Atom-Photon Physics - Coursework 3 - Solutions

1. The Keldysh parameter is given by $\gamma = \sqrt{I_P/2U_P}$ where I_p is the ionization potential and U_P the
3 ponderomotive energy (the mean quiver energy of an electron in a field).
1 This parameter delimits the tunneling regime and the multiphoton regime. In the tunneling regime ($\gamma < 1$), the electron reaches the continuum by tunnel ionization.
4 In this case, the external field is assumed to be quasi-static and the electron tunnels through the potential barrier $V_{eff} = V(r) - \vec{r}\cdot\vec{E}(t)$. In this
1 regime, intermediate resonances are not important.
2 This corresponds to high driving-field intensities and/or low frequencies.
In the multiphoton regime ($\gamma > 1$), the electron reaches the
2 continuum by a multiphoton transition. The atom is being driven too quickly by the field for tunneling
3 to occur. In this case, atom-laser resonances play an important role. This corresponds to not so high driving-field intensities and/or high frequencies.

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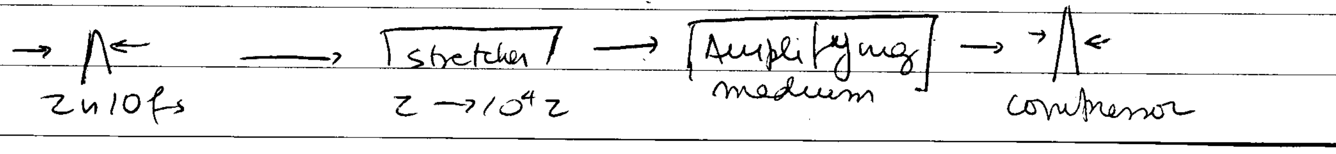
2. Chirped pulse amplification

(a) In chirped pulse amplification, a short laser
1 pulse is stretched, amplified and then compressed again. The stretcher is built employing lenses and/or diffraction gratings in such a way that
3 the "red" and "blue" components of the pulse are separated, i.e., a chirp is introduced in the pulse.

A schematic representation of this process is provided below

1 

(b) Chirped pulse amplification made it possible to overcome an important obstacle to high-intensity laser fields: self focusing. Self focusing is caused by the fact that:

1 (i) the beam profile varies in the direction perpendicular to the propagation direction

1 (ii) The active medium has an intensity dependent refractive index $n = n_0 + n_1 I + n_2 I^2 + \dots$

1 Hence, the center and the edges of the active material exhibit different refractive indices, and it behaves like a lens. This leads to the distortion of the pulse and to the destruction of the material. By stretching the pulse so that it carries the same energy but has a lower intensity, one reduces the effects caused by self-focusing and allows light amplification.

(c) Ideally, at the end of the process the spectral distribution of the pulse should be equal to that it had before being ~~streathed~~ stretched. This means that the chirp introduced by the stretcher should be compensated by the compressor. In reality, the residual chirp must be minimized.

The Zeeman Tuning Technique

Given: Average radiation pressure force on a two-level atom

$$F = -\frac{\hbar \Omega^2 \Gamma k}{\Gamma^2 + 2\Omega^2 + 4[\omega - \omega_0 + kv]^2} \qquad (*)$$

(a) The inhomogeneous magnetic field will shift the transition frequency by $\gamma B(z)$ (hint.)

1 Hence $\omega - \omega_0 + kv \longrightarrow \omega - \omega_0 - \gamma B(z) + kv$ in $(*)$

1 F is maximal if $\omega - \omega_0 - \gamma B(z) + kv = 0$ (1)
This gives the constant acceleration

1
$$\boxed{a_{max} = \frac{F_{max}}{M} = \frac{\hbar k}{M}\,\frac{\Omega^2 \Gamma}{\Gamma^2 + 2\Omega^2}}$$

2
- Low laser intensity: $\Omega << \Gamma \Rightarrow$ the acceleration is proportional to Ω^2 and thus increases linearly with the intensity
- High laser intensity: $\Omega >> \Gamma$: a_{max} ~~ma~~ saturates to

2
$$a_s = \frac{\hbar k}{M}\,\frac{\Gamma}{2}$$ Hence, it cannot increase indefinitely

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(b) Integrating a_{max} twice in time we find

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$$v = v_0 + a_{max} t \qquad (2)$$
$$z = v_0 t + \frac{a_{max}}{2} t^2 \qquad (3)$$

We want, however, v as a function of z. Hence we must eliminate t.

