Further mathematics for economists

Exercise Sheet 8 - Functions of several variables I

1. Compute the gradient and the Hessian matrix of the following functions of several variables:

   (a) $f(x, y) = x^{\alpha} y^{\beta}$.

   (b) $f(x, y) = x^2 \sin(2xy)$.

   (c) $f(x, y, z) = e^{3xy^2}(x^2 + 1)\cos(3z)$.

2. Find the differential df and the **total** derivative df/dx of the functions in 1.a and 1.b., with $y = 2x$.

3. Find df/dt for $f(x(t), y(t)) = xy + y^2$, with $x = e^t$ and $y = (1 + t)$ using the chain rule.

4. A company's output is modelled by the Cobb-Douglas production function

$$Q(K, L) = 20K^{1/2}L^{1/3}.$$

   (a) Show that

$$K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = cQ(K, L)$$

   for some constant c. What is the value of c?

   (b) Write down the gradient vector of Q in terms of K and L.

   (c) How could you have proven (a) without computing the partial derivatives? Discuss.

5. Determine whether the following functions are homogeneous.If so, of what degree?

   (a) $f(x, y) = x^3 - xy + y^3$

   (b) $f(x, y) = 2x + y + 3\sqrt{xy}$

   (c) $f(x, y) = \exp[x^2 + y^2]$