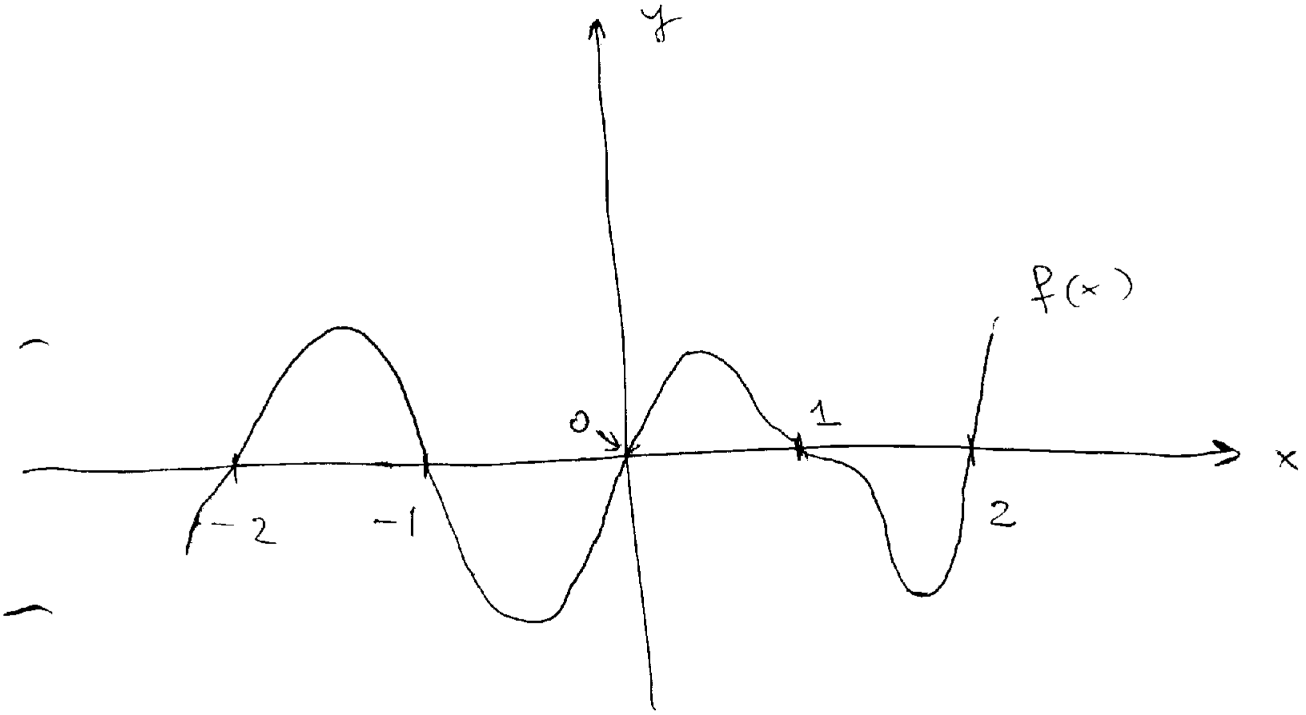
1) $f(x) = (x+2)(x+1)\,x\,(x-1)^3(x-2)$



(a) $[-3, 2.5]$

Midpoint: $-0.25 \rightarrow f(a).f(p_0) > 0$

Interval will be taken as $[-0.25, 2.5]$ ($a = p_0$)

Midpoint: $1.125 \rightarrow f(a).f(p_1) > 0$

Interval will be taken as $[1.125, 2.5]$ ($a = p_1$)

→ the procedure will converge to $p_n = 2$

(b) $[-2.5, 3]$

Midpoint $0.25 \rightarrow f(a).f(p_0) < 0$

Interval will be taken as $[-2.5, 0.25]$ ($b = p_0$)

Midpoint: $-1.125 \rightarrow f(a).f(p_1) < 0$

Interval will be taken as $[-2.5, -1.125]$ ($b = p_1$)

→ the procedure will converge to $p_n = -2$.

(c) $[-1.75, 1.5]$

Since $-1.75 > -2$ it will not converge to -2

Since $1.5 < 2$ it will not converge to 2

(both roots are outside the bracketing interval)

Midpoint: $-0.125 \to f(a) \cdot f(p_0) < 0$ (set $b = p_0$)

Interval will be $[-1.75, -0.125]$

$\to$ the procedure will converge to $p_n = -1$

(d) $[-1.5, 1.75]$

Since $-1.5 > -2$ and $1.75 < 2$ it will not converge to the roots -2 and 2

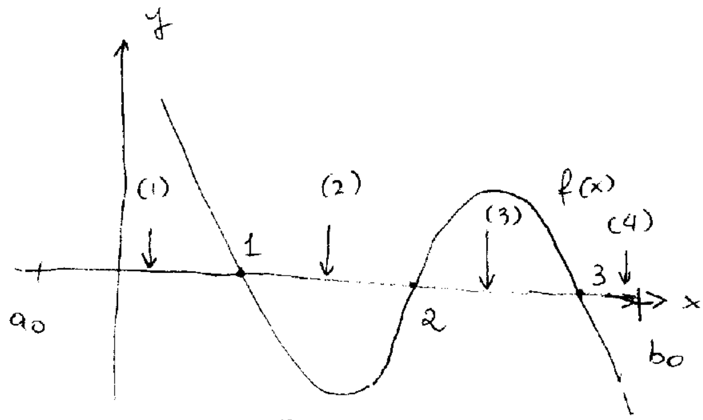
Midpoint: $0.125 \to f(a) \cdot f(p_0) > 0 \to a = p_0$

New interval will be $[0.125, 1.75]$

$\to$ the procedure will converge to $p_n = 1$

(2) The polynomial $f(x) = (x-1)^3(x-2)(x-3)$ has three zeros : $x = 1$ (multiplicity 3), $x = 2$ (multiplicity 1) and $x = 3$ (multiplicity 1). If a_0 and b_0 are two real numbers such that $a_0 < 1$ and $b_0 > 3$ then $f(a_0)f(b_0) < 0$. Thus, on the interval $[a_0, b_0]$ the bisection method will converge to one of the three zeros. If a_0 and b_0 are selected such that

$c_n = \frac{(a_n + b_n)}{2}$ is not equal to 1, 2 or 3 for n



(1) $\frac{\overbrace{a_0 + b_0}^{p}}{2} < 1$ (could happen if i.e.g. $a_0 < -1$ ...)

