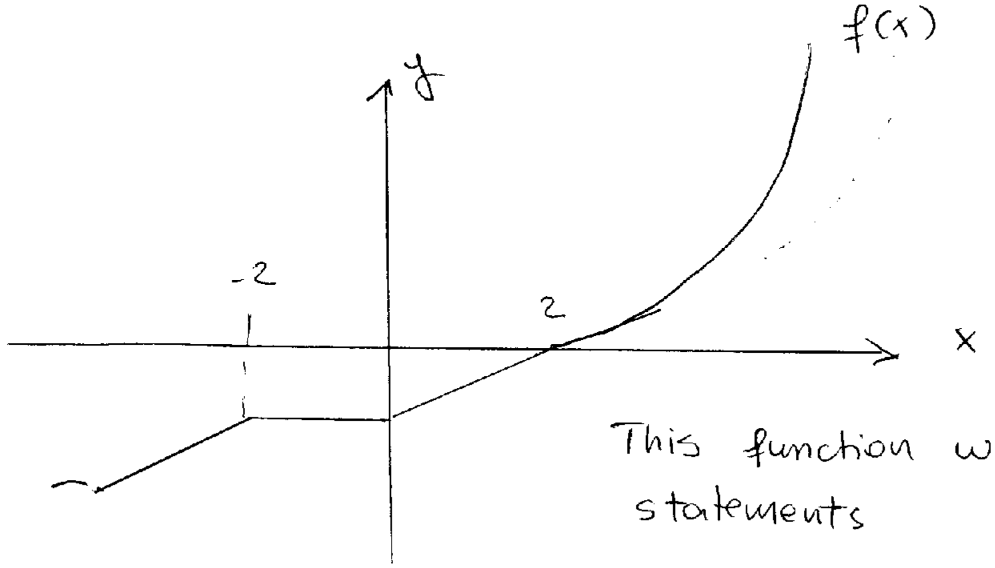
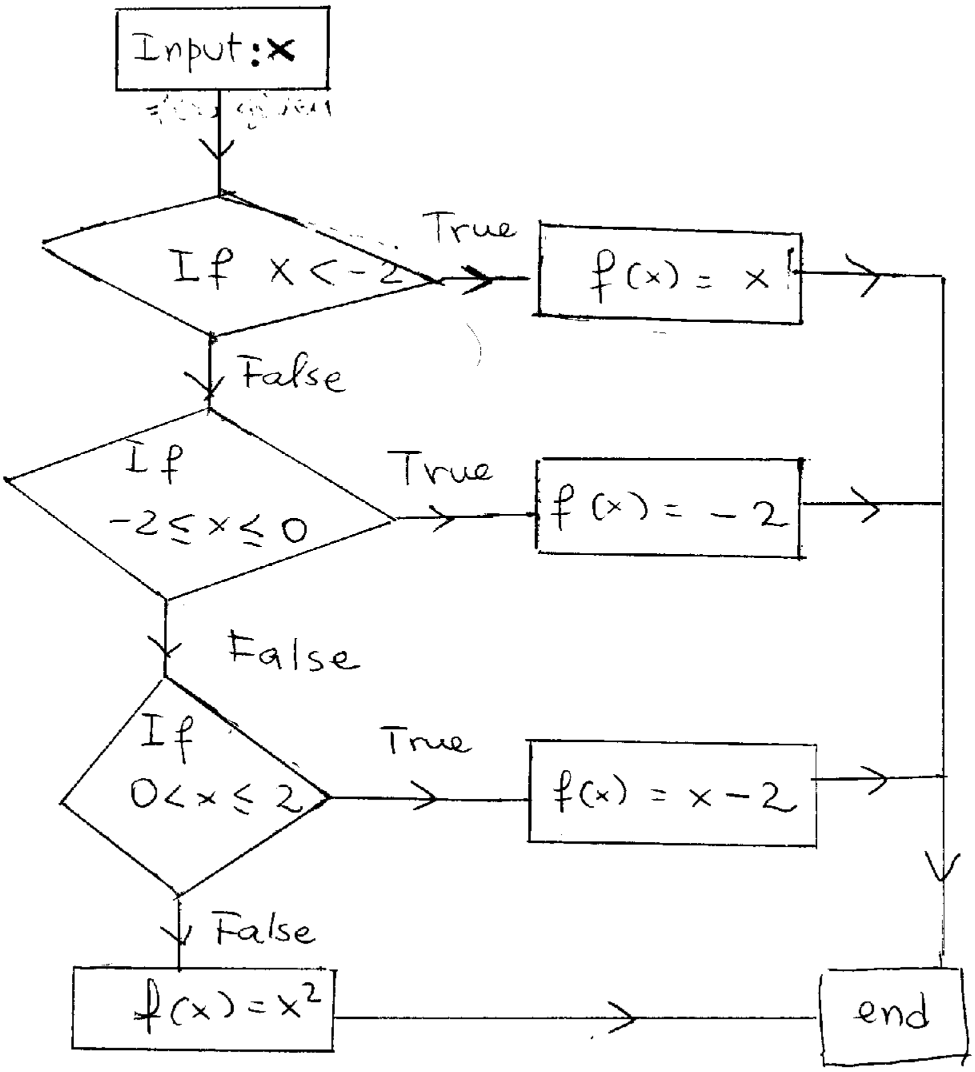
1)
$$f(x) = \begin{cases} x, & x < -2 \\ -2, & -2 \le x \le 0 \\ x-2, & 0 < x \le 2 \\ x^2-4, & x > 2 \end{cases}$$



