potential clinical applications in computer aided interventions. The needle probe is suited to computer-assisted minimally invasive drug delivery, biopsy, cyst drainage, and electrode placement. The position and orientation of the flexible instrument tip, such as endoscopes and catheters can be tracked accurately within the body. EM trackers use the attenuation of oriented Electro-magnetic signals to determine the absolute position and orientation of a tracker relatively to a source. The induced voltage level provides information about both the distance from the transmitter to the receiver and the axis-alignment between them. Boundaries of equal accuracy are found along a hemisphere or sphere around the transmitter.

Although EM trackers are susceptible to ferromagnetic interference they can achieve a high application accuracy after careful in situ calibration [152]. The potential clinical applications of EM systems have already extended well beyond optical and mechanical positional systems. Electro-magnetic systems that provide simultaneous electrophysiological and spatial information have been approved for patient care and are used in clinical practice. Such systems have been used in cardiac interventions, endonasal sinus surgery and bronchoscopy [152-154]. EM systems also facilitate the development and expansion of new diagnostic techniques and imaging modalities, such as Laparoscopic Ultrasonography and neuorsurgical applications [155, 156]. This paragraph presents a brief overview of the clinical applications and studies and the related challenges in EM tracking.

Electro-magnetic tracking technology (InstaTrack® CAS system) has been employed for endonasal sinus surgery [152]. Endonasal sinus surgery is challenging because of the presence of a variety of endangered vascular and neural structures in a very confined space of the anterior skull base. For this kind of operations, computer-aided surgery is an important tool for increasing the accuracy of orientation and becomes mandatory for specific patient cases. For these systems, two EM sensors provide positional information relative to the CT images. One sensor is located to the standard suction chamber and the
other is located on a headset worn by the patient during the surgical procedure. Similarly, the possibility of extending the use of EM trackers in neurology has been shown in a number of feasibility studies [156]. In the work by Zaaroo et al., the accuracy of an EM system (Magellan system, Biosense Webster) is measured for image-guided neurosurgical applications. The Magellan system was developed to provide real-time tracking of the distal tips of flexible catheters, steerable endoscopes, and other surgical instruments, using ultra-low electro-magnetic fields and a novel miniature position sensor for image-correlated intra-operative navigation and mapping applications.

This EM system has been also tested for anatomically targeted ablations of the fossa ovalis, which is found in the right atrium of the heart, and the left atrium [157]. Transvenous catheter ablation is a treatment for many arrhythmias that involves the surgical removal part of body tissue. Anatomical knowledge has important role in treating arrhythmias, such as atrial fibrillation and non-idiopathic ventricular tachycardia. Although fluoroscopy is the main intra-operative imaging modality, it provides limited information to localise the catheter position in relation to the cardiac structures and expose both the patient and medical staff to high doses of ionising radiation. Furthermore, the arteries can only be made visible by inhaling a contrast agent, which subjects the patient to additional risks. Dickfeld et al. [157] suggests stereotactic catheter guidance for cardiac applications. The EM system is superimposed on three-dimensional MR images using fiducial markers. This allowed the dynamic display of the catheter position on the true anatomy of previously acquired MRI in real-time.

In surgery, EM tracking technology has also been used to provide surgeons with important spatial cues during Laparoscopic Ultrasonography (LUS), so they can orient oblique images with few constrains on positioning the laparoscopic port [155]. The key component of this visualisation technique is that it shows how the ultrasonography plane interacts with the aorta and thoracic skeletal anatomy. LUS is the most sensitive method for detecting hepatic metastasis and it is also a mean of identifying stones during laparoscopic cholecystectomy. However, one of the major problems is in interpreting LUS images because they are not obtained in conventional orientations, such as transverse or sagittal planes. This limits the adaptation of LUS in clinical practice, since both novice and advanced ultrasonographers experience difficulty.
6.2.1 *Accuracy Assessment and Metal Calibration of EM Trackers*

There are a number of evaluation criteria for EM trackers:

1. **Resolution** - It is related to the ability of the tracker to measure different points in space, and subsequently, the numerical accuracy of the system. The accuracy of a system is usually affected by whether the sensor moves or not and thus, it is described as static or dynamic, respectively.

2. **Registration** - It refers to the correspondence between actual position and orientation and reported position and orientation. Calibration processes are used to measure and adjust for the differences between reported and actual position.

3. **Drift** - It is a gradual increase in tracking error over time.

4. **Jitter** - It is a fluctuation in the reported position/orientation of the sensor.

5. **Phase Lag** - It is the time that takes the sensor to report its position and it is related both with the data generation and the transmission process.

6. **Update Rate** - It is the number of tracker position/orientation samples per second that are transmitted to the receiving computer. Higher update rates do not necessarily imply better accuracy. An upper bound is determined by the communications rate between computer and tracker and the number of bits that encode the pose information.

7. **Range** - Positional range refers to the working volume of the system, since the accuracy of EM tracking systems decreases with distance. Orientation range refers to the set of sensor orientations that the tracking system reports with a given resolution.

8. **Interference and noise.** Inaccuracies might occur due to external object and Electromagnetic fields. Random variations in an otherwise constant reading may also occur and they are related to the static accuracy of the system.

9. **Physical dimensions** - This refers to the size, mass and any other special characteristic that affects the feasibility and function of the tracking system under a specific application.
(9) *Multiple sensors* - This is related with the ability of the system to track multiple points within the same volume.

Although AC magnetic trackers are accurate and can provide high update rates, they are particularly sensitive to eddy-currents and ferromagnetic environment. The Aurora EM system has been evaluated by a number of research groups [158-163] and has been used in numerous clinical as well as phantom studies of image-guided applications [154, 164-166]. It represents a new generation of DC magnetic trackers with increased accuracy and stability even in ferromagnetic environments [164]. The basic system consists of a control unit, sensor interface devices and field generator and it enables the coils to be tracked in five degrees-of-freedom (three spatial and two rotational coordinates). The sensors can be as small as 0.9mm in diameter and 8mm in length. According to manufacturer’s specifications, the sensors have a positional accuracy of 1-2mm and angular accuracy of 0.5-1 degree. The system’s dynamic rate is its maximum data rate, which is equivalent to 45Hz. The measurement volume is 50 by 50 cm, located 50mm from the field generator.

Typically, in the operating room there are various materials, such as closed metallic loops, wire guides, catheters and monitors that interfere and distort the EM sensor measurements. In addition, their accuracy depends on the distance between the transmitter and the sensor. As opposed to optical tracking, the useful field-of-view is rather small. One possible approach to overcome this problem is to design a hybrid tracking system, using an additional optical tracking tool to monitor displacements of the emitter [158]. Nevertheless, the susceptibility of EM tracking systems to EM interference is by far the greater obstacle in the acceptance of these systems in standard clinical practice. Subsequently, there are numerous studies testing the nature of the EM distortion caused by the presence of different ferromagnetic materials in the operating room, as well as the development of techniques that improve the measurement accuracy.

Hummel *et al.* [158] has tested the Aurora system to examine its accuracy under the influence of: (1) closed wire loops with different radii, (2) guiding wires (endoscopes) with different configurations of loops, (3) ultrasound scan probes near to the emitter and sensor, (4) a mobile C-arm fluoroscopy unit, (5) an optical tracking tool, (6) an endoscope with a radial ultrasound scan head, and (7) a needle holder, biopsy forceps and a drill handle. Wilson *et al.* [163] also tested Aurora within a CyberKnife radiosurgery suite. CyberKnife is a non-invasive, frameless, stereotactic radiosurgery device used to ablate tumours within the brain, spine, chest and abdomen. One of the aims of the work was to determine the accuracy and device behaviour with the presence of ionising radiation. Although, there are noticeable trends in the Aurora system performance with and without the presence of
ionising radiation, the overall effect of measurement accuracy is minimal. On the other hand, the tracker is more sensitive to distortions caused close to the emitter rather than to the sensor. The deviations also depend on the relative orientation between emitter and sensor, with the worst orientation having the sensor directed towards the field generator and the best orientation being orthogonal. In the case of a biopsy tool close to the sensor the distortion is negligible.

Collecting precise information on the nonlinear local error and calibrating the EM tracking system can compensate the influence of the metallic equipment. An effective way to collect reliable positional information is by using an optical tracking system, such as Polaris, which determines the orientation and position of tracked objects relative to a camera system [160, 167]. Robotic arms can be also used but they carry the risk that their metal bases interfere with the EM tracking measurements. Once the data have been collected, several tracking calibration methods can apply to find a mapping function between the gold standard data provided from optical tracking and the EM data distorted from EM interference. Kindratenko [168] presents a good survey on Electro-magnetic position tracker calibration techniques, which are mainly used in virtual reality applications, such as the CAVE (Cave Automatic Virtual Environment). Most methods, however, assume that position displacements depend only on position and do not change with orientation.

Wu et al. [167] indicates that this assumption may be true for smooth distortions, but it does not hold for the Aurora in a clinical setup. Chung et al. [160] also suggests that it may not be adequate to obtain static measurements when performing the calibration, since the accuracy of EM trackers is reduced when tracking objects are in constant motion. Subsequently, Chung et al. [160] proposes a freehand calibration technique that incorporates these requirements. It allows for free movements of the calibration object and provides a mechanism of interdependence between positional and angular correction. To ensure smooth interpolation of $C^2$ continuity over the six degrees-of-freedom that describes the position of the EM sensor, a radial basis spline framework is extended to cater for the non-Euclidean nature of orientation space. While current calibration techniques offer a factor of improvement of around 1.5, the second generation of extra-immune EM systems has also become available recently [161]. These systems report accuracy of sub-millimiter in the presence of ferromagnetic objects common in the operative room and open the way of wider use of EM systems in computer aided surgery.
6.2.2 *EM Tracking under Tissue Deformation and Patient Movements*

Crucial to the success of EM endoscope tracking is the alignment between the EM data and the pre-operative images. When the anatomical structure depicted on the pre-operative images does not deform during operation then a rigid transformation between the EM tracking coordinate system and 3D data can easily established based on landmark correspondences. Subsequently, a least-square approach is used similar to [169, 170], which offers an optimum in least-square sense closed-form solution and it requires a minimum number of three non-collinear point correspondences. This methodology is common in orthopedics and neurosurgery and generally in optical tracking, because the related anatomical structures are relatively rigid and apparent landmarks in both pre-operative data and intra-operatively are easily identifiable. Furthermore, the position of the patient can be easily tracked, by introducing additional sensors, for example attached to the head and so on.

