Coursework 3 - Atom - Photon Physics - 2013

Solution

1) The ponderomotive energy is the mean quiver energy of an electron in a field [1]. It is given by $U_p = \frac{\langle A^2(t) \rangle_t}{2m}$, where $A(t)$ is the vector potential and $\langle \; \rangle_t$ denotes the temporal average [2]. The ponderomotive energy may also be associated with the mean kinetic energy transfer from the field to the electron [1].

The ponderomotive energy is related to the term in A^2 in the laser-atom Hamiltonian $H = \frac{p^2}{2m} + \frac{A^2(t)}{2m} + \frac{\vec{P}\cdot\vec{A}(t)}{m} + V$ [1]. If the field is weak,

$$\frac{A^2}{2m} \ll \frac{\vec{P}\cdot\vec{A}(t)}{m}$$

and this term can be ignored in the interaction Hamiltonian $H_{int} = \frac{\vec{P}\cdot\vec{A}(t)}{m} + \frac{A^2(t)}{2m}$ [2]. In contrast, if the field is strong, it can no longer be neglected [1].

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2 A