Foundations of Numerical Methods
(2nd term 2005)

Exercise Sheet 4 - Systems of linear equations

1. Factorize the matrix below usng the LU factorization algorithm with $l_{ii} = 1$ for all i.

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ -1 & 3 & 3 & 0 \\ -2 & -2 & 1 & 4 \\ -2 & 2 & 2 & 5 \end{bmatrix}$$

2. Find the permutation matrix for the matrices below so that PA can be factorized into the product LU, where L is lower triangular with 1s on its main diagonal and L is upper triangular.

(a) $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

3. Solve the linear system

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

4. Solve the linear system below using the LU factorization algorithm

$$\begin{array}{rrrl} 2x_1 & -x_2 & +x_3 & = -1 \\ 3x_1 & +3x_2 & +9x_3 & = 0 \\ 3x_1 & +3x_2 & +5x_3 & = 4 \end{array}$$