However, relating the EM tracking data to a static pre-operative scan can be particularly challenging when flexible instruments and catheters are tracked inside the body. This is due to tissue deformation and global shift of the position of the patient during operation. Tissue deformation can result from tissue-tools interaction, cardiac contraction, respiratory motion, organ function and internal shift of the organ as the patient changes position. This problem has been reported to numerous research studies [154, 157, 171] and various motion compensation techniques have been developed [166]. For example, Dickfeld et al. investigated whether stereotactic catheter guidance based on EM tracking can place radiofrequency (RF) ablations at well-defined anatomical targets, within the lateral wall of left atrial appendage, identified on MRI. In the work by Dickfeld et al. [157] it is recognised the effects of respiratory motion to the accuracy of the technique and attached a reference catheter on the chest of the animals to improve the accuracy of image registration during respiratory or animal position changes. Respiratory motion compensation is well known in radiotherapy, where a certain dose of radiation needs to be delivered in the cancerous target with minimal effect on the surrounding healthy tissue.

In the work by Timinger et al. [166] also used an EM system (Aurora) to locate the catheter during navigation within the coronary tree. The image-guided system was extended to compensate for cardiac contraction and respiratory motion. Cardiac contraction is compensated by gating the catheter position. In this process only those images acquired during the rest phase of the heart are used for the image-guided system. An affine model is used to compensate for the respiratory motion. This model is driven by a parameter, which indicates the current state of the motion. Such a parameter can be the location of the edge of
the diaphragm, which moves along with respiration. However, a correlation between this parameter and the resulting effect on the organ has to be identified. Assuming that this correlation can be described by an affine model, point correspondences of the cardiac phantom between fully exhaled phase and fully inhaled phase was extracted and subsequently a least square minimisation approach was employed to recover the parameters of the affine model. The work by Timinger et al. [166] based their methodology on the fact that the movements of the heart and those of the diaphragm are similar due to the anatomical neighbourhood. Subsequently, they simulated respiratory motion induced to the artificial coronaries by a rotation.

There is high correlation between the motion of the needle placed in the liver and the motion reported from a sensor attached on the sternum of the patient [172]. Subsequently, an affine model was used to compensate for the error. However, it was also indicated that there is the need to create more sophisticated models that account for tissue warping. In general, motion of internal human organs may be complex with high non-rigid components. This implies that simple mathematical models may fail to predict the deformation of more complex anatomical structures, such as the tracheo-bronchial tree. For this reason it may neither be adequate nor straightforward to associate the motion of an externally tracked point to the internal motion of an organ. Nevertheless, the importance of motion compensation in successful integration and clinical application of EM medical devices has been widely recognised.

6.3 EM Tracking in Image-Guided Bronchoscopy

Miniaturised EM trackers have been also used in image-guided bronchoscopy to accurately display the position of the tip of the flexible endoscope in pre-operative CT images [74, 154]. Bronchoscopists mainly rely on the endoscopic view to navigate in the airways and acquire tissue samples. However, this results in poor diagnosis when transbronchial needle aspiration is performed to detect extraluminal lesions. Moreover, during transbronchial forceps biopsy the bronchoscopist relies on a C-arm fluoroscopy for guidance. Two-dimensional fluoroscopy cannot accurately provide the 3D position of the lesion, which frequently results in missing the target. EM tracking has the potential to facilitate bronchoscopy procedures by tracking the tip of the flexible instruments.
It has been postulated that in the presence of airway deformation, one can readily use the EM tracking data and subsequently apply image registration to recover the unknown axial camera rotation (AURORA) [75]. In the work by Solomon et al. an additional EM tracker is placed on the patient chest to partially compensate for respiratory motion [154]. Measurements taken repeatedly at the same point showed that an error of $3.6 \pm 2.6$ mm still exists, indicating that further improvements to the method should be considered. This issue will be investigated more in the following chapter, where the tracheo-bronchial tree reconstructed from a CT scan, will be deformed to accommodate anatomical changes due to respiration. However, before any solution can apply global motion of the patient should be monitored and decoupled from the EM data.

6.4 Decoupling of Global and Respiratory Motion

![Figure 6.1](image)

**Figure 6.1:** A diagrammatic illustration of the EM tracking sensor setup, where one catheter tip sensor is inserted into the patient’s lungs and another is attached on the chest of the patient to monitor global and respiratory motion.

Additional positional sensors can be introduced to extract respiratory motion by tracking the position of external fiducial markers attached on the skin of the patient. However, this technique compromises the accuracy of the system, since the human skin is easily deformable, especially in obese individuals [154]. Moreover, the introduction of extra tracking devices complicates the experimental procedure and may not be practical in clinical examinations. Due to the different frequency characteristics of the respiratory and global motion, it is possible to simultaneously acquire and decouple these motions by using a single sensor. This is not trivial as global and respiratory motion is intertwined depending
on the projection axes. In the subsequent sections, we will present a new technique that uses wavelet analysis to identify and filter out episodes related to coughing and extreme breathing patterns, and a principal component analysis based method to retrieve the hidden respiratory pattern of the EM position sensor.

6.4.1 Principal Component Analysis

Given that the dominant variation of the signal is due to respiratory motion and it has a constant direction during a given time window, then principal component analysis can be employed to decouple it. The principal vector is expected to lie on the same plane over time and, thus projecting the signal on this plane can isolate the signal variations due to respiration. We define as $f$ the positional vector over time of the MagTrax (Traxtal Technology) reference sensor attached to the chest. Typically, the respiratory pattern has a higher frequency than positional drift of the patient, a box kernel $\delta(t)$ is convolved to each axis of the motion sensor such that

$$f^*(t) = f(t) - f(t) * \delta(t) \quad (6.1)$$

The width of the kernel $\delta(t)$ is chosen to be approximately half respiration cycle (3sec) so that the DC component is filtered out. Here the DC component includes the static position of the patient, as well as the positional drift and the motion of the bronchoscope with lower frequency than respiration.

In discrete form, the samples over time can be represented as:

$$f^*(t-w \cdot \Delta t), \ f^*(t-w \cdot \Delta t + \Delta t), \ \ldots \ \ f^*(t + w \cdot \Delta t) \quad (6.2)$$

where $\Delta t$ expresses the sampling time interval and $w$ is an integer that corresponds to each sample in a kernel that has a typical length of 30 sec.

Subsequently, a matrix $h$ is defined for each time sample as:

$$h(t) = \left[ f^*(t-w \cdot \Delta t), \ f^*(t-w \cdot \Delta t + \Delta t), \ \ldots \ \ f^*(t + w \cdot \Delta t) \right]^T \quad (6.3)$$
$h$ has dimension of $(2w+1)\cdot 3$, because it includes positional information of the bronchoscope in a 3D coordinate system $(x,y,z)$. Note that $\bar{h}(t)$ is the mean of $h(t)$. Therefore, the covariance matrix is produced as:

$$M(t) = \left(h(t) - \bar{h}(t)\right)\cdot \left(h(t) - \bar{h}(t)\right)^T$$  \hspace{1cm} (6.4)

The principal vector $p(t)$ is estimated from the eigen-decomposition of the covariance matrix. Finally, $r$ is a scalar vector, which represents the decoupled respiratory motion. Each element of $r$, $r(t)$, is estimated as the projection of $f^*(t)$ on the principal vector $p(t)$.

$$r(t) = f^*(t) \cdot p(t)$$  \hspace{1cm} (6.5)

### 6.4.2 Wavelet Analysis

During bronchoscope examination, coughing involves major distortion of the airway thus those video sequences should be isolated during image based modelling. In order to identify these episodes wavelet analysis was used [173]. Wavelet analysis provides a localised frequency analysis and has the potential to analyse signals that contain multiple non-stationary or transitory signal characteristics, such as drift and abrupt changes. On the other hand, Fourier transform assumes that a signal is stationary, which implies that it has constant statistical parameters over time. The issue of stationarity depends on the size of the sampling window. During bronchoscope examinations, the signal cannot be approximated as stationary and thus the wavelet analysis is inherently more suitable. The main advantages of the method are both its ability to perform local analysis and to preserve time information. In the other words, wavelets represent a windowing technique with variable-sized regions that allow the use of long time intervals when low frequency information is available, and shorter intervals when high frequency information is present.

Let $\psi(t)$ be a function in the Hilbert space $L^2(R)$ of measurable, square-integrable one-dimensional functions with an average of zero. The wavelet transform of a function $f(t)$ at scale $j$ and position $i$ is given by the convolution product:

$$W_j f(t) = f \ast \psi_j(t)$$  \hspace{1cm} (6.6)
Figure 6.2: Wavelet analysis: X, Y and Z positional data has been acquired from an EM tracker attached on the skin of a healthy subject in order to assess the wavelet technique of filtering sudden movements similar to those that are introduced in a bronchoscopy session when the patient coughs. We used Daubechies (db10) mother wavelets to analyze the signals.

In multiscale edge analysis, $\psi(t)$ is usually chosen to be the derivative of some smoothing function, and thus the local maxima of $|Wf(t)|$ indicate the positions where sharp signal
variations occur. In order to identify these variations, we define an energy function $e$ with threshold $\xi$.

$$e(t) = \sum_{j < 10} W_j f(t) > \xi$$

(6.7)

6.5 Experimental Design

Two experiments have been conducted in order to validate our technique. Firstly, the EM tracker was attached on the subject chest to monitor normal breathing and coughing. The subject was asked to cough at a particular time, while slight global motion was introduced. An example is demonstrated in Figure 6.2 and has been used to validate the wavelet technique. Secondly, positional data has been acquired from two Aurora EM trackers. The first sensor was placed on the chest of the immobile volunteers to be used for respiratory gating. The second one was placed on the chair, in order to monitor the global motion used for validation. In this case, the positional difference of the two sensors should be purely due to respiration. Subsequently, we applied the decomposition method mentioned above to extract respiration from just one sensor attached to the chest. The aim is to see the extent to which the respiratory motion can be recovered from this sensor alone. Eight subjects have been recruited for this study. During the experiment, they were asked to keep their body completely immobile, while they were breathing normally. Their respiration has been monitored for about 2 minutes. Global motion was introduced either by moving the EM field emitter or the base of the chair smoothly.

6.6 Results

Figure 6.2 and Figure 6.3 demonstrate the wavelet analysis of the X, Y and Z positional data that have been acquired from an EM tracker attached on the skin of a healthy subject. The wavelet coefficients are plotted for each signal component. Figure 6.3 represents the energy function of the X-wavelet coefficient graph that has been defined within equation (6.7). The time interval where the patient coughs shows up clearly as a pick at around 1000 time samples that corresponds to 88 sec.
Figure 6.3: The energy function after analyzing the X-motion signal. Similar results have been acquired from analyzing either the Y or the Z component.

In Figure 6.4, the respiratory motion decoupling technique is demonstrated. Figure 6.4 (b) illustrates the x, y, z positional component of the data received from the EM tracker attached on the chest of one of the subjects, while Figure 6.4 (a) illustrates the x, y, z positional component of the data received from the EM tracker attached on the chair. Global motion has been introduced by moving slightly the field emitter (Aurora). Figure 6.4 (c) represents the recovered respiratory component after applying the proposed technique of Section 6.4 to the data received from the EM tracker on the subject’s chest. In order to validate our technique the correlation between the recovered waveform (Section 6.4) and the waveform estimated by applying principal component analysis to the vector between the two EM tracker tool tips has been calculated for eight subjects. Table 6.1 demonstrated the results, which indicate a high degree of correlation.