From (3) $\frac{a_{max}}{2} t^2 + v_0 t - z = 0$

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[1] $t = -\frac{v_0}{a_{max}} \pm \frac{1}{a_{max}}\sqrt{v_0^2 + 2a_{max}z}$

Inserting into (2),

$$v = \cancel{v_0} + a_{max}\left[-\frac{\cancel{v_0}}{\cancel{a_{max}}} \pm \frac{1}{a_{max}}\sqrt{v_0^2 + 2a_{max}z}\right]$$

[1] $$\Rightarrow \boxed{v = \pm\sqrt{v_0^2 + 2a_{max}z}}$$

[1] We will consider only the positive root as the negative root implies a change in the direction of motion

This gives $v = \sqrt{v_0^2 - \frac{\hbar\Omega^2 k z}{M(\Gamma^2+2\Omega^2)}}$

[1] Note that, for v to be real, the ~~term~~ expression inside the square root must be positive. Hence,

$$v_0^2 \geq \frac{\hbar\Omega^2 k z}{M(\Gamma^2+2\Omega^2)}$$

[1] $$\Rightarrow \boxed{z_{MAX} \leq \frac{(\Gamma^2+2\Omega^2)}{\Omega^2}\frac{M}{\hbar k}v_0^2}$$

[1] This implies that there is an upper bound on z, which will depend on the initial ~~velocity~~ kinetic energy of the atom considered

To encounter the shape of the field we will use (1).

[1] $$\Rightarrow \boxed{B(z) = \frac{1}{\gamma}\left[\Delta + k\sqrt{v_0^2 + 2a_{max}z}\right]}$$

[1] Physically, apart from making the atom always resonant with the decelerating field, $B(z)$ guarantees a maximal, constant deceleration. 10

Optical molasses

[1] (a) F_{z_1} is related to the co-propagating beam and F_{z_2} to the counter-propagating beam. This can be seen by inspection of the general expression for the radiation pressure force. Since the
[2] Doppler term is $\vec{k}\cdot\vec{v}$, a positive or negative sign indicates that $\vec{k}$ and $\vec{v}$ are in the opposite, or in the same direction, respectively. 3

[1] (b) Small velocities: $kv_z \ll \Delta$

Hence, we will write

$$F_{z_n} = \frac{\hbar\Gamma}{\underbrace{\Gamma^2+2\Omega^2}_{a}+\underbrace{4\Delta^2}_{b}\left[1 \pm \underbrace{\frac{kv_z}{\Delta}}_{x}\right]^2} \qquad (*)$$

[1] Expansion in series of (*)

[2] $$F_{z_1} \simeq \frac{1}{a+b} - \frac{2bx}{(a+b)^2} + \frac{(-ab+3b^2)x^2}{(a+b)^3} + O(x^3) + \ldots$$

[2] $$F_{z_2} \simeq \frac{1}{a+b} + \frac{2bx}{(a+b)^2} + \frac{(-ab+3b^2)}{(a+b)^3}x^2 + O(x^3) + \ldots$$

Cumbersome if done by hand; give up to 2 marks each depending on the effort.

