Coursework 3 - Atom-Photon Physics

Deadline: First day of 2^{nd} term (January 2013)

1. (15/100) Give the Keldysh parameter and discuss the different physical regimes delimited by it. Focus on their similarities and differences, and to what parameter ranges of the external driving field they correspond.

2. (14/100) **Chirped pulse amplification**. Address the following issues:

   (a) Explain the main idea behind Chirped Pulse Amplification, provide a schematic representation and discuss where the chirp comes from.

   (b) Why was it important for obtaining intense laser fields?What obstacles could CPA overcome?Discuss these obstacles in detail.

   (c) What are the important issues concerning the stretching and the compression of the light pulse?

3. **The Zeeman tuning technique.** In the Zeeman tuning technique, a two-level atom is kept resonant with a counter-propagating laser field by employing an inhomogeneous magnetic field. The key idea is that the Zeeman effect counterbalances the Doppler effect during the deceleration process. Considering the average radiation pressure force on a two-level atom,

$$F = -\frac{\hbar\Omega^2\Gamma\mathbf{k}}{\Gamma^2 + 2\Omega^2 + 4[\Delta + kv]^2}, \quad (1)$$

   where $\Delta = \omega - \omega_0$ is the difference between the laser frequency ω of the laser and the transition frequency ω_0, $\Omega = -eE_0 \langle 2| r |1\rangle$ is the Rabi frequency, E_0 is the driving-field amplitude, $\mathbf{k}$ is the wave vector of the radiation, Γ is the linewidth of the transition in question and v is the atom velocity,

   (a) (7/100) By employing an inhomogeneous magnetic field $B(z)$ and assuming that the transition frequency will shift from ω_0 to $\omega_0 + \gamma B(z)$ find the condition for which F is maximal and the expression for the maximal (constant) acceleration $a_{\max} = F_{\max}/M$. How this acceleration behave for low and high driving-field intensity? Can it increase indefinitely? Why or why not?
   Hint: Note how the magnetic field changes the denominator in Eq. (1) and use a similar argument to that employed in class to establish the maximum of a Lorentzian when discussing natural line broadening.

   (b) (10/100) Determine the velocity v as a function of the coordinate z. Thereby, assume that the atom's intitial velocity is v_0 and that it starts its motion from $z_0 = 0$. Is there any limitation upon the length employed for the deceleration? Why? and, from there, the shape of the magnetic field $B(z)$. Physically, what makes this field so special, apart from keeping the atom resonant?

4. **Optical molasses.** Consider now two forces such as (1) along the z axis such that the resulting force is

$$F_z = F_{z1}+F_{z2} = \hbar\Gamma \left[\frac{1}{\Gamma^2 + 2\Omega^2 + 4[\Delta - kv_z]^2} - \frac{1}{\Gamma^2 + 2\Omega^2 + 4[\Delta + kv_z]^2} \right]. \tag{2}$$

   (a) (3/100) Which of the above stated forces is related to the co-propagating beam and which one to the counterpropagating beam? Why?

   (b) (10/100) Show that, for small velocities this force is of viscous type, i.e., of the form -βv.
   Hints: this derivation has been sketched in class.

   (c) (15/100) Briefly explain how to construct a set up for obtaining optical molasses and sketch the optical-molasses laser-beam configuration. Would you say that an optical-molasses laser-beam configuration confines atoms? Why or why not?Would you need to modify the optical-molasses setup to confine atoms? If so, how?

5. **The curse of the displaced atom.** In the strong-field approximation (SFA), the continuum states are approximated by field-dressed plane waves $|\mathbf{p} + \mathbf{A}(t)\rangle$. A well-known shortcoming of this approximation is loss of translation invariance (the curse of the displaced atom). Below you will show that this is indeed the case. Consider the SFA transition amplitude for direct above-threshold ionization

$$\begin{aligned} M^{(SFA)}(p) = & \int_{t_0}^{\infty} dt \exp[-i \int_t^{\infty} \frac{[p + A(\tau)]^2}{2} d\tau]\mathbf{E}(t) \cdot \langle \mathbf{p} + \mathbf{A}(t)| \, \mathbf{r} \, |g\rangle (3) \\ & \times \exp[-iE_g(t - t_0)]. \end{aligned}$$

Let $\psi_g(\mathbf{r}) = \langle \mathbf{r}| \, g\rangle$ be the ground-state wavefunction in the position representation centered at the origin $\mathbf{r} = 0$ and $\psi_g(\mathbf{r} - \mathbf{r}_0) = \langle \mathbf{r}| \, g_{r_0}\rangle$ the ground-state wavefunction centered at an arbitrary position $\mathbf{r} = \mathbf{r}_0$. Consider now the matrix elements $\mathbf{d}_1$=$\langle \mathbf{p} + \mathbf{A}(t)| \, \mathbf{r}|g >$ and $\mathbf{d}_2 = \langle \mathbf{p} + \mathbf{A}(t)| \, \mathbf{r}|g_{r_0} >$ between the approximated continuum and the ground state.

   (a) (13/100) Write these matrix elements in position space. Show that they differ by a time-dependent phase and by term proportional to $\mathbf{r}_0$ ($\mathbf{d}_2 = e^{-i[\mathbf{p}+\mathbf{A}(t)]\cdot\mathbf{r}_0}[\mathbf{d}_1 + \mathbf{r}_0 \langle \mathbf{p} + \mathbf{A}(t)| \, g\rangle]$ You will need to use a closure relation

$$\int d^3r \, |\mathbf{r}\rangle \, \langle \mathbf{r}| = I \tag{4}$$

in r, a coordinate shift in one of the matrix elements and the fact that

$$\langle \mathbf{p} + \mathbf{A}(t) | \, \mathbf{r} \rangle = \frac{1}{(2\pi)^{3/2}} \exp[-i[\mathbf{p} + \mathbf{A}(t)] \cdot \mathbf{r}], \tag{5}$$

$$\langle \mathbf{p} + \mathbf{A}(t) | \, g \rangle = \frac{1}{(2\pi)^{3/2}} \int d^3 r \exp[-i[\mathbf{p} + \mathbf{A}(t)] \cdot \mathbf{r}] \psi_g(\mathbf{r}). \tag{6}$$

(b) (13/100) Why are these extra terms artifacts and how is the translation invariance broken? Provide a semi-quantitative explanation in which you argue that $\left| M^{(SFA)}(p) \right|^2$ computed using $\mathbf{d}_1$ and $\mathbf{d}_2$ lead to different results. Would the terms be present if the exact continuum states were used? Why or why not? Focus on the exact inner product $\langle \phi_c \, | g \rangle$, where $|\phi_c\rangle$ is an exact continuum state when providing your explanation. What should happen to this product if an orthogonal basis is used?

Please note: for a lot of extra hints see Sec. 6 in Smirnova et al, J. Mod. Opt. **54**, 1019 (2007). Focus on the standard SFA result.