<table>
<thead>
<tr>
<th>Subjects</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>0.918</td>
<td>0.962</td>
<td>0.805</td>
<td>0.794</td>
<td>0.879</td>
<td>0.791</td>
<td>0.786</td>
<td>0.687</td>
</tr>
</tbody>
</table>

Table 6.1: Correlation of the waveform estimated by the suggested respiratory decoupling motion technique with the waveform estimated by relative position of two sensors.
Figure 6.4: Example position traces sampled by the EM tracker and the extracted respiratory motion pattern.

6.7 Discussions and Conclusions

Thus far, tracking devices have been integrated into a number of image-guided surgery applications. The most successful examples are in neurosurgery and orthopaedic surgery. Accuracy and stability of these systems is critical to patient health and the outcome of the
operation. Currently, the most reliable and precise position sensors available in surgical applications are optical tracking systems. The main advantage of these methods is that they are not affected from ferromagnetic distortions, while they have a large operative volume. Unfortunately, optical trackers are limited to the line-of-sight applications and thus they are inappropriate for tracking flexible instruments inserted in patients.

Recently, the technological advances of miniaturised EM tracking sensors have allowed their insertion in the biopsy channel of flexible endoscope and permit real-time tracking of the endoscope tip inside the body cavity. Although the inherent mechanism of EM tracking devices is inferior to the corresponding mechanism of optical trackers, it does not imply that their application accuracy is always inferior. In fact, optical trackers can only be used to track fiducial markers that are always rigidly attached on the skin or on the rigid surgical tools. Nevertheless, this is compromised by the deformable nature of soft tissue and the non-perfect rigidity of the tools. Furthermore, continuous tracking is also compromised from the line of sight constraint.

It is important to realise that mechanical accuracy of a positional system is different from application accuracy. For example, an EM system with manufacturing specifications of sub-millimetre accuracy, under the influence of metal and tissue deformation, it may provide measurements that are several centimetres away from the correct positions. This indicates that the limiting factors in the use of these systems in practice are not their inherent inaccuracy but the lack of appropriate methods to exploit their capabilities. Most of the current image guided systems rely on a preoperative-static scan without any update to allow for tissue deformation with relation to patient movements during the procedure. Therefore, methods should be developed to compensate for tissue deformation and to monitor the global position of the underlying anatomy with relation to a fixed coordinate system. This, however, is not a trivial task, particularly in bronchoscopy. The internal motion of the airways is difficult to be modelled adequately and it is not straightforward how it correlates with the motion of externally tracked points. There are two goals to be achieved. The first aims to decouple global from respiratory motion and the second is to model the tracheo-bronchial tree motion during respiration.

In the work presented in this chapter, we have developed a novel method of decoupling respiratory motion from the signal received from an EM tracker attached to the chest. We used wavelet analysis to filter out episodes due to coughing and other extreme breathing patterns. Principal component analysis has been employed to decompose the respiratory cycle from global motion. It should be noted that the basic assumption used for respiratory
motion decoupling is that the global positional drift has a relatively low frequency in comparison to respiratory cycles. When there is sudden motion involving large acceleration, it is likely that rapid changes in sensor readings will be introduced. In this case, the algorithm based on principal component analysis will lead to detection errors. Since we have used wavelet analysis to sense rapid sensor movement, these events will also be isolated along with coughing and extreme respiratory motion. The results in this study indicate that episodes related to coughing can easily identified and filtered out and the respiratory motion component can be reliably extracted. The suggested technique can facilitate a 2D/3D registration regime under deformation and increase the accuracy of the registration process. Further clinical approval needs to be done to investigate its efficiency. In the next chapter, we will demonstrate that it is also possible to capture the airways shape variability during respiration and use this prior knowledge to facilitate the non-rigid 2D/3D registration framework. In this way, the EM devices can be utilised directly in the registration process to further enhance the registration accuracy of the proposed $pq$-space 2D/3D registration method.
In the previous chapter, we have demonstrated the potential value of EM tracking for 2D/3D registration. We have shown that continuous tracking of the bronchoscope is difficult with image-based registration algorithms alone and the method can fail if extreme breathing patterns are encountered. Image artefacts caused by mucosa, bubbles and bleeding during a typical bronchoscopy examination can also pose significant challenges. The introduction of in vivo catheter tip EM tracking means that it is now possible to explicitly incorporate airway deformation into the registration framework.

The incorporation of airway deformation to the existing 2D/3D registration framework, however, is a difficult task. This is due to the potentially very large degrees of freedom involved in simultaneous tracking of the camera pose and structural changes. At first glance, it may seem that the use of a 5 DoF catheter tip EM tracker reduces the original 2D/3D registration to only a 1D problem. It has been postulated that in the absence of airway deformation, one can readily use the EM tracking data to recover the unknown axial camera rotation [75]. For situations where large non-linear deformation is encountered in vivo experiments, however, the nature of the problem is much more difficult than what it first appears. This is because the EM tracker reports only the position and orientation of the bronchoscope camera in relation to the fixed, world coordinates. When this information is used to guide the registration of the airway structure reconstructed from the tomographic data, a large displacement due to airway deformation can be observed. When this EM tracking information is used to initialize the 2D/3D registration, it has been found that the
initial view can often be completely outside of the bronchial tree. This initialization is in fact inferior to the traditional image based approach, as in this case the camera view is always in synchrony with the moving frame of reference of the deforming airways. For this reason, direct mapping of the EM tracking measurements to the static preoperative CT images is not appropriate, and its practical use must be considered with the deformation of the airways.

In this chapter, we will present a non-rigid 2D/3D registration framework and its phantom validation for subject-specific bronchoscope simulation. The method makes use of a 5 DoF miniaturised catheter tip EM tracker such that the position and orientation of the bronchoscope can be accurately determined. This allows the effective recovery of unknown camera rotation and airway deformation, which is modelled by an Active Shape Model (ASM). It is demonstrated that ASM can capture the intrinsic variability of the tracheo-bronchial tree during breathing. ASM is specific to the class of motion it represents and this enables the reduction of the number of parameters that control the deformation. Subsequently, the optimisation procedure is greatly simplified and pq-based registration as described in Chapter 4 is performed to recover both the camera pose and parameters of the ASM. Detailed assessment of the algorithm is performed on a deformable airway phantom, with the ground truth data being provided by an additional 6DoF EM tracker to monitor the level of simulated respiratory motion.

7.1 Deformable 2D/3D Registration

Figure 7.1 provides a schematic illustration of the proposed registration framework. Given a number of volumetric images at different respiratory phases of the tracheo-bronchial tree, an ASM capturing the intrinsic deformation of the airway is created. Based on the control points of the ASM, Radial Basis Functions (RBFs) are used to warp the 3D mesh to reflect the structural deformation induced. 2D/3D registration is then used to recover both the parameters of the ASM and the pose of the bronchoscope camera. We demonstrate that by modelling the deformation of the airways with ASM, it is possible to capture intrinsic variabilities of the tracheo-bronchial tree based on images acquired at different phases of the respiratory cycle. In this way, the dimensionality of the non-rigid 2D/3D registration problem can be significantly reduced, thus leading to a more robust registration framework that can be deployed to patient bronchoscope examinations.
7.1.1 Deformation Modelling of the Airway Tree

In order to design an efficient non-rigid 2D/3D registration, prior knowledge about the airways deformation during respiratory motion is required. Deformable models can accommodate the variability of biological structures over time and across different individuals. As it has already been mentioned in Chapter 3, deformable models have been proved to be effective in tracking anatomical structures and they are used extensively in non-rigid registration for formulating transformation models. Deformable models are constructed under the influence of geometry, physics and approximation theory [91]. Geometry serves to represent object shape, physics imposes constraints on how the shape may vary over space and time, and optimal approximation theory provides the mechanisms for fitting the models to the measured data.

As an example, prior knowledge about the deformation of the lungs can be derived by monitoring the tracheo-bronchial tree during respiration in order to determine how the shape changes over time. This can be accomplished by scanning lungs in different respiratory phases [95]. Alternatively, anatomical and physiological models of respiration can be incorporated [174, 175]. In the work by Kaye et al. [50], a simulation system that combines anatomy and respiratory mechanics has been developed based on a physical model. The lungs are considered as a single-compartment, viscoelastic structure, and the classic mechanical model based on observable variables such as pleural pressure change, lung volume change, and airway pressure is used. However, the contraction of the intra-thoracic tree depends on the surrounding structure and the biomechanical properties of the
elastic tissue. In practice, biomechanical modelling of the airways is not trivial because it requires in vivo measurements of the tissue properties of the vessel lumen.

In the work by Garrity et al. [95], a dynamic model for the lung lobes and the airway tree has been developed. A model of the first four to five generations of the airway tree has been created by segmenting the visible human CT data. The algorithm assumes that each branch is a rigid cylindrical tube with a constant diameter and a dichotomous subdivision system. Parameters for the daughter branches are determined by equations defining the relationships between the air flow and the diameter, angle, and length of the branches. Once the lung lobes and airway tree are finalized, the respiratory motion is modelled based on high-resolution 4-D respiratory-gated CT data obtained from a normal human subject. The data consists of two CT scans taken at two different time points of the respiration cycle. One scan was taken at 5% of total lung capacity (TLC) and the other scan was taken at 40% of TLC. The limitations of this approach are that the parameterisation is based on splines and requires a large amount of parameters. Furthermore, the airways are over-smoothed and the method does not offer a patient specific deformation model of the lungs.

7.1.2 Active Shape Models

In this thesis, we suggest using the Point Distribution Model (PDM) of the ASM for the modelling of airway deformation to ensure that a compact representation of the deformation of all the control points can be captured by the principal modes of shape variation. In this way, the degree-of-freedom during deformable 2D/3D registration is only dictated by the number of modes chosen plus the extra degree-of-freedom introduced by the unknown axial rotation of the endoscope.

ASMs are parametric deformable models that deform in ways that are consistent with the class of shape and motion they represent [40]. This is defined by the boundary point distribution of a number of shapes, called a training set. To ensure that the ASM captures the intrinsic shape rather than the corresponding landmark point variations, all shapes in the training set should be aligned so as to minimise the distance variance of corresponding control points. Subsequently, Principal Component Analysis (PCA) is applied to the PDM to extract the statistical modes of variations that describe these shapes.