This function will require nested if statements

<u>Flow Chart</u>



Algorithm

Input : x

Output : $f(x)$

Step 1 If $x < -2$ then

$f(x) = x$

else

Step 2 If $x \leq 0$ then

$f(x) = -2$

else

Step 3 If $x \leq 2$ then

$f(x) = x - 2$

else

$f(x) = x^2 - 4$

endif

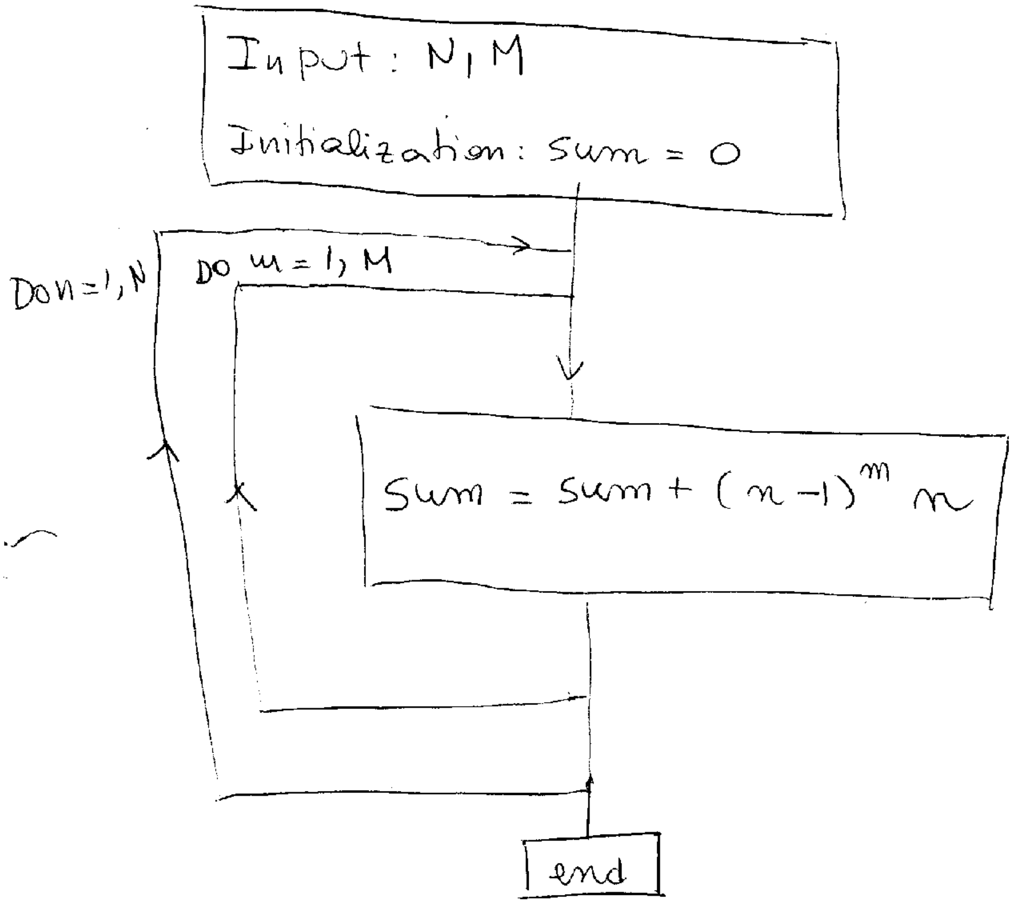
endif

endif

(*) Please note: The "pseudo-language" for the algorithms varies in the literature

