Further mathematics for economists

Exercise Sheet 7 - Second-Order Differential Equations

1. Find the specific solution of the differential equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$$

so that $y(0) = 0$ and $dy/dt = 1$ when $t = 0$

2. Find the specific solution of the differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 10y = 0$$

so that $y(0) = 0$ and $y(\pi/2) = 1$. Discuss this solution qualitatively (Hint: note that $\sin u = \frac{1}{2i}(e^{iu} - e^{-iu})$)

3. Find the specific solution of the differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 0$$

so that $y(0) = 1$ and $y(1) = 0$

4. Find the general solution of the following second-order differential equations:

(a)

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = 3e^t + 5$$

(Hint: use $y_p = ae^t + b$)

(b)

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = 3e^{-t}$$

(Hint: use $y_p = ate^{-t}$)

(c)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 26t + 21$$