The intuitive benefit of ASM derives from the fact that for most deformable models the landmark points are not moving independently, so the intrinsic dimension is significantly
lower than the number of control points. Let \( s_i \) be a vector describing the \( n \) control points of the \( i^{th} \) shape of the deformable model.

\[
s_i = (x_{i0}, y_{i0}, z_{i0}, \ldots, x_{ik}, y_{ik}, z_{ik}, \ldots, x_{in-1}, y_{in-1}, z_{in-1})^T
\] (7.1)

Each sample in the training set can therefore be represented by a single point in a \( 3n \)-dimensional space. A set of \( N \) deformable shapes (6 for this study) results in \( N \) points in the \( 3n \)-dimensional space. These points in general lie within a region of the shape space, which is called the ‘allowable shape domain’. Given that every \( 3n \)-dimensional point within this domain gives a set of landmarks whose shape is similar to those in the original training set, the variability of the shape can be modelled in a systematic way in the remapped space.

By assuming that the allowable shape domain is approximately ellipsoidal, its centre can be calculated as the mean shape \( \bar{s} \), i.e.,

\[
\bar{s} = \frac{1}{N} \sum_{i=0}^{N} s_i
\] (7.2)

Subsequently, the principal axes of the ellipsoid fitted to the data are calculated by applying PCA to the data. Each axis gives a mode of variation, which represents the way that the landmark points tend to move together as the shape varies. For each shape in the training set, the deviation from the mean \( ds_i \) is calculated to form the covariance matrix \( S \):

\[
S = \frac{1}{N} \sum_{i=0}^{N} ds_i ds_i^T \quad \text{with} \quad ds_i = s_i - \bar{s}
\] (7.3)

In this way, any point in the allowable shape domain can be modelled by taking the mean shape and adding a linear combination of the eigenvectors of the covariance matrix, i.e.,

\[
s = \bar{s} + Pb
\] (7.4)

where \( P = (p_1, \ldots, p_t) \) is the matrix of the first \( t \) eigenvectors. Therefore, new shapes are generated by varying the parameters \( b_k \) within the suitable limits for them to fit to the training set. The limits for \( b_j \) are derived by examining the distributions of the parameter values required to generate the training set. In this study, the set of the parameters \( \{b_1, \ldots, b_t\} \) is chosen by assuming that the plausible shape variations lie within three standard deviations of the mean.
where $\lambda_k$ is the $k^{th}$ eigenvalue. This choice covers the plausible shapes between the two extreme shapes at maximum ‘exhalation’ and ‘inhalation’ of the phantom.

7.1.2.1 Point Correspondence for the Active Shape Model

As mentioned earlier, in practical implementations the quality of a statistical shape model relies heavily on the accuracy of the landmark correspondence across the 3D model with different levels of structural deformation. In order to establish correspondence between a set of control points across the 3D meshes, the skeleton of the airway was first extracted from the 3D data. The skeleton points were estimated from each slice as the geometric centroid of the surface extracted from the marching cube algorithm. Manual interaction was required to define the bounding box of each branch. The bifurcations of the airway tree were then used as the landmarks for establishing the correspondence across the skeletons.

This technique is demonstrated in Figure 7.2, where the $x$, $y$ and $z$ values are measured in $\text{mm}$. In Figure 7.2 (a), the extracted geometrical centroids of each airway branch of each CT slice (red dots) are shown for six 3D airway meshes that correspond to different phases of the movement cycle as illustrated in Figure 7.4. An algorithm has been developed to automatically connect the skeleton points (red dots) and identify bifurcation and terminal points. The blue line represents the extracted skeletons, whereas the bifurcation and terminal points are indicated by yellow markers. In Figure 7.2(a), the green lines show the correspondence between the extracted points defined from the bifurcation points. A graphical user interface has been designed to permit manual interaction and efficient 3D visualisation. Each skeleton can be extracted in less than twenty minutes.

For each of the skeleton points, the surface perpendicular to the skeleton axis was subsequently defined and its intersection with the 3D mesh was estimated. A Gaussian filter has been used to smooth the skeleton line and improve the quality of the correspondence. For simplicity, four surface control points corresponding to each skeleton landmark were used Figure 7.2 (b). This in general is sufficient to define a branching generalised cylinder for the airways.
Figure 7.2: a) The extracted geometrical centroids of the airway branch for each CT slice (red dots) for six 3D airway meshes that correspond to different phases of the simulated respiratory cycle of the phantom. The blue line represents the extracted skeletons, whereas the bifurcation and terminal points are indicated by yellow markers. The green line shows the correspondence between the extracted points. b) For each of the skeleton points, the surface perpendicular to the central axis is subsequently defined and its intersection to the 3D mesh is estimated. For simplicity, four surface control points corresponding to each skeleton landmark are used.
7.1.3 Mesh Warping with Radial Basis Functions

The application of ASM to the skeleton and surface control points results in intermediate, deformed airway structures. To generate a smooth 3D mesh of the airways, Radial Basis Functions (RBFs) are used. An RBF is a continuous function, of at least $C^1$ continuity that provides smooth, controllable deformation. It maps each control point in one domain to the corresponding control point in the other with interpolation of the mapping at intermediate points. An RBF spatial transformation in $d$ dimensions, denoted $T(x, y)$, is composed of $k = 1, \ldots, d$ mapping functions [176], such that:

$$T(\vec{x}) = [f_1(\vec{x}), \ldots, f_k(\vec{x}), \ldots, f_d(\vec{x})]$$  \hspace{1cm} (7.6)

With RBFs, each of the mapping functions can be decomposed into a global component and a local component, and this decomposition enables a family of transformations to be defined where the influence of each control point can be determined. Given $n$ pairs of corresponding control points, each of the $k$ mapping functions of the RBF takes the following form:

$$f_k(\vec{x}) = P_{mk}(\vec{x}) + \sum_{i=1}^{n} A_{ik} g(r_i)$$  \hspace{1cm} (7.7)

where $\vec{x} = (x, y, z), P_{mk}(\vec{x})$, is a polynomial of degree $m$, $g(r_i)$ is a radial basis function, $r_i$ denotes the Euclidean norm defined by

$$r_i = \|\vec{x} - \vec{x}_i\|$$  \hspace{1cm} (7.8)

and $A_{ik}$ corresponds to the $i^{th}$-column of the parameter array $W$. It is worth noting that the polynomial term has been omitted and $g(r_i)$ is a linear radial basis function that expresses the contribution of each control point on the original vertex.

In the 3D case represented in this study and ignoring the polynomial term of Equation 7.7, the RBF transformation is determined by $n$ coefficients in each dimension. The coefficients of the function $f_k(\vec{x})$ are determined by constraining $f_k(\vec{x})$ to satisfy the interpolation conditions and solving the following linear system of equations:
where \( G(i, j) = g(r_j) \), and \( Y \) is a matrix with the deformed points. Intuitively, \( g(r_j) \) measures the effect of the \( j^{th} \) control point on the transformation at the \( i^{th} \) control point. The above equation is typically solved by using Singular Value Decomposition (SVD) and the new position of each vertex is then calculated from Equation 7.7. During calculation, since the vertices have been changed the normals also need to be recalculated. In this study, the normals for each rendered scene are implicitly estimated as part of the registration procedure.

### 7.1.4 A pq-based Similarity Measure

The basic principle of constructing a similarity measure for the 2D/3D registration is based on the original pq-based framework suggested in Chapter 4. In this technique, surface normals of the video endoscopic image are extracted by using a linear shape-from-shading algorithm, which exploits the unique endoscopic camera/lighting special configuration. During registration, these vectors are matched to the surface normals of the 3D model with its deformation controlled by the ASM and axial camera rotation. The similarity measure of the two images was determined by evaluating the dot product \( \varphi \) of the corresponding pq-vectors. By incorporating the mean angular differences and the associated standard deviations \( \sigma \), the following similarity function is used:

\[
S = \left( \sum \sum (\varphi_w) \cdot \sum \sum (1 - \sigma(\varphi_w) \cdot \|\widehat{n}_{3D}\|) \right)^{-1}
\]  

(7.10)

where \( \widehat{n}_{3D} \) is the surface normals derived from the 3D model and \( \varphi_w \) stands for the weighted dot product. With this scheme, neither the lighting parameters nor the other rendering parameters need to be adjusted during registration. The similarity measure is weighted according to the 3D model surface vectors, such that it is relatively immune to texture discrepancies that are common to the endoscopic images. For optimising the above similarity measure, the Simplex algorithm by Nelder and Mead was used [177].
## 7.2 Incorporating EM Tracking Devices

In previous chapters, we have emphasised the fact that an EM tracker reports only the position and orientation of the bronchoscope camera in relation to fixed, world coordinates. For this reason, direct mapping of the EM tracking measurements to the static preoperative CT images is not appropriate, and its practical use must be considered in relation to both the deformation of the airways and the global motion of the patient. In the last chapter, it has been demonstrated how to utilise the measurements of a positional sensor attached on the patient chest to decouple global and respiratory motion. Subsequently, this information can be used to remove the global motion component from the EM data. In this chapter, ASM is used to model the motion of the airways during respiration.

The next step of this approach is to transform the EM data from the EM coordinate system to the preoperative image coordinate system. To this end, three landmarks have been added to the phantom such that can be accurately identified in the pre-operative scans. By marking them with the tip of the EM tracker, their positions can be determined in the EM tracker coordinate system. In a real-case scenario, three or more reliable landmarks should be identified during a bronchocopy session and then matched to the pre-operative images.

Subsequently, a rigid transformation between the two coordinate systems can be determined based on a close-form solution. In this method, a rigid-body transformation is expressed as a combination of a rotation $R$ and a translation $t$. So a point $r_A$ from $A$ coordinate system is transformed to $r_B$ in $B$ coordinate system by the following formula:

$$ r_B = R_{AB}r_A + t_{AB} \quad (7.11) $$

Consequently, a rigid-body registration seeks the values of $R$ and $t$ that minimize the mean square error given 3D point correspondences $r_A$ and $r_B$, i.e.,

$$ \min_{R,t} \sum_{i=1}^{N} \left\| r_{B_i} - (R \cdot r_{A_i} + t) \right\|^2 \quad (7.12) $$

In the work by Horn et al. [169, 170] showed that it is possible to reformulate the problem to decouple the computation of $R$ and $t$ and solve it with a closed form solution. This can be done by referring the coordinates to the respective centroids of each point set, leading to the minimization of the following equation:
Subsequently, the translation is given by the difference of centroids:

\[ t = \frac{1}{N} \sum_{j=1}^{N} r_{Bi} - R \left( \frac{1}{N} \sum_{j=1}^{N} r_{Ai} \right) \]  

(7.16)

Subsequently, the rotation can be represented by either orthonormal matrices or quaternions [169, 170]. However, quaternions offer a more stable solution than orthonormal matrices as for the latter the orthonormality constraint should be maintained in each step. Once the quaternion representation has been adopted, the solution is obtained by the most positive eigenvalue of the matrix \( N \).

\[
N = \begin{bmatrix}
S_{xx} + S_{yy} + S_{zz} & S_{yx} - S_{zy} & S_{zx} - S_{xz} & S_{xy} - S_{yx} \\
S_{yx} - S_{zy} & S_{xx} - S_{yy} - S_{zz} & S_{zy} + S_{yx} & S_{xz} + S_{zx} \\
S_{zx} - S_{xz} & S_{zy} + S_{yx} & -S_{xx} + S_{yy} - S_{zz} & S_{yz} + S_{zy} \\
S_{xy} - S_{yx} & S_{zx} + S_{xz} & S_{yz} - S_{zy} & -S_{xx} - S_{yy} + S_{zz}
\end{bmatrix}
\]  

(7.17)

where the elements of the matrix \( S \) are estimated as:

\[
S_{xx} = \sum_{i=1}^{n} x_{Bi} \cdot x_{Ai}' \quad \text{and} \quad S_{xy} = \sum_{i=1}^{n} x_{Bi} \cdot y_{Ai}'
\]  

(7.18)

### 7.3 Incorporating Temporal Tracking

The use of ASM allows the incorporation of deformable modelling into the registration process. This enables the efficient use of EM tracking data to further improve the registration accuracy. In Chapter 5, it has been emphasised that both image-based and EM tracking techniques have certain limitations. A probabilistic tracking framework based on
the Condensation algorithm is used to allow the use of multi-modal probability distributions, which is essential to track the camera position under airway deformation and image artefacts. This framework provides an effective way of combining the results from \textit{pq}-space registration with EM tracking to enhance the robustness of the non-rigid registration processing. In essence, temporal tracking uses a prediction model to evolve a number of particles and a measurement model to assign a weight to each particle. Subsequently, by evaluating the given moments of the state density, it estimates the most probable state.

