Coursework 2 - Atom-Photon Physics
Deadline: 6th of December 2013

1. Explain Doppler broadening. In your explanation, address the following issues:

   (a) (5/100) Why does Doppler broadening occur? How does the frequency of an atomic transition change?

   (b) (10/100)What shape does a Doppler-broadened line have? Why? Provide a semi-quantitative explanation based on the Maxwell distribution. Take ω_0 and ω as the angular frequencies of the atomic transition and of the light emitted, respectively, and v as the velocity of an emitter in the beam.

2. (15/100) Explain why population inversion is essential in order to build a laser. Can it be encountered in nature, in a system in thermal equilibrium? Why or why not? Provide a qualitative explanation of how this can be achieved in a three and four level atom. Which scheme is more efficient? Why?

3. (15/100) What were the key ingredients employed in the construction of the ammonia maser? How did they fit together?Focus on the importance of "quantum state selection", on why ammonia was chosen as an active medium and on why a microwave transition was more convenient at the time.

4. (15/100) What kind of modes may a laser cavity have?Where do they come from? Explain these modes, and place particular emphasis on the differences between them. Based on the propagation of a beam, how are these modes related to the length and the curvature of the cavity? Why?

5. (10/100) Provide a semi-quantitative explanation of quantum beat spectroscopy. Explain why it can be used for measuring closely spaced energy levels and focus on the importance of coherence.

6. **Saturation in a two-level atom.** Consider the two-level atom depicted in Fig. 1.

   (a) (5/100) Write the rate equations for $N_1(t), N_2(t)$ and show that

   $$d\frac{\Delta N(t)}{dt} = -\left(W_{12} + W_{21}\right)\Delta N(t) - (\gamma_{12}+\gamma_{21})\left(\Delta N(t) - \frac{\gamma_{21}-\gamma_{12}}{\gamma_{12}+\gamma_{21}}N\right), \tag{1}$$

   where $\Delta N(t) = N_1(t) - N_2(t)$ and $N = N_1(t) + N_2(t)$.

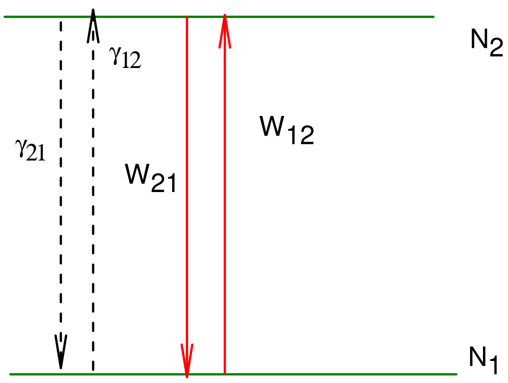

Figure 1: Schematic representation of a two-level atom, with energies E_1, E_2 and bound-state populations $N_1(t)$ and $N_2(t)$. The stimulated transition rates per atom are W_{12} and W_{21} (absorption and stimulated emission, respectively). The relaxation rates per atom from level 2 to level 1 and vice-versa are γ_{21} and γ_{12}, respectively, with $\gamma_{12}/\gamma_{21} = \exp[-(E_2 - E_1)/(k_b T)]$ for a system in thermal equilibrium. Thereby, k_b and T are the Boltzmann constant and the absolute temperature, respectively. The total population is $N = N_1(t) + N_2(t) = const.$

(b) (10/100) Rewrite the above-stated equation as

$$d\frac{\Delta N(t)}{dt} = \underbrace{-2W_{12}\Delta N(t)}_{(i)} - \underbrace{\frac{\Delta N(t) - \Delta N_0}{T_1}}_{(ii)}, \tag{2}$$

where T_1 is defined as the recovery time

$$\frac{1}{T_1} = \gamma_{12} + \gamma_{21}$$

and

$$\Delta N_0 = N_{10} - N_{20} = \frac{\gamma_{21} - \gamma_{12}}{\gamma_{12} + \gamma_{21}} N$$

is the population difference in the system at thermal equilibrium with no applied signal. What is the physical interpretation of the terms (i) and (ii) in Eq. (2)?Why?

(c) (15/100) Show that

$$\Delta N(t) = \frac{\Delta N_0(1 + 2T_1 W_{12} \exp[-t(1 + 2T_1 W_{12})/T_1])}{1 + 2T_1 W_{12}} \tag{3}$$

if the system is initially at thermal equilibrium.

i. What happens if $W_{12} \rightarrow \infty$?What is the physical meaning of this result, with regard to the possibility of obtaining population inversion in a two-level atom?Can this ever be achieved?Why or why not?

ii. What happens if $t \rightarrow \infty$? Compare such results to the stationary solution of Eq. (2) obtained setting $d\frac{\Delta N(t)}{dt} = 0$and to ΔN_0. Is it above or below ΔN_0? Why?What role does the stimulated transition rate play?