Coursework 1 - Atom-Photon Physics
Deadline: 15th of November 2013

1. Let us consider the states $|n, l, m\rangle$ of Hydrogen, where n, l and m denote the principal, orbital and magnetic quantum numbers, respectively.

   (a) (20/100) Consider an initial state $|n_1, l_1, m_1\rangle$ coupled to a final state $|n_2, l_2, m_2\rangle$ by an electric dipole transition induced by circularly polarized light. Prove that the selections rules $\Delta l = l_2 - l_1 = \pm 1$ and $\Delta m = m_2 - m_1 = \pm 1$ hold.
   Hints: You will need to use the fact that $Y^l_m(\Omega) = e^{im\phi} P^m_l(\cos\theta)$ and some relations involving associated Legendre polynomials.

   (b) Indicate whether the following transitions are allowed or forbidden. Justify your answer in terms of selection rules.

      i. (5/100) $2S_{1/2}(m = -1/2) \to 2P_{3/2}(m = +1/2)$, electric dipole transition, linearly polarized light.

      ii. (5/100) $3S_{1/2}(m = +1/2) \to 3D_{5/2}(m = -3/2)$, magnetic dipole transition.

      iii. (15/100) $1S_{1/2} \to 3D_{5/2}$, electric quadrupole transition. Indicate what m sublevels are coupled if we consider the components xz, z^2 and xy of the quadrupole operator.

2. Consider the electric quadrupole operator x^2, coupling an arbitrary initial s state of principal quantum number n and a final state with angular momentum quantum number l and magnetic quantum number m.

   (a) (20/100) Show that the transition matrix element is non-vanishing if $\Delta m = \pm 2, \Delta l = \pm 2$. In the derivation, neglect spin-orbit coupling and fine structure effects.
   Hints:

   - Expand x in terms of spherical harmonics
   - Use the fact that $\left(Y_1^{\pm 1}(\theta, \phi)\right)^2 \propto Y_2^{\pm 2}(\theta, \phi)$.

   (b) (15/100) Consider now a transition from an initial state $2s$ to a final state $4d$. Provide a diagrammatic representation of the sublevels coupled by the x^2 quadrupole operator in terms of the quantum numbers j, m_j associated with the total angular momentum operator $\mathbf{J} = \mathbf{L} + \mathbf{S}$.
   Hint: They are written in the form $nL_j(m_j =$ magnetic quantum number) (for instance, $2p_{3/2}(m = +1/2)$ corresponds to $n = 2$, $l = 1$, $j = 3/2$ and $m_j = +1/2$)

3. (20/100) Give examples of particular properties of laser light, compared to an incandescent light bulb. Explain where the physical differences come from and why they are useful to applications such as nonlinear optics. Base

your discussion on the similarities and differences between spontaneous and stimulated emission.
Hints:

- What happens to the direction, polarization and spectral range of the photons emitted by each process?
- How are the spectral ranges of both sources?Why?