$f(p) \cdot f(a_0) > 0$ → will take $a_0 = p$ and continue bisecting ... until one ends up in (2) or (3)

(2) $1 < \frac{\overbrace{a_0 + b_0}^{p}}{2} < 2$

$f(p) \cdot f(a_0) \cancel{>} 0$ → will take $b_0 = p$ and converge to 1
("squeeze" to the left)

(3) $2 < \frac{\overbrace{a_0 + b_0}^{p}}{2} < 3$

$f(p) \cdot f(a_0) > 0$ → will take $a_0 = p$ and converge to 3
("squeeze" to the right)

(4) $\frac{a_0 + b_0}{2} > 3$ (if. e.g. b_0 is very large)

$f(a_0)\ f(p) < 0$ → will take $b_0 = p$ and continue bisecting until one ends up in (2) or (3).

$\Rightarrow$ It will never converge to 2.

(3) $f(x) = x e^{-x}$

a) Newton-Raphson formula:

$$p_k = p_{k-1} - \frac{f(p_{k-1})}{f'(p_{k-1})}$$

$$f'(x) = -x e^{-x} + e^{-x}$$

$$f'(p_{k-1}) = -p_{k-1} e^{-p_{k-1}} + e^{-p_{k-1}} \quad (*)$$

$$f(p_{k-1}) = p_{k-1} e^{-p_{k-1}} \quad (**)$$

$$\frac{(**)}{(*)} = \frac{p_{k-1}}{1 - p_{k-1}}$$

$$p_k = \frac{\cancel{p_{k-1}} - p_{k-1}^2 - \cancel{p_{k-1}}}{1 - p_{k-1}} = \frac{p_{k-1}^2}{p_{k-1} - 1}$$

b) $p_0 = 0.2$

$p_1 = -0.05$

$p_2 = -0.002381$

$p_3 = -0.0005537$

$\lim_{n \to \infty} p_n = 0$

c) $p_0 = 20$

$p_1 = 21.0526$

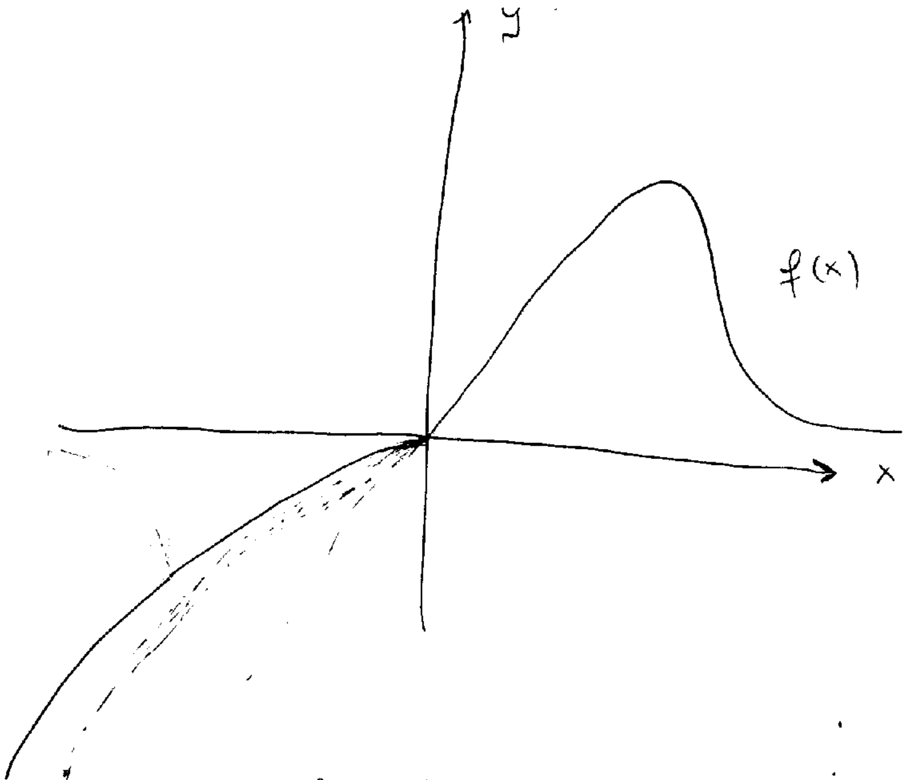
$p_2 = 22.1025$

$p_3 = 23.1499$

$\lim_{n \to \infty} p_n = \infty$

(5) d) What is happening:

The function f has a zero at $x=0$ and decays exponentially for large x (see plot)



In case (a), the Newton method is converging to the zero of the function, since the initial guess has been taken close enough to this point.

In case (b) the Newton method is converging to infinity, for which $f(x) \sim 0$. This is due to the initial guess, which is very large.

(*) Please note: This would not happen, e.g. for the bisection method, since in this case, you have to define the interval in x where the root is.