Foundations of Numerical Methods
(2^{nd} term 2005)

Exercise Sheet 1 - Control structures, error analysis and root finding

1. Construct a flow chart, an algorithm and a MatLab code to compute the function

$$f(x) = \begin{cases} x, x < -2 \\ -2, -2 \leq x \leq 0 \\ x - 2, 0 < x \leq 2 \\ x^2 - 4, x > 2 \end{cases}$$

   - Hint: you will have to use nested if statements

2. Construct a flow chart, an algorithm and a MatLab code to perform the following double sum

$$\sum_{n=1}^{N} \sum_{m=1}^{M} (n-1)^m n$$

   - Hint: you will have to use two nested do loops

3. Construct a flow chart, an algorithm and a MatLab code to compute the sum

$$\sum_{n=1}^{N} \frac{(-1)^n}{n}, \ N \ \text{large}$$

   so that the relative error with respect to $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = -\ln(2)$ is smaller than 10^{-3}

   - Hints:
     - You will have to use a do loop and if statements
     - Note that even numbers are divisible by two
   - How does the sum converge to this limit?

4. Suppose two points (x_0, y_0) and (x_1, y_1) are on a straight line with $y_1 \neq y_0$. Two formulae are available to compute the x-intercept of the line:

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \text{ and } x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0}$$

   (a) Show that both formulae are equivalent

(b) Use the data $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$ and three-digit rounding arithmetic to compute the intercept in both ways (the true three-digit value is $x = -0.0116$). Which formula is better and why?

5. The formulae below introduce loss of significance in a code, due to the fact that they involve a subtraction of nearly equal numbers. Find equivalent formulae which avoid this problem

   (a) $\ln(x+1) - \ln(x)$, x large

   (b) $\sqrt{x^2+1} - x$, x large

   (c) $\cos^2 x - \sin^2 x$, $x \simeq \pi/4$

6. Write a flow chart to find an approximation to $\sqrt{2}$ up to 3 significant digits using the bisection method (Hint: Consider $f(x) = x^2 - 2$ and take an interval for which the root is positive). Try to write a MatLab code for that.

Elements of MatLab syntax

- Do loops:

```
for index=start:increment:end
statements
end
```

Example:

```
for i=2:6
    sum=sum+1
end
```

(Please note: 1 is the default increment)

- If statements:

```
if (logical expression)
    statements
else
    statements
end
```

Nested ifs:
if (logical expression)

```
    statements
elseif (logical expression)
    else
        statements
    end
    statements
end
```

Example:

```
If rem(a,2)==0
    disp('a is even')
else
    disp('a is odd')
end
```

- Comparison operators:

  == equal to
  ∼= not equal to
  < less than
  <= less than or equal to
  > greater than
  >= greater than or equal to

- Printing something on the screen: disp('something')

- Defining a function (with if's inbetween):

```
function y=f(x)
if (logical expression)
    y=this
else
    y=that
end
```

- Check if a number is divisible by 2: If rem(a,2)==0

- Arithmetic operators:

  + addition
  - subtraction
  * multiplication
  / division
  ^ power