It should be noted, however, the observation model of Chapter 5 was based only on the image-based registration results. In this chapter, the observation/measurement model needs to combine both the EM tracker measurements and the image-based registration results. Generally, the observation density \( p \left( z_t \mid x_t \right) \) defines the likelihood that a state \( x_t \) causes the measurement \( z_t \). This is typically achieved by assuming that the measurements are statistically independent both mutually and with respect to the process model. Furthermore, both the EM tracker and the image-based registration measurements are imprecise. In this study, the uncertainty is quantified with a Gaussian probability density function \( p \sim N \left( 0, \sigma \right) \), where the variance \( \sigma \), depends on the reliability of the different sensors measurements. The observation density for a particular state is estimated by integrating over the space of sensors measurements:

\[
p \left( z_t \mid x_t \right) = \sum_m p_{z_t}^{(m)} \tag{7.19}
\]

The state of the system has been defined as the 6 DoF pose of the camera. For the process model, a second order auto-regressive model was used to describe the motion of the endoscopic camera. A detailed description of this model is provided in Section 5.3. Training of this model is based on the algorithms also described in Chapter 5 and the training sequences have been constructed based on the EM tracking data. since EM tracking data are available only for 5 out of 6 DoF, the last degree-of-freedom was tracked independently.
7.4 Experimental Design

![Image depicting bronchoscope navigation paths](image)

**Figure 7.3:** The bronchoscope navigation paths involved in this study, where the camera travels from the right branch (frame-9780) to the trachea (frame-10845) and then continues to the left branch (frame-12501).

In order to assess the registration accuracy of the proposed algorithm, a dynamic phantom was constructed to imitate the airway deformation during respiration. An optical bronchoscope has been used to navigate inside the phantom tree. **Figure 7.3** illustrates the path that has been followed during the experiment. The bronchoscope travels from the end of the right branch back to the trachea and then continues to the left branch. A 5DoF EM tracker was inserted in the biopsy channel to facilitate the 2D/3D registration. The ground truth data for the non-rigid parameters of the 2D/3D registration has been provided from an EM sensor attached on the phantom to monitor the deformation phase during navigation.

To validate the accuracy of the non-rigid 2D/3D registration technique two experiments have been designed. Firstly, the 5DoF’s of the rigid registration that define the position and two of the rotation angles have been directly recovered from the 5DoF EM tracker. The sixth DoF, which defines the rotation of the camera around its axis, as well as the deformation parameters have been recovered by optimising a $pq$-based similarity measure.
Quantitative results have been provided for the recovered deformation parameters in Section 7.5.4. This section also provides with qualitative results of the overall accuracy of the non-rigid 2D/3D registration.

A second experiment has been designed to show the effectiveness of the temporal tracking. Temporal tracking has been applied to only the rigid motion of the camera and not to the deformation parameters. The EM tracking data have not been used directly to define the pose of the camera, but they were combined probabilistically with the image-based registration results to provide the optimal pose.

### 7.4.1 Video Preprocessing

Prior to 2D/3D registration, pre-processing of the videos was necessary in order to alleviate the effects of interlacing, lens distortion, and image artefact. Preprocessing of the endoscopic frames is similar to Section 4.3.5. De-interlacing can be achieved by separating odd/even scan lines, which is particularly important to minimise the comb effect during fast advancement or retraction of the bronchoscope. To correct for barrel distortion due to the wide angle bronchoscope camera used, the camera model suggested by Heikkil, which includes both radial and tangential distortion of the video image, was used. The initialization of the calibration parameters follows the work by Zhang et al. Finally, to improve the SNR of the video image, a structure adaptive anisotropic filter was used [136]. The method uses a local orientation and anisotropic measure to control the shape and extent of the filter kernel and thus ensures that salient image features are well preserved whilst removing image artefacts. The method generally provides a smoother optimisation landscape, and brings improved convergence of the 2D/3D registration algorithm.

### 7.4.2 Phantom Setup

In this study, the airway phantom was made of silicon rubber and painted with acrylics. The inner surface was coated with silicon-rubber mixed with acrylic to give it a textured and specular finish that is similar to the bronchial lumen. The phantom surface was finished with red colour splashes to imitate the blood and texture discrepancies during a bronchoscope examination. The distal ends of the airway-phantom have been fixed and a balloon is located below the main bifurcation to simulate respiratory-induced airway
deformation as shown in Figure 7.4. The navigation paths used for validation are also indicated in Figure 7.3. The extent of the airway motion is regulated by a valve controlling the amount of air in the balloon. The tomographic model of the phantom was scanned with a Siemens Somaton Volume Zoom 4-channel multi-detector CT scanner with a slice thickness of 1 mm and in-plane resolution of 1 mm. A total of six CT volumes were acquired at different stages of the motion cycle.

**Figure 7.4:** Illustrations of the phantom setup during two different deformation stages. a) The maximum level of deformation is achieved when the balloon is full of air, and b) shows the resting state when the balloon is empty.
7.4.3 Ground Truth Data

To track the pose of the bronchoscope camera, an NDI (Northern Digital Inc, Ontario) Aurora EM tracker was used. The Aurora system is particularly suited for tracking flexible endoscopes during minimally invasive intervention. This typically employs a single coil sensor due to size restrictions imposed by the biopsy channel. Having only a single coil, however, the sensor is limited to reporting position and direction only (i.e. 5 DoF with unknown axial rotation). The working volume of the Aurora system is within 500mm cube located 50mm from the field generator. In an environment that is free from Electromagnetic interference, the Aurora has a static positional accuracy of 0.9-1.7mm and an orientation accuracy of 0.5 degrees.

In order to obtain the ground truth deformation data, a real-time, 6 DoF EM tracker tool was used to track the deformation of the balloon so that the deformation of the phantom airway can be indexed back to the 3D CT data. An Olympus bronchoscope (Olympus BF Type, with a field of view 120°) operating in PAL recording mode (25fps) was used to capture the 2D bronchoscope video.

In this study, the ground truth data used was provided by the 6-DoF EM tracker attached to the balloon controlling the level of simulated respiratory motion. This permits the direct association of the deformation of the airway phantom during bronchoscope examination with the exact 3D geometrical shape captured by CT. The projections of the deformation as mapped onto the principal axes defined by the ASM were used in this study to provide a quantitative measure of the effectiveness of the registration process, i.e.,

\[ b_i = \hat{M} \cdot p_r \cdot \left( p_r \cdot p_r^T \right)^{-1} \]  \hspace{1cm} (7.20)

where \( p_r \) is the eigenvector that corresponds to the point \( s_i \) of the 3D mesh that is nearest to the position of the 6-DoF EM tracker, and \( \hat{M} \) is the vector between these two points.
7.5 Results

For the results presented in this chapter, a conventional PC with an AMD Athlon™, 2.19GHz, 1GB RAM and a graphic card NVIDIA GeForce 6800 Ultra was used. The rendering was based on OpenGL and the graphical interface is designed using FLTK (Fast Light ToolKit). The airway mesh consisted of 13,101 vertices and 26,101 triangles. The code has not been optimised and each frame took around 3 minutes to optimise. For the ASM, 301 control points have been used and the RBF has been implemented such that the initialisation step is calculated only once to estimate the inverse of $G$ matrix of Equation (7.9), which represents the topology of the initial control points. The rest part of the calculation was almost completely dedicated to the deformation of the vertices based on Equation 7.7.

7.5.1 Assessment of Phantom Visual Reality

Figure 7.5: Example bronchoscope video frames acquired during the phantom experiment where the images have been pre-processed to alleviate the effect of de-interlacing and lens distortion.
To demonstrate the general visual appearance of the phantom airway model, Figure 7.5 illustrates 9 example frames of the video sequence after pre-processing, which includes de-interlacing, lens distortion, and anisotropic smoothing. Red stains have been added on the inside phantom walls to imitate vessels and texture discrepancies. The accuracy of the registration scheme does not degrade in the parts of the phantom where these features are eminent. This indicates that the registration algorithm is relatively immune to texture discrepancies as it has been also demonstrated in Chapter 4.

7.5.2 Phantom Deformability

Figure 7.6: The normalised mesh displacement between successive deformation stages of the phantom.

To demonstrate the amount of deformation introduced, the normalised mesh displacement map at five different deformation positions of the phantom are shown in Figure 7.6, illustrating the non-linear, localised deformation at different branches of the airway. By reference to the navigation paths shown in Figure 7.3, it is evident that we have chosen the most deformed part of the airway phantom for the validation purpose.

In this study, the main purpose of the phantom design is to assess the extent of free-form deformation and its effect on the accuracy of 2D/3D registration. It should be noted that the phantom is not a complete representation of the airway structure and its physiological response. The air flow patterns and the force distribution on the airway conduits are patient-
specific and they vary significantly across different subjects [178]. Since the ASM model is to be constructed on a patient-specific basis, the proposed phantom validation framework should provide a good indication of the achievable accuracy of the algorithm for *in vivo* applications.

### 7.5.3 Modelling of the Airways Respiratory Motion

*Figure 7.7:* The result of applying the ASM to the reconstructed 3D volumes, illustrating the warped 3D mesh by using the RBF along the first principal mode of the shape variation. The red, green, and blue meshes correspond to varying $-3\sqrt{\lambda_1}$, $0$, and $+3\sqrt{\lambda_1}$ from the mean shape.

