Sheet 5 - trigonometry

(1) • $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\beta\sin\alpha$

$\alpha=\beta \Rightarrow$ (a) $\sin 2\alpha = 2\sin\alpha\cos\alpha + \sin\alpha\cos\alpha = 2\sin\alpha\cos\alpha$

(b) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

(c) $\sin 3\alpha = \sin(\alpha+2\alpha) = \sin 2\alpha\cos\alpha + \sin\alpha\cos 2\alpha$

$= \cos\alpha[2\sin\alpha\cos\alpha] + \sin\alpha[1-2\sin^2\alpha]$

$= 2\sin\alpha\cos^2\alpha + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha[1-\sin^2\alpha] + \sin\alpha - 2\sin^3\alpha$

$= 3\sin\alpha - 4\sin^3\alpha$

(2) $y = A\sin mx + B\cos mx$

$\frac{dy}{dx} = mA\cos mx - mB\sin mx$

$\frac{d^2y}{dx^2} = -m^2 A\sin mx - m^2 B\cos mx$

$= -m^2[\underbrace{A\sin mx + B\cos mx}_{y}]$

$\Rightarrow \boxed{\frac{d^2y}{dx^2} + m^2 y = 0}$

(3) $f(x) = 5\cos x + 12\sin x = R\cos(x-\alpha)$

α acute.
$\cos\alpha > 0$
$\sin\alpha > 0$

$5\cos x + 12\sin x = R[\cos\alpha\cos x + \sin\alpha\sin x]$

$R\cos\alpha = 5$ $\quad \tan\alpha = \frac{12}{5}$ $\quad \Rightarrow \alpha = \text{Arc Tan}\left[\frac{12}{5}\right] \simeq 0.37\pi$

$R\sin\alpha = 12$ $\quad R = \frac{12}{\sin\alpha} = 13$