Foundations of Numerical Methods
(2^{nd} term 2005)

Exercise Sheet 2 - Root finding

1. Let $f(x) = (x+2)(x+1)x(x-1)^3(x-2)$. To which zero of f does the bisection method converge when applied to the following intervals?

   (a)[-3,2.5]
   (b)[-2.5,3]
   (c)[-1.75,1.5]
   (d)[-1.5,1.75]

2. The polynomial $f(x) = (x-1)^3(x-2)(x-3)$ has three zeros: $x = 1$ (multiplicity 3), $x = 2$ (multiplicity 1) and $x = 3$ (multiplicity 1). If a_0 and b_0 are two real numbers so that $a_0 < 1$ and $b_0 > 3$ then $f(a_0).f(b_0) < 0$. Thus, on the interval $[a_0, b_0]$ the bisection method will converge to one of the three zeros. If a_0 and b_0 are selected such that $c_n = (a_n + b_n)/2$ is **not** equal to 1,2,3 for any $n \geq 1$ then the bisection method will **never** converge to which zero?Why?

3. Consider the function $f(x) = xe^{-x}$

   (a) Find the Newton-Raphson formula $p_k = g(p_{k-1})$
   (b) If $p_0 = 0.2$, then find p_1, p_2 and p_3. What is $\lim_{k\to\infty} p_k$?
   (c) If $p_0 = 20$, then find p_1, p_2 and p_3. What is $\lim_{k\to\infty} p_k$?
   (d) Discuss the results found