[1] $$F_{z_1} - F_{z_2} = \frac{4bx}{(a+b)^2} + O(x^3) \quad \text{(even } \text{powers cancel out)}$$

[1] $$\boxed{F_z \simeq \frac{\hbar \Delta k v_z}{[\Gamma^2 + 2\Omega^2 + 4\Delta^2]}}$$

[2] This force is linearly dependent on the velocity, and it is viscous for red-detuned laser beams ($\Delta<0$)

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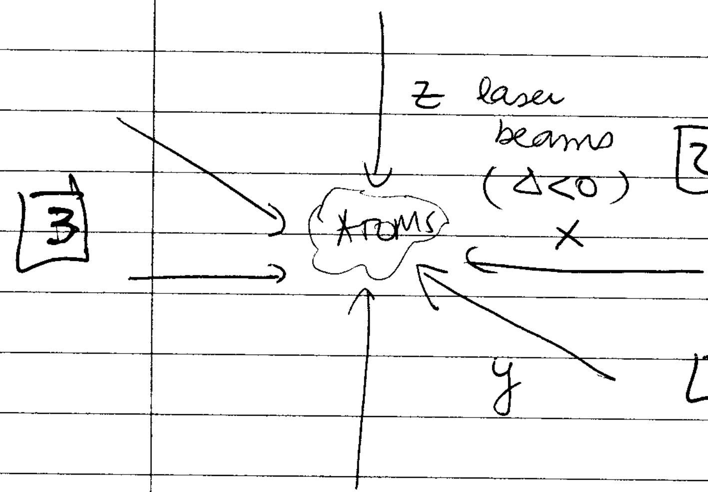
(c) The laser-beam configuration in order to obtain

[3] optical molasses consists of three pairs of ~~counter-pro~~ red-detuned laser beams in the x, y and z direction. In each pair, there is one counterpropagat-

[1] ing and one co-propagating beam with regard to the atoms.



In each direction, because of the red detuning, the [2] atoms will experience a viscous force, which will effectively cool them.

[1] Three pairs of beams guarantee this cooling is nearly isotropic.

[1] The above set up cools, but not traps atoms.

In order for the atoms to be trapped, it would

[2] be necessary to confine them in a potential minimum by using, for instance, a restoring force.

[2] This can be achieved by ~~applying~~ adding an inhomogeneous magnetic field and adequately polarized laser beams, ~~forming magneto-optical trap~~. to the optical-molasses set up. This would give rise to a magneto-optical trap.

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The curse of the displaced atom

(a) Let us consider the matrix elements
$\vec{d}_1 = \langle \vec{p}+\vec{A}(t) | \vec{r} | g \rangle$ and $\vec{d}_2 = \langle \vec{p}+\vec{A}(t) | \vec{r} | g_{r_0} \rangle$.

We will first write d_1, d_2 in the position space. For that purpose, we will need the closure relation

$\boxed{1}$
$$\int d^3r \, |\vec{r}\rangle\langle\vec{r}| = \mathbb{1}$$

Inserting this in d_1 yields

$$\vec{d}_1 = \int d^3r \, \langle \vec{p}+\vec{A}(t) | \vec{r} \rangle \, \vec{r} \, \langle \vec{r} | \vec{g} \rangle =$$

$\boxed{3}$
$$= \frac{1}{(2\pi)^{3/2}} \int \frac{e^{-i[\vec{p}+\vec{A}(t)]\cdot\vec{r}}}{(2\pi)^{3/2}} \, \vec{r} \, \Psi_g(\vec{r})$$

(1 mark for each correct step)

Similarly,

$\boxed{3}$
$$\vec{d}_2 = \int d^3r \, \langle \vec{p}+\vec{A}(t) | \vec{r} \rangle \, \vec{r} \, \langle \vec{r} | \vec{g} \rangle = \int d^3r \, \frac{e^{-i[\vec{p}+\vec{A}(t)]\cdot\vec{r}}}{(2\pi)^{3/2}} \, \vec{r} \, \Psi_g(\vec{r}-\vec{r}_0)$$