*Table 7.1* shows the total modelling error associated with each eigenvalue. It is evident that the first principal component captures most of the airway deformation, which means that even by varying the first mode alone, most of the structural deformation as reconstructed by RBF can be used to control the subsequent 2D/3D registration process. This was also convenient for validating the deformation parameters with ground truth data derived from one sensor attached on the phantom. However, further investigation needs to decide whether in clinical practice more parameters should be taken into consideration to adequately model the deformation of an *in vivo* tracheo-bronchial tree during respiration. This is not expected to change the overall accuracy of the suggested non-rigid registration framework but it will considerably increase the time of the optimisation convergence.
The result of applying the ASM to the reconstructed 3D volumes are also shown in Figure 7.7, illustrating the warped 3D mesh by using the RBF along the first principal mode of the shape variation in the sagittal, coronal and axial views. The red, green, and blue meshes correspond to varying $-3\sqrt{\lambda_1}$, 0, and $+3\sqrt{\lambda_1}$ from the mean shape, respectively. These are the extreme values of the allowable shape domain. The quality of the ASM depends on the extraction of correspondence features between the avaluable set of 3D CT data. The overall results indicate that the ASM adequately captures the variability induced in the phantom airways. Most, errors are consentrated in the bifurcation points because the angle between the skeleton and the perpendicular plane at these points is not well defined.

<table>
<thead>
<tr>
<th>EigenValues in Descending order</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error in mm</td>
<td>4.3542</td>
<td>0.7723</td>
<td>0.6170</td>
<td>0.1482</td>
<td>0.055</td>
<td>0.0</td>
</tr>
<tr>
<td>Error in percentage (%) of the phantom diameter</td>
<td>10.88</td>
<td>1.93</td>
<td>1.54</td>
<td>0.3706</td>
<td>0.1392</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 7.1: Modelling error associated with omitting each eigenvalue individually

7.5.4 Accuracy of Deformable 2D/3D Registration

The performance of the proposed registration algorithm by using the EM tracking data combined with the deformable model as determined by ASM can be examined in Figure 7.8. In this figure, the top row is the bronchoscope view of the phantom airway, whereas the middle row illustrates the corresponding view of the 3D model determined by the 5 DoF catheter tip EM tracker. Significant mis-registration is evident due to the unknown rotation and airway deformation. It is evident that the position of the camera changes rapidly due to the respiratory motion in the world coordinates, and a direct use of the EM data can result in initial camera position located completely outside of the bronchial tree.

The result of applying the proposed registration algorithm is shown in the bottom row of Figure 7.8, demonstrating the visual accuracy of the method. The frames shown have been acquired at different parts of the tracheo-bronchial tree as it is depicted in Figure 7.3. Generally, the algorithm performs better when prominent features are apparent in the endoscopic images. Such features can visually describe bifurcations or protuberances on the phantom airway walls due to simulated lesions or natural surface folding. The larger inaccuracies occur when the camera is facing the wall and the light diffuse in such a way that it is not able to distinguish any surface details.
In order to provide a detailed quantitative validation of the method, Figure 7.9 illustrates the recovered deformation as projected onto the first principal axis of the ASM model. In other words, this reflects how parameter $b_i$ controls the shape of the mesh based on Equation (7.5). The corresponding ground truth value as determined by the 6 DoF EM tracker from Equation (7.21) is also provided for comparison. It is evident that the reconstruction result followed the ground truth reasonably well. Further details of the error analysis are illustrated in Figure 7.10, which provides a Bland-Altman plot of the errors of the recovered deformation as compared to the ground truth. The Bland-Altman plot reveals how well two datasets agree with each other.

Figure 7.8: The performance of the proposed registration algorithm by using the EM tracking data combined with the deformable model as determined by ASM. Left-column: the bronchoscope view of the phantom airway; Mid-column: the corresponding view of the 3D model determined by the 5 DoF catheter tip EM tracker where significant mis-registration is evident due to unknown rotation and airway deformation; Right-column: the result of applying the proposed registration algorithm
demonstrating the visual accuracy of the method.

**Figure 7.9:** The recovered deformation as projected onto the first principal axis of the ASM model where the corresponding ground truth value (dotted curve) as determined by the 6 DoF EM tracker from Equation (7.21) is provided for comparison.

**Figure 7.10:** Bland-Altman plot of the non-rigid registration results as compared to the ground truth.
7.5.5 Temporal Tracking

![Graph showing temporal tracking](image)

**Figure 7.11**: Euclidean distance between the first and subsequent camera position as it has been predicted from the *Condensation* algorithm with relation to the EM tracking data.

**Figure 7.11** illustrates the results by applying the *Condensation* algorithm to a video segment as compared to the ground truth data recovered from the EM data. The results show that this scheme was effective because it has the ability to correct errors and failures of both techniques. It is able to recover even if either of them fails. Tracking of the camera based only on the EM tracker device’s results in errors due to inaccuracies in the initial coordinate transformation. This is more prominent when the airways deform dynamically due to the bronchoscope itself. Currently, the suggested registration scheme cannot recover these deformation parameters. Subsequently, temporal tracking improves the overall accuracy by relying on both measurements.

7.6 Discussions and Conclusions

In this chapter, we have proposed a deformable 2D/3D registration framework based on ASM. Despite the prevalence of tissue deformation in most surgical navigation and planning applications, its incorporation into the 2D/3D registration framework has been limited. This is due to the large degrees-of-freedom involved in the registration parameters, which make a stable implementation of the algorithm difficult.
The recent advance of miniaturised catheter tip EM trackers has greatly simplified the 2D/3D registration process. For rigid registration, existing 5 DoF EM tracker inserted into the biopsy channel of the endoscope can provide much improved results as the remaining 1 DoF is relatively easy to optimise. For deformable airways, however, existing research has shown that complex deformation of the anatomical structure still represents a major problem to the registration process despite the use of the catheter tip EM tracker [75]. In this case, direct application of local deformation to the 3D structural mesh to match to that of the bronchoscope video is computationally impractical due to the large number of control points involved. Furthermore, issues such as mesh folding also need to be carefully addressed. Although it is possible to use the finite element model to provide biomechanically plausible model to define airway deformation, its practical implementation can be difficult due to the general lack of detailed tissue mechanical property, as well as the prohibitive computational cost involved.

In this study, we have demonstrated that the use of ASM provides a simple yet effective framework for a compact representation of the airway deformation. This allows deformation to be modelled by the limited principal modes of variation such that conventional optimisation processes can be effectively applied. Statistical shape models and PDMs in particular, provide an effective means of capturing statistically plausible variations of the shape of the object. The ASM combines a model-based approach similar to template models that allow for large variability with a prior knowledge incorporated through a number of previous observations. It has a number of advantages for its use in a non-rigid 2D/3D registration framework. The method describes both typical shape and intrinsic deformation and the motion variability is accommodated through prior knowledge incorporated through the 3D data. Therefore, they can effectively model the deformation of the structure even though the motion mechanisms are not sufficiently understood, or too complex to derive. Furthermore, they are specific to the class of motion they represent, and hence can be implemented in a subject specific manner.

It should be noted, however, that the effectiveness of the method depends on the ability of the 3D models in capturing possible structural deformations during video bronchoscope examination. For the phantom experiment presented in this chapter, we have used CT to provide the structural information of the airways. This, however, may not be suitable for patient studies as the capture of airway deformation at different phases of the respiratory cycle for constructing the ASM model can involve significant ionising radiation. A more practical choice would be to use the recent developments of MRI that permits the visualisation of the lung parenchyma and airways during normal physiological motion [51].
The combination of respiratory navigator echoes will allow the imaging of the intra-thoracic tree at different stages of the respiratory motion, and therefore the construction of a complete deformable model of the airways on a subject-specific basis.

It is also important to note that with the proposed registration framework, the global motion of the patient must be monitored during bronchoscope examination such that it is removed from the catheter-tip EM tracking data for localising the catheter tip in relation to the CT scan volume. This is an issue not encountered in the traditional image registration framework as global motion of the patient will result in a synchronous movement of the bronchoscope and the airways. This problem, however, can be addressed by tracking externally the position and orientation of the patient during bronchoscope examination [154]. The advantage of the proposed method is that it provides a practical way of handling airway deformation and continuation of the registration process when extreme breathing patterns and artefacts caused by mucosa, bubbles, bleeding are encountered. In a typical examination, these effects account for 20-30% of the procedure and they can reach almost 100% in extreme cases, such as profuse haemoptysis. These effects interleave during common bronchoscope procedures, thus providing only short video sequences where image-based registration approaches are successful. When the registration fails it is not trivial to recover the camera position and re-initialise the algorithm. EM tracking, however, is much more immune to these problems [154]. The use of 5 DoF catheter tip EM tracker combined ASM can provide a closer approximation of the camera pose such that the exact solution to the problem can be limited to a localised search space. Patient data also indicates that at the bifurcation/turning points, a rigid model is relatively difficult to smoothly follow the video frames, since the pre-operative and intra-operative 3D structures may not be in correspondence.

In terms of localised airway deformation, one additional problem that deserves consideration is that in typical tomographic imaging the patient is in a supine position, whereas in bronchoscope examinations the patient typically adopts an upright posture. It is feasible that the deformation of the intra-thoracic airways due to the gravity effect may be different due to the variation in pose. This, however, is a problem that is encountered by all 2D/3D registration techniques.

It should also be noted, that the success of ASM depends on the number and quality of correspondent landmarks across the observable shapes. A simplistic method was used to extract the skeleton of the tracheo-bronchial tree and subsequently correspondence was established based on the matching of the bifurcation points. The identification of optimal
control points for ASM for complex topological shapes, however, is a significant challenge [179].

Since the ASM is based on landmark correspondence, an effective way of smoothly interpolating the mesh points with regard to the motion of the control points is necessary. RBFs have been used for a number of reasons. Firstly, the distribution of the control points can be both sparse and irregular. RBF provides an easily controllable behaviour that can be tailored to meet specific requirements. It may be purely deformable, or contain some forms of linear component to allow for both localised and global deformations [176]. Other techniques such as the one presented by Garrity et al. [95] uses NURBS based surface representation are also possible for creating the dynamic model of the airway. The actual computational complexity for estimating each new vertex with RBFs is $O(nN)$, where $n$ is the number of control points and $N$ is the number of vertices. The solution of the equation system in the pre-processing step requires an additional $O(n^3)$. Here we calculate the SVD only at the initialisation stage of the algorithm since the topology of the points does not change. Therefore, the time required depends mainly on the number of vertices of the 3D model, which are usually much larger than the number of control points. These time requirements can also be optimised. For example, the mesh can be deformed only locally since the bronchoscope only provides a localised view of the airways. Other improvements such as to bind locally the radial basis functions and use of parallel processing can also be introduced [176].

