

# Further mathematics for economists

## Coursework 2

**Deadline:** Tuesday, April 04, 2006.

1. (10/100) Are the functions below homogeneous? If so, of which degree? Justify your answer

(a)

$$f(x, y, z) = \sqrt{\frac{3x^2/z + z\sqrt{x/y}}{x^3 + 3xyz + y^2x}}$$

(b)

$$f(x, y, z) = e^{x/y} \ln\left(\frac{2x^3}{xyz}\right)(x^2y + yz^2 + x^2z^2)$$

2. (30/100) Find the specific solution of the second-order differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = (1 - 5x)e^{-2x}$$

so that  $dy/dx = -3/5$  when  $x = 0$  and  $y(0) = 4/5$ . Express your answer in terms of trigonometric functions.

3. (20/100) Given the function

$$f(x, y, z) = \cos(x^2y)(x + 2e^{yz} + 1),$$

(a) Compute its gradient and its Hessian matrix

(b) Compute the total derivative  $df/dx$ , taking  $y = 2x$  and  $z = x^3$ .

4. (20/100) Given the implicit relations

$$F_1 : \quad \alpha \ln(x) + C\beta \ln(y) + \gamma \ln(z) = 0$$

and

$$F_2 : \quad x^\alpha y^\beta z^\gamma = 5$$

Turn over...

- (a) Find the Jacobian matrix of  $F_1, F_2$  with respect to  $x, y$ .
  - (b) Compute the total derivatives  $dx/dz, dy/dz$ .
  - (c) Are there any values of the constant  $C$  for which such derivatives do not exist? Justify.
5. (20/100) Identify and classify all critical points  $(x_c, y_c, f(x_c, y_c))$  of the function

$$f(x, y) = 1 - x(x - \alpha)^2 - y(y + 2\alpha) + 2xy$$

where  $\alpha = \text{const}$  and  $\alpha \neq 0$ . Consider

- (a)  $\alpha = 3$ .
- (b)  $\alpha = -1$ .