$\boxed{1}$ By shifting $\vec{r} \to \vec{r}-\vec{r}_0$, d_2 becomes

$$\vec{d}_2 = \frac{1}{(2\pi)^{3/2}} \int d^3r \, e^{-i[\vec{p}+\vec{A}(t)]\cdot(\vec{r}+\vec{r}_0)} \, (\vec{r}+\vec{r}_0) \, \Psi_0(\vec{r})$$

$\boxed{2}$
$$= \frac{1}{(2\pi)^{3/2}} e^{-i[\vec{p}+\vec{A}(t)]\cdot\vec{r}_0} \left[\overbrace{\int d^3r \, e^{-i[\vec{p}+\vec{A}(t)]\cdot\vec{r}} \, \vec{r} \, \Psi_0(\vec{r})}^{(*)} + \vec{r}_0 \overbrace{\int d^3r \, e^{i[\vec{p}+\vec{A}(t)]\cdot\vec{r}} \, \Psi_0(\vec{r})}^{(**)} \right]$$

$\boxed{2}$ Note that $(**) = (2\pi)^{3/2} \langle \vec{p}+\vec{A}(t) | \vec{g} \rangle$, $(*) = (2\pi)^{3/2} \vec{d}_1$

This gives

2 $$\vec{d_2} = \underbrace{e^{-i[\vec{p}+\vec{A}(t)]\cdot\vec{r_0}}}_{\text{overall phase}} [\vec{d_1} + \underbrace{\vec{r_0} \langle \vec{p}+\vec{A}(t)|g\rangle}_{\text{term dependent on } r_0}]$$

(b) Physically, both terms in $\vec{d_2}$ will lead to changes in the SFA transition probability They are clearly artifacts, as it should not matter how the origin of a coordinate system is chosen.

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One can readily see that both transition probabilities are different:

Transition probability obtained with $\vec{d_1}$:

1 $$|M^{(1)}_{SFA}|^2 = \left| \int_{t_0}^{\infty} dt\ \vec{d_1}\cdot\vec{E}(t)\ e^{\underbrace{-\frac{i}{2}\int_t^{\infty}[\vec{P}+\vec{A}(\tau)]^2 d\tau - iE_g(t-t_0)}_{i\phi(\vec{P},t,t_0)}} \right|^2$$

Transition probability obtained with $\vec{d_2}$

3 $$|M^{(2)}_{SFA}|^2 = \Big| \underbrace{e^{-i\vec{P}\cdot\vec{r_0}}}_{\text{just a phase}} \int_{t_0}^{\infty} dt\ e^{-i\vec{A}(t)\cdot\vec{r_0}} [\vec{d_1} + \vec{r_0}\langle\vec{p}+\vec{A}(t)|g\rangle]\cdot\vec{E}(t)\, e^{i\phi}$$

$$= \left| \int dt\ e^{-i\vec{A}(t)\cdot\vec{r_0}}\ \vec{d_1}\cdot\vec{E}(t)\ e^{i\phi(P,t,t_0)} \right|^2 +$$

$$+ \left| \int \vec{r_0}\cdot\vec{E}(t)\langle p+A(t)|g\rangle\ e^{-i\vec{A}(t)\cdot\vec{r_0}}\ e^{i\phi} \right|^2 +$$

Interference terms

$$\neq |M^{(1)}_{SFA}|^2$$

The time dependent phase $e^{-i\vec{A}(t)\cdot\vec{r_0}}$ will lead to
[2] spurious oscillations. It is directly related with the SPA's lack of gauge dependence.

[2] The $\vec{r_0}$ dependent term is an artifact which appears because in the SFA the continuum and the bound states are not orthogonal.

[2] In fact, for an exact continuum state $|\phi_c\rangle$, $\langle \phi_c | g\rangle = 0$. This would eliminate this term.

On the other hand,

[1]
$$\langle \vec{p}+\vec{A}(t) | g\rangle = \frac{1}{(2\pi)^{3/2}}\int e^{-i[\vec{p}+\vec{A}(t)]\cdot\vec{r}}\,\psi_g(\vec{r}) \neq 0 .$$

This causes the above-stated problems