In conclusion, we have developed a non-rigid 2D/3D registration framework that models the respiratory motion of the intra-thoracic tree which incorporates the EM tracking data for improved robustness and accuracy. ASM has been used to capture the intrinsic variability of the airways across different phases of the respiratory motion and it also constrains the motion they represent to be specific to the subject studied. This allows the subsequent non-rigid registration implemented in a much reduced dimensional space, and thus greatly simplifies the 2D/3D registration procedure. The detailed phantom validation of the method in this study demonstrates the potential clinical value of the technique.
In this thesis, we have developed a registration framework for constructing subject-specific simulation models for virtual bronchoscopy. By fusing real bronchoscopy video with 3D tomographic data of the same patient, it is possible to generate photorealistic models that allow high fidelity, patient specific bronchoscope simulation. In order to match video bronchoscope images to the geometry extracted from 3D reconstructions of the bronchi, robust registration techniques have to be developed. This is a challenging problem as it implies 2D/3D registration with certain degrees of deformation and different physiological responses. In this thesis, we have proposed a new pq-space based 2D/3D registration method for camera pose estimation in endoscope tracking. The proposed technique involves the extraction of surface normals for each pixel of the video images by using a linear local shape-from-shading algorithm derived from the unique camera/lighting constrains of the endoscopes. We demonstrate how to use the derived pq-space distribution to match to that of the 3D tomographic model. The registration algorithm is further enhanced by introducing temporal constrains based on particle filtering. For motion prediction, a second-order auto-regressive model has been used to characterize camera motion in a bounded lumen as encountered in bronchoscope examination. The proposed method provides a systematic learning procedure with modular training from ground truth data such that information from different subjects are integrated for creating a dynamic model that accommodates the learnt behaviour. To cater for airway deformation, an active shape model (ASM) driven 2D/3D registration has been proposed, which captures the intrinsic variability of the tracheo-bronchial tree during breathing and it is specific to the class of motion it represents. The method reduces the number of parameters that control the deformation, and thus greatly simplifies the optimisation procedure. The method also exploits the recent development of 5 DoF miniaturised catheter tip electro-magnetic...
trackers such that the position and orientation of the bronchoscope can be accurately determined under dis-occlusion and bleeding. The accuracy of the proposed method has been assessed by using both a specially constructed airway phantom with an electromagnetic tracker, and in vivo patient data. The original contribution of this thesis includes the following aspects.

8.1 Main contributions of the thesis

8.1.1 A novel pq-space based registration scheme

A novel similarity measure has been introduced in Chapter 4 based on the angular differences between pq-vectors derived from the video image and the 3D model. Surface information is extracted from a robust linear shape-from-shading technique that exploits the camera and lighting configuration of bronchoscope. These pq-vectors are used to match to the normals specified from the 3D model. For the similarity function developed, it also involves a weighting factor that accounts for texture variations of the tracheo-bronchial tree, which are absent from the 3D CT model. The main advantages of the proposed technique are that there is no need to adjust the illumination conditions in 3D rendering and the method does not require feature extraction and matching. The method is also superior to the alternative approach of surface-to-surface registration. This is because surface reconstruction from pq-vectors requires an integration step, which is error prone. The technique was validated with phantom data with known ground truth. In vivo bronchoscope data has also been used to assess the practical value and potential pitfalls of the technique. It has been shown that handling large tissue deformation and occlusion artefacts are major obstacles of the proposed method when applied to in vivo bronchoscope sequences. To this end, temporal information must be integrated into the proposed registration framework for it to be practically useful.
8.1.2 *Incorporation of temporal constrains based on particle filtering*

The incorporation of temporal constraints is based on particle filtering in this thesis. A probabilistic framework has been developed in Chapter 5 to utilise the inherent in bronchoscope camera motion between successive video frames to improve the accuracy of the 2D/3D registration algorithm. Although the concept of particle filtering itself is not new, we have tackled the difficult problem of dynamic model building and modular learning. A second order autoregressive (ARG) model has been used to evolve the underlying model through time, and the method weights the propagated hypotheses for accurate pose estimation. A systematic training procedure has been developed to tune the parameters of the ARG model based on modular learning. There are two main advantages of the probabilistic tracking algorithm suggested. Firstly, there are no restrictions on the measurement models used and the distribution of error sources. This allows the handling of non-linear dynamics during endoscope navigation. Secondly, modular training facilitates the incorporation of multiple sequences from different patients. In this way a generic motion prediction model can be developed.

8.1.3 *Miniaturised EM tracking and motion decoupling*

The challenges involved in incorporating EM trackers into bronchoscope navigation have been investigated in Chapter 6. Although, EM tracking devices have the potential to enhance real-time 2D/3D registration, ferromagnetic interference and tissue deformation can limit their practical accuracy in clinical environments. For the proposed 2D/3D registration framework, it is also necessary to decouple local deformation from global motion. To this end, we have developed a new motion decoupling scheme based on principal component analysis to identify the variation of the EM signal due to respiration and global patient movements. The method has also been used to identify extreme breathing patterns so that these segments can be treated separately during 2D/3D registration.

8.1.4 *Deformation modelling based on statistical shape models*

Conceptually, the development of statistical shape models for deformation modelling during 2D/3D registration is the most significant part of the thesis. Currently, most of
2D/3D registration methods are based on rigid or affine models and the high degrees of freedom involved in free-form deformation models are both technically and computationally prohibitive. In Chapter 7, we have used statistical shape modelling combined with EM tracking to significantly reduce the number of degrees-of-freedom involved in the registration process. In this thesis, deformation modelling has been implemented by extracting the skeleton of the tracheo-bronchial tree and establishing surface point correspondence based on bifurcation points. Radial Basis Functions (RBFs) have been used to warp the mesh in intermediate positions. Motion compensation is attained with the recovery of the deformation of the tracheo-bronchial tree along with the pose estimation. In this way, EM tracking data can be directly applied to guide the registration process. We have shown that for the phantom model that involves extensive non-linear deformation, the degrees of freedom has been reduced to only two – one for the unknown rotation of the camera and the other for the principle mode of variation describing the deformation of the airways. For in vivo data, the principle modes of variation are expected to be higher. Nevertheless, this is likely to be limited to only a few rather than the otherwise thousands of DoFs.

### 8.2 Discussion and Conclusions

Despite the achievements made, there are several major improvements that can be made for future research. In Chapter 4, we suggested a \( pq \)-space similarity measure. However, by the nature of the shape-from-shading algorithms, specularities and inter-reflections have been ignored. In bronchoscopic images, strong reflections are common due to mucosa and surface properties of the lumen. Conventional shape-from-shading techniques can mistake the presence of local specularities as high curvature surface features [180]. Although we have directly used \( pq \) distribution during registration to minimise this effect, its explicit consideration needs to be taken into account. For human observers, we in fact rely on specular reflection to provide an additional visual cue for the 3D surface, such information can potentially be utilised to further enhance the 3D structural reconstruction process.

In practice, the detection of specular highlights can be achieved by a dichromatic model but it is also possible to factor in the reflectance properties into the shape-from-shading algorithm itself. There are several ways in which surface reflectance properties can be approximated. If a surface is too difficult to be described by a particular model then a general approach can be used to acquire empirical estimates of the Bidirectional
Reflectance Distribution Function (BRDF). This is an expensive and time consuming process because BRDF is described from a large number of degrees of freedom. But empirical and physical-based methods can be used to simplify its derivation. In bronchoscope procedures, the light coincides with the viewer and this can simplify the estimation of the surface properties.

Another issue that deserves extensive future research is the effective handling of local minima of the registration process. Although the use of global optimisation techniques can help the overall consistency of the registration process, a systematic approach needs to be adopted to combine the construction of the similarity measure with the subsequent optimisation landscapes. In this thesis, the construction of the similarity measure is empirically designed but the parameters involved are not determined systematically. The integration of temporal information partly alleviates the problem of local minima but further investigation of using a learning based framework is necessary so that the overall robustness of the algorithm can be further enhanced.

In this thesis, the integration of temporal information for the registration scheme is presented. One of the major advantages of this probabilistic framework is that it incorporates modular training. This allows the learning of the dynamic motion characteristics of the endoscope from several training sets, so that a more representative dynamic model can be created. However, a problem arises when more complex dynamics involving multiple classes of motion [141]. In this work, the dynamic motion characteristics are modelled as a second order autoregressive model. This is a versatile tool that can impose strong constraints on the camera movements and it allows the motion to change rapidly both in speed and in direction. However, modelling of multi-class dynamics has yet to be fulfilled. The models developed assume that the training sequences have similar statistical patterns, which may not be true when the endoscope camera explores different parts of the tracheo-bronchial tree. Further investigation is required to overcome this problem.

For the ASM used in deformation modelling, we extracted the tracheobronchial skeleton manually in order to establish the correspondence based on the matching of the bifurcation points. However, manual segmentation of each tracheo-bronchial branch in each slice is tedious and time consuming. A fully automated method that incorporates structural features such as mediality and homotopy needs to be developed. This can be particularly useful for segmenting distal airways as consistent manual segmentation is difficult, if not impossible. Another important issue to consider is point correspondence of the ASM model. Furthermore, it is well-known that the success of the ASM depends on the quality of the
control point correspondences. In this work, we have used bifurcation landmarks to achieve the overall structural correspondence of the 3D models. Alternatively, optimal and dense shape correspondence can be achieved by optimising the correspondence on a parametric space with respect to some objective functions. This implies that the tracheo-bronchial surface should be parameterised onto a manifold with minimal distortion. These techniques have been successfully applied in myocardium motion modelling but their application in more complex structures, such as the airways is a challenging research issue [179].

In conclusion, we have addressed several major issues related to the construction of patient-specific models for bronchoscope simulation. Although the technical components developed in this thesis are only validated with airway phantom and \textit{in vivo} bronchoscope data, its applicability to general MIS simulation is evident. It must be noted that the work presented in this thesis is part of a larger project within the Royal Society/Wolfson MIC Laboratory where other issues related to photorealistic modelling, finite-element tissue deformation modelling, and visual perceptual behaviour of MIS simulation are addressed.
Appendix A

Construction of the Projection Matrix

2D/3D registration of the endoscopic video with the 3D operative data relies on the comparison between several projections of the 3D data with the endoscopic image. In order to obtain a correct match, a projection should be recovered by calibrating the endoscopic camera and this should be applied in rendering the 3D surface tracheobronchial data. In this paragraph, we will derive the equations that directly link the OpenGL rendering parameters with the intrinsic parameters calculated with OpenCV camera calibration toolbox for MATLAB.