Sometimes, for clarity, it is useful to write the ends of the ifs and "mark" them

2) Flow chart



Algorithm

Input : N, M

Output : $\sum_{n=1}^{N} \sum_{m=1}^{M} (n-1)^{m}\, n$

Step 1 (Initialize sum) sum = 0

Step 2 For n = 1, N do Step 3 – Step 6

Step 3 For m = 1, M do Step 4 – Step 5

Step 4 $sum = sum + (n-1)^m\, n$

Step 5 m = m + 1

Step 6 n = n + 1

Step 7 Write ('sum=') sum.

2. To check this, we will perform Gaussian elimination in the matrices below

(a)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix}$$

$(E_2 - 2E_1) \to (E_2)$

$$A^{(2)} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

Rows 2 and three must be interchange

Permutation matrix:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

interchanging rows

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Problem: $a_{11} = 0$

Rows 1 and 2 must be interchanged

Are there further problems ? Let us check

$$\tilde{A} = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix}$$

$(E_3 - E_1) \to (E_3)$

$$\tilde{A} = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

No problems!

3. $$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}_{} \underbrace{\begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

- Forward substitution:

$$y_1 = -1$$

$$y_2 = 3 - 2y_1 = 5 \qquad \Rightarrow y = \begin{bmatrix} -1 \\ 5 \\ -1 \end{bmatrix}$$

$$y_3 = 0 + y_1 + 0 \cdot y_2 = y_1 = -1$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ -1 \end{bmatrix}$$

- Backward substitution

$$x_3 = -1/3$$

$$x_2 = \frac{-1}{2}\left[5 + \frac{1}{3}\right] = -\frac{8}{3}.$$

$$x_1 = \frac{1}{2}\left[-1 - 3x_2 + x_3\right] = \frac{1}{2}\left[-1 - 8 - \frac{1}{3}\right] = -\frac{14}{3}$$

$$\vec{x} = \begin{bmatrix} -14/3 \\ -8/3 \\ -1/3 \end{bmatrix}$$

4 Matrix of the system:

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix} \begin{array}{l} E_1 \\ E_2 \\ E_3 \end{array}$$

LU Decomposition:

$$l_{11} = l_{22} = l_{33} = 1$$

$$u_{11} = 2; \; u_{12} = -1; \; u_{13} = 3$$

$$\left(E_2 - \tfrac{3}{2} E_1\right) \to (E_2) \qquad l_{21} = 3/2$$

$$\left(E_3 - \tfrac{3}{2} E_1\right) \to (E_3) \qquad l_{31} = 3/2$$

$$A^{(2)} \begin{array}{r} E_1 \\ = E_2^{(2)} \\ E_3^{(2)} \end{array} \begin{bmatrix} 2 & -1 & 3 \\ 0 & 9/2 & 9/2 \\ 0 & 9/2 & 1/2 \end{bmatrix} \begin{array}{l} \leftarrow \text{1st row of } U \\ \leftarrow \text{2nd row of } U \\ \end{array}$$

$$\left(E_3^{(2)} - E_2^{(2)}\right) \to \left(E_3^{(2)}\right) \qquad l_{32} = 1$$

$$A^{(3)} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 9/2 & 9/2 \\ 0 & 0 & -4 \end{bmatrix} \begin{array}{l} \\ = U \\ \leftarrow \text{third row of } U \end{array}$$

relative error : $\dfrac{|0.00658 - 0.0116|}{|0.0116|} \approx 4.32 \times 10^{-1}$

- Considering (1):

$x_0 y_0 = 4.25$

$(1) \Rightarrow 1.31 - \dfrac{(6.25 - 4.24)}{1.52} = 0.0100$

$\therefore$

relative error: $\dfrac{|0.0100 - 0.0116|}{|0.0116|} \approx 1.38 \times 10^{-1}$

(four times smaller!)

Hence, (1) is a better formula than (2).
This is due to the fact that in (2), one is subtracting nearly equal numbers, whereas in (1) this problem does not occur.

5) (a) $\ln\left[\dfrac{x+1}{x}\right]$ or $\ln\left[1 + \dfrac{1}{x}\right]$ (using properties of logarithms)

(b) $\dfrac{x^2+1-\cancel{x}}{\sqrt{x^2+1}+x} = \dfrac{1}{\sqrt{x^2+1}+x}$ (we rationalized the numerator)

(c) $\cos(2x)$