Further mathematics for economists

Exercise Sheet 1 - Integration

1. Compute the following integrals and sketch them as areas under a curve. Discuss the similarities and differences between the results obtained

   (a) $\int_{-2}^{2} x\,dx$

   (b) $\int_{-2}^{2} 3x^2\,dx$

2. Find the area between the following curves and the vertical lines $x = 1$ and $x = 4$

   (a) $y(x) = x^2$

   (b) $y(x) = 2x + x^2 - 1$

   (c) $y(x) = \frac{1}{x^2}$

3. Find the area below the curve

$$f(x) = \begin{cases} 4x^3 - 2, & x < 1 \\ 2, & x \geq 1 \end{cases}$$

   for $0 < x < 2$

4. Find the area above the parabola $y = x^2$ and below the line $y = k$, where $k = const$ and $k > 0$.

5. Find the area above the parabola $y = x^2$ and below the line $y = kx$, where $k = const$ and $k > 0$.

6. Compute the following indefinite integrals

   (a) $\int \frac{x^2+1}{x}\,dx$

   (b) $\int e^{ax}\,dx, a = const$ and $a \neq 0$

   (c) $\int 2x e^{x^2}\,dx$

   (d) $\int \left(\frac{6}{x^5} + \frac{1}{x^3} + \frac{2}{\sqrt{x^3}} \right) dx$