Further mathematics for economists

Exam (Duration: 3 hours)

Section **A**

Answer **all three** questions from this section.

1. (10/100) Compute the area confined by the parabola $f_1(x) = x^2+2x+1$ and the line $f_2(x) = 2 + 2x$.
2. (10/100) Compute $\int_a^b \ln(2x)dx, 0 < a < b$, by parts.
3. (15/100) Find the gradient and the Hessian matrix of $f(x, y) = x^2 e^{2y} \sin y$.

Section **B**

Answer either part a) or b) of the following three questions from this section.

1. (20/100)

   (a) Find the specific solution of the differential equation

   $$\frac{dy}{dt} + 3y = 2e^{-t} + 3$$

   so that $y(0) = 1$.

   Hints:

   - Take $y_p = ae^{-t} + b$ for the particular solution.
   - Alternatively to using the particular solution you can also use the integrating factor.

   (b) Find the general solution of the differential equation

   $$\frac{dy}{dx} + \frac{1}{x}y = -3\sqrt{1 - x^2}.$$

   Hints:

- Use the integrating factor.
- You will need to solve $-3\int x\sqrt{1-x^2}dx$ by substitution in the end, with $u = 1 - x^2$.

2. (15/100)

   (a) Show that $z = ((1/\sqrt{2} + i/\sqrt{2}))^8 + (1/\sqrt{2} - i/\sqrt{2})^4 = 0$.

   (b) Find the roots of $z^3 = 8$ in algebraic form.
   (Hint: you can compute one root less if you remember a property of complex roots of real numbers)

3. (30/100)

   (a) Find the general solution of the second-order differential equation

   $$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 6e^t.$$

   (b) Find the specific solution of the second-order differential equation

   $$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$$

   so that $y(0) = 1$, $dy/dt(t = 0) = 0$.