(1) $y = A + Bx + Cx^2$ passing through $(1,4)$, $(2,7)$ and $(3,14)$

$$4 = 1 \cdot A + 1 \cdot B + 1 \cdot C$$
$$7 = A + 2B + 4 \cdot C$$
$$14 = A + 3B + 9C$$

$$\Rightarrow A = \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix} \left(\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 7 \\ 1 & 3 & 9 & 14 \end{array}\right)$$

<u>Gaussian elimination</u>:

$(E_2 - E_1) \to (E_2)$:
$$A = \left(\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & 3 \\ 1 & 3 & 9 & 14 \end{array}\right)$$

$(E_3 - E_1) \to (E_3)$:
$$A = \left(\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & 3 \\ 0 & 2 & 8 & 10 \end{array}\right)$$

$(E_3 - 2E_2) \to (E_3)$:
$$A = \left(\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right)$$

Backward substitution:

$$C = \frac{4}{2} = 2.$$

$$B = 3 - 3C = -3$$

$$A = 4 - B - C = 4 + 3 - 2 = 5$$

(2) (a)

$$A = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array}\right) \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix}$$

$(E_2 - E_1) \to (E_2)$

$$A = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 0 & 5 & 1 \\ 2 & -1 & 2 & 3 \end{array}\right) \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix}$$

(E_2, E_3) need to be swapped

(b)

$$A = \left(\begin{array}{ccc|c} 0 & 1 & 1 & 6 \\ 1 & -2 & -3 & 4 \\ 1 & -1 & 1 & 5 \end{array}\right) \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix}$$

(E_1, E_2) need to be swapped since $a_{11} =$

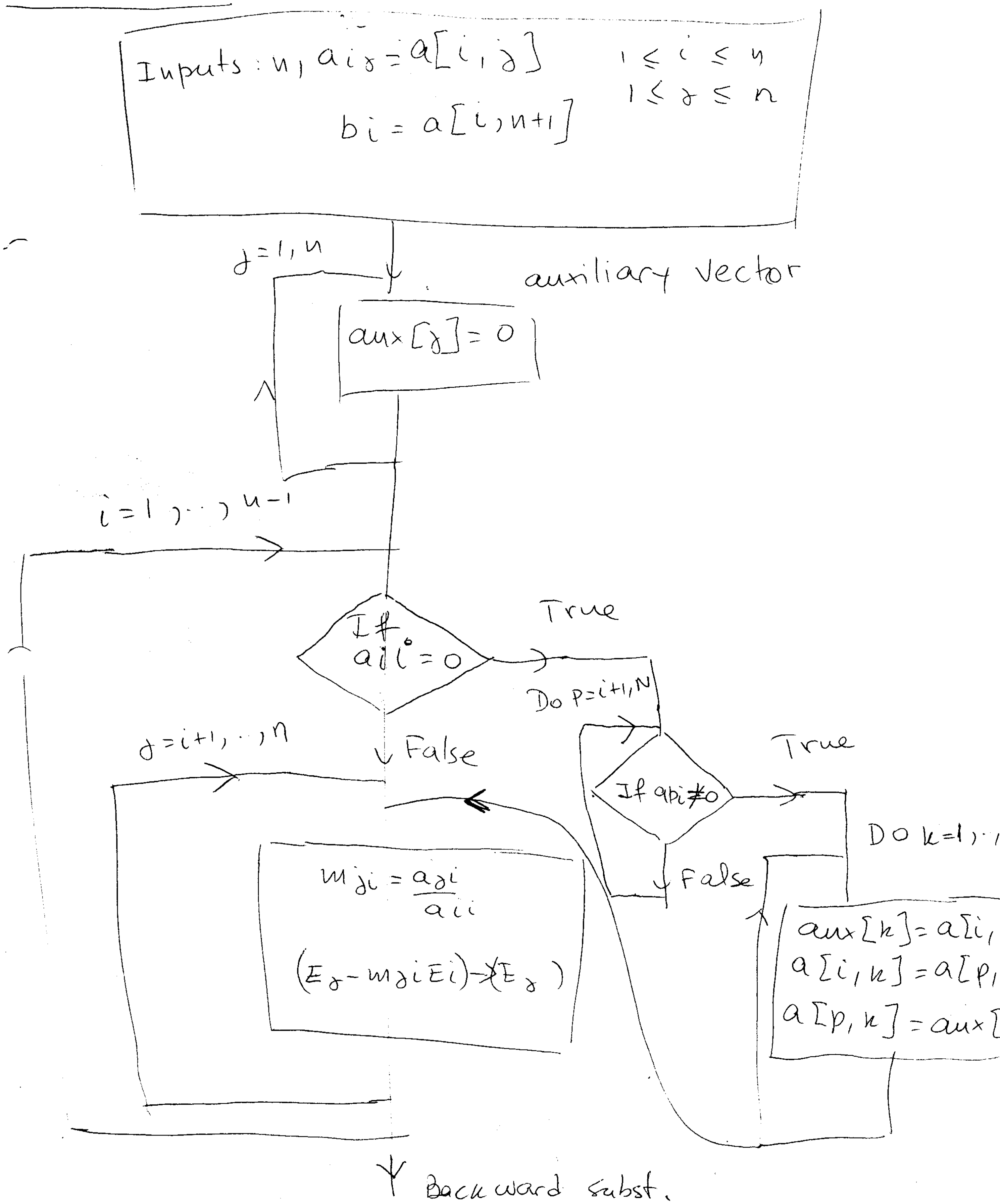
$$A = \left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & 1 & 1 & 6 \\ 1 & -1 & 1 & 5 \end{array}\right)$$

