Foundations of Numerical Methods
(2^{nd} term 2005)

Coursework

1. (10/100)Can the Newton-Raphson method be used to find the root $f(x) = 0$ if

   (a) $f(x) = \begin{cases} x-1, x \geq 1 \\ 1-x, x < 1 \end{cases}$, with the initial guess $p_0 = 1.2$?

   (b) $f(x) = (x-1)e^{-2x}$, with the initial guess $p_0 = 1.2$?

   (c) Same function as in (b), with the initial guess $p_0 = 15$?
   Justify your answers

2. (30/100)A parabolically terminated cubic spline satisfies the boundary conditions $S'''(x) \equiv 0$ on the interval $[x_0, x_1]$ and $S'''(x) \equiv 0$ on the interval $[x_{n-1}, x_n]$. Show that such boundary conditions, together with the recurrence relations seen in class, lead to the tridiagonal system $A\vec{x} = \vec{b}$, with

$$A = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ h_0 & 2(h_0+h_1) & h_1 & \ddots & \vdots \\ 0 & h_1 & 2(h_1+h_2) & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & h_{n-1} \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix},$$

$$\vec{x} = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix}$$

and

$$\vec{b} = \begin{bmatrix} 0 \\ 3/h_1(a_2-a_1) - 3/h_0(a_1-a_0) \\ \vdots \\ 3/h_{n-1}(a_n-a_{n-1}) - 3/h_{n-2}(a_{n-1}-a_{n-2}) \\ 0 \end{bmatrix}.$$

Find the parabolically terminated cubic spline that passes through the points (-3,2), (-2,0), (1,3) and (4,1)

3. (30/100) Show that

$$\int_{x_0}^{x_4} f(x)dx \simeq \frac{2h}{45}[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)].$$

Use this approximation and Simpson's composite method (dividing into 4 subintervals), to compute the integral

$$\int_0^2 e^{2x}\sin(3x)dx = -14.214.$$

Compare both results with the exact value.

4. (20/100) An animal walks in the directions north, south, east and west with equal probabilities, and steps of length Δl. Write an algorithm to simulate such a motion using uniformly distributed random numbers, and a total number n of steps. As an output, such an algorithm should give the total length, and the total number of steps walked in each direction (you do not need to subtract the lengths walked in opposite directions). Hints:

   - Divide the interval $[0, 1]$ within which the numbers are generated into subintervals, and assign one subinterval to each direction.
   - You will need a do loop and if statements

5. (10/100)Show how a random number with frequency function

$$p(x) = \begin{cases} 4/[\pi(1+x^2)], & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

can be obtained from uniformly distributed random numbers. Employ the transformation method.