Let’s assume that the focal length is defined as \( f_c \) and it is described in pixels, such as 
\[
\begin{bmatrix}
  f_c_x \\
  f_c_y 
\end{bmatrix}
\]
Similarly, the principal point is also expressed in pixels as 
\[
\begin{bmatrix}
  c_x \\
  c_y 
\end{bmatrix}
\]
Let \( P = [X_c, Y_c, Z_c]^T \) be a point in space in the camera reference frame and \( X_n \) is its normalized pinhole image projection:
\[
X_n = \begin{bmatrix}
  X_c / Z_c \\
  Y_c / Z_c 
\end{bmatrix} \tag{A.1}
\]

If we ignore lens distortion and assume skewing coefficient is equal to zero then the space point \( P \) is projected to the pixel coordinates \( X_p = [x_p, y_p]^T \) through the camera matrix:
OpenGL, however, follows different notation, so that the coordinate axes are flipped around the horizontal axis, as it is shown in Figure A.1.

Therefore, the camera matrix takes the form:

\[
\begin{bmatrix}
{x_p} \\
y_p \\
1
\end{bmatrix} =
\begin{bmatrix}
f_{cx} & 0 & c_{cx} \\
0 & f_{cy} & c_{cy} \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X_c/Z_c \\
Y_c/Z_c \\
1
\end{bmatrix}
\]  
(A.2)

Subsequently, the space point \( P_1 = [\text{left}, \text{top}, z_{near}]^T \) corresponds to \( X_{p1} = [0, 0]^T \) and the above equation is written as:

\[
\begin{bmatrix}
0 \\
n_y - 1 \\
1
\end{bmatrix} =
\begin{bmatrix}
f_{cx} & 0 & c_{cx} \\
0 & f_{cy} & n_y - 1 - c_{cy} \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\text{left}/z_{near} \\
\text{top}/z_{near} \\
1
\end{bmatrix}
\]  
(A.4)

Similarly, the space point \( P_2 = [\text{right}, \text{bottom}, z_{near}]^T \) corresponds to \( X_{p2} = [n_x - 1, n_y - 1]^T \) and the projection equation is written as:
\[
\begin{bmatrix}
   n_x - 1 \\
   0 \\
   1
\end{bmatrix}
= \begin{bmatrix}
   f_{cx} & 0 & ccx \\
   0 & f_{cy} & n_y - 1 - ccy \\
   0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
   \frac{right}{z_{near}} \\
   \frac{bottom}{z_{near}} \\
   1
\end{bmatrix}
\]

(A.5)

Based on the later two equations, we have:

\[
\begin{aligned}
\left\{ \begin{array}{c}
left = z_{near} \cdot \frac{-ccx}{f_{cx}} \\
top = z_{near} \cdot \frac{ccy}{f_{cy}} \\
right = z_{near} \cdot \frac{n_x - 1 - ccx}{f_{cx}} \\
bottom = z_{near} \cdot \frac{ccy - n_y + 1}{f_{cy}}
\end{array} \right.
\]

(A.6)

Subsequently the OpenGL command glFrustum can be used directly to render the 3D model.
Appendix B

Kalman Filtering

Kalman Filtering solves the problem of estimating the state of a linear stochastic system by using measurements that are linear functions of the state. Differential equations have provided concise mathematical models for many dynamic systems of importance to humans. The dependent variables of the differential equations become state variables of the dynamic system. Essential to Kalman Filtering is the development of the state-space representation for dynamic systems described by a set of linear differential equations.

The state, $X \in \mathbb{R}^n$, of a discrete-time controlled process can be described by the linear stochastic equation:

$$X_k = AX_{k-1} + Bu_{k-1} + w_{k-1}$$  \hspace{1cm} (B.1)

with a measurement $Z \in \mathbb{R}^m$ that is:

$$Z_k = HX_k + v_k$$  \hspace{1cm} (B.2)

The $n \times n$ matrix $A$ in the difference equation relates the state at the previous time-step to the state at the current time-step. The $n \times l$ matrix $B$ relates the optional control input $u \in \mathbb{R}^l$ to the state $X$. The $m \times m$ matrix $H$ is the measurement sensitivity matrix, which relates the state to the measurement, $z_k$, and $v_k$ represents the process measurement noise. $n$, $m$ and $l$ are the dimensionalities of the state, measurement and optional input. 
vector, respectively. The random variables \( w \) and \( v \) represent the process and measurement noise, correspondingly. They are assumed to be independent, white and with normal probability distributions of covariance \( Q \) and \( R \), respectively.

Kalman filtering can be also interpreted as a mechanism of propagating and combining Gaussian densities. We define \( \hat{X}_{k|k-1} \) to be the \textit{a priori} state estimate at step \( k \) given knowledge of the process up to the \((k - 1)\) step. Furthermore, \( \hat{X}_{k|k} \) is the \textit{a posteriori} state estimate at step \( k \) given measurement \( Z_k \). In order to derive the equations for the Kalman filter, we compute the \textit{a posteriori} state estimate \( \hat{X}_{k|k} \) as a linear combination of the \textit{a priori} estimate \( \hat{X}_{k|k-1} \) and a weighted difference between an actual measurement \( Z_k \) and a measurement prediction \( H\hat{X}_{k|k-1} \):

\[
\hat{X}_{k|k} = \hat{X}_{k|k-1} + K \left( Z_k - H\hat{X}_{k|k-1} \right)
\]

(B.3)

The \( n \times m \) matrix \( K \) is called the \textit{gain} or \textit{blending factor} and it minimizes the \textit{a posteriori} error covariance.

The last equation elucidates that the Kalman Filter uses a form of feedback control to estimate the process state at a particular time. The feedback is in the form of noisy measurements. Therefore, the filter is composed of two parts: (1) the time update equations that are responsible for propagating the current state and error covariance measurements, and (2) the measurement update equations that implement the feedback mechanism. It is also apparent that a number of parameters need to be calculated before the actual operation of the filter.

These parameters are the measurement and process noise covariance, \( R \) and \( Q \), respectively and the process of training is called system identification and it is about inferring mathematical models from observations of a system. In general, it is easier to estimate the measurement error covariance rather than the process error covariance, since usually it is not possible to directly observe the estimated process. It is important, however, to note that usually a relatively simple process model can produce acceptable results if the uncertainty of the process has been tuned properly via the selection of the process noise covariance \( Q \). This fact highlights the significance of a successful system identification procedure and generally applies to any probabilistic model. In this section the main
assumptions of Kalman Filtering is reviewed and it is underlined their importance in tracking the endoscopic camera.

### B.1 Limitations of Kalman Filtering

Kalman filtering assumes that the system is described from a set of linear equations. A linear system model is adequate in most applications and it is preferable because of its simplicity. When a system cannot be modelled as linear, Kalman filter concept can be extended to some nonlinear applications by linearized them about some nominal points or trajectory. A Kalman filter that linearizes about the current mean and covariance is called an extended Kalman filter (EKF) [181]. Generally, a non-linear model will be governed by a non-linear stochastic difference equation of the form:

\[
    x(t) = f(x(t-1), u(t-1), w(t-1))
\]  

with a measurement \( z \) that is

\[
    z(t) = h(x(t), v(t))
\]

Linearization can be performed about the current state estimate using the partial derivatives of the process and measurement functions to compute predictions in face of non-linear relationships:

\[
    x(t) \approx \tilde{x}(t) + A(x(t-1) - \tilde{x}(t-1)) + Ww(t-1)
\]

with a measurement \( z \):

\[
    z(t) \approx \tilde{z}(t) + H(x(t) - \tilde{x}(t)) + Vv(t)
\]

\( A \) is the Jacobian matrix of partial derivatives of \( f \) with respect to \( x \), \( W \) is the Jacobian matrix of the partial derivatives of \( f \) with respect to \( w \), \( H \) is the Jacobian matrix of partial derivatives of \( h \) with respect to \( x \), and \( V \) is the Jacobian matrix of partial derivatives of \( h \) with respect to \( v \).

The square of the difference between measured value \( z(t) \) actually obtained at the current time and measurement estimate calculated from the estimated state variable is normalized
with the error variance of measurements. Then the square of the difference between the state variable \( x(t-1) \) at the current time calculated from the state variable estimated at a previous time and the estimated state variable \( x(t) \) is normalized with the error covariance matrix of \( x(t-1) \). The sum of these two values is the cost function, and the state variable is estimated such that the cost function becomes the minimum. Therefore, the prediction error takes the form of:

\[
\tilde{e}_{x(t)} = x(t) - \hat{x}(t)
\]  

(B.8)

with a measurement \( z \) that is

\[
\tilde{e}_{z(t)} = z(t) - \hat{z}(t)
\]  

(B.9)

This approach gives a simple and efficient algorithm to handle a nonlinear model. In general, the extended Kalman filter is not optimal [182].

Various improvements have been introduced by iteratively updating the linearization, starting with an initial guess. In other words, a linear Taylor approximation of the system function is considered at the previous state estimate and that of the observation function at the corresponding predicted position. However, convergence may not be achieved if the initial guess is poor or if the disturbances are so large that the linearization is inadequate to describe the system, introducing accumulated errors. The difference between the estimate and minimum position of the cost function is amplified with the increase of the function’s non-linearity. In the case of the cost function of EKF, past measurements are not accumulated; instead information on the cost function is stored as the estimate. Therefore, as the error of the estimate increases, the cost function itself contains an error. If estimation is repeated in this status, the error gradually accumulates. Even if new measurements are acquired, information on the cost function looses accuracy, influenced by the estimate that has accumulated the error thus far [183]. There are also non-linear algorithms, which involve collections of EKF. Although, these approaches are more powerful, there are also more complex.

Kalman Filtering also assumes a white noise source. This implies that the noise value is not correlated in time and thus it has equal power at all frequencies. This results in a noise with infinite power, which obviously cannot exist in real systems. However, any physical system of interest has a certain frequency bandwidth, which is a frequency range of inputs to which it responds. Above this range, the input either has no effect, or the system so severely attenuates the effect that is essentially does not exist. Within the bandwidth of the system of
interest the assumed white noise would look identical to the real wideband noise. Moreover, it turns out that the mathematics involved in the filter is vastly simplified. There are cases in which the noise power level is not constant over all frequencies within the system bandwidth, or in which the noise is in fact time-correlated. For such instances, a white noise put through a small linear system can duplicate virtually any form of time-correlated noise. This small system, called a ‘shaping filter’, is then added to the original system, to achieve an overall linear system driven by white noise.

Finally, the last assumption states that the noise follows a Gaussian probability density. The probability density of Gaussian noise amplitude takes on the shape of a normal bell-shaped curve. This assumption can be justified physically by the fact that a system or measurement noise is typically caused by a number of small sources. Mathematically, when a number of independent random variables are added together, the summed effect can be described very closely by a Gaussian probability density, regardless of the shape of the individual densities. Furthermore, in most applications, the first and second order statistics (mean and variance or standard deviation) of a noise process are known. In the absence of any higher order statistics, there is no better form to assume than the Gaussian density. The first and second order statistics completely determine a Gaussian density, unlike most densities that require an endless number of orders of statistics to specify their shape entirely. Therefore, the Kalman filter, which propagates the first and second order statistics, includes the whole information contained in the conditional probability density.
### Bibliography


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