$(E_3 - E_1) \to (E_3)$

③ Strategy 1: Exchange rows i and p if

$a_{ii} = 0$

• Flow chart (Gaussian elimination)



Algorithm : Inputs : n, $a_{ij} = a[i, j]$ $1 \leq i \leq n$
$1 \leq j \leq n+1$

Output : $x_1, \dots, x_n$ or message of failure

Step 1 For $j = 1, \dots, n$
$aux[j] = 0$

Step 2 For $i = 1, \dots, n-1$ DO steps 3 - 6

Step 3 If $a_{ii} = 0$ then

Step 4 For $p = i+1, n$ (Do while $a_{pi} = 0$)

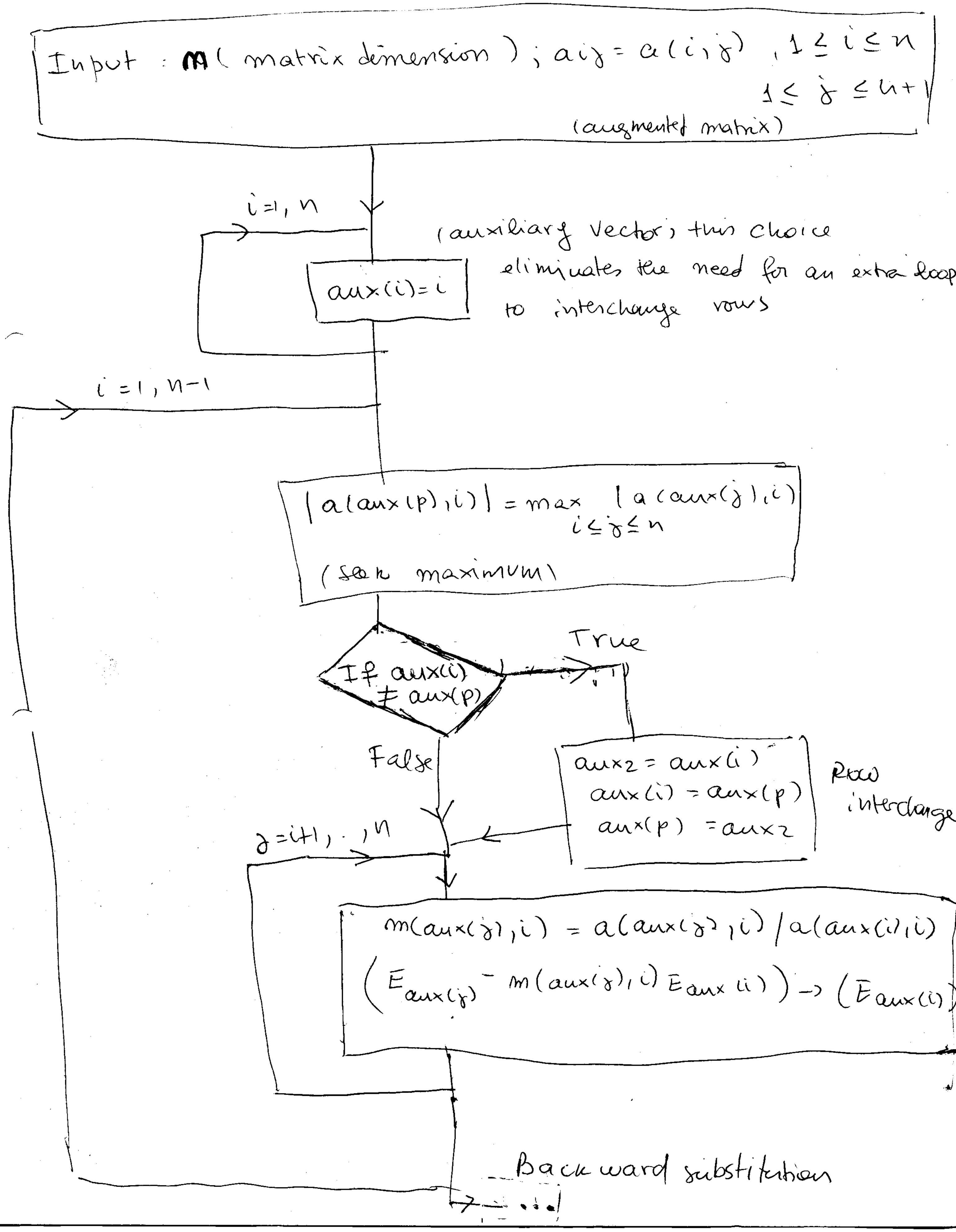
Step 5 For $k = 1, \dots, n$
$aux[k] = a[i, k]$
$a[i, k] = a[p, k]$
$a[p, k] = aux[k]$

Step 6 For $j = i+1, \dots, n$

$m_{ji} = \frac{a_{ji}}{a_{ii}}$

$(E_j - m_{ji} E_i) \to E_j$

Strategy 2: Seek the largest element at the main diagonal during the Gaussian elimination



Algorithm:

Input : number of eqs, unknowns: n
augmented matrix

Output solution $x_1, \dots, x_n$ or message that the linear syst. has no unique solution

Step 1 : For $i = 1, \dots, n$ set $aux(i) = i$

Step 2 : For $i = 1, \dots, n-1$ do Steps 3–6

Step 3 $|a(aux(p), i)| = \max_{i \le j \le n} |a(aux(j), i)|$

Step 4 If $a(aux(p), i) = 0$ then OUTPUT ('no unique solution exists')
STOP

Step 5 If $aux(i) \neq aux(p)$ then set $aux_2 = aux($
$aux(i) = aux($
$aux(p) = aux$

Step 6 For $j = i+1, \dots, n$ do Steps 7 and 8

Step 7 $m(aux(j), i) = a(aux(j), i) / a(aux(i),$

Step 8 $(E_{aux(j)} - m(aux(j), i) \cdot E_{aux(i)}) \rightarrow E_{aux(j)}$

Step 9 if $a(aux(n), n) = 0$ then OUTPUT ('no unique solut exists')

STOP

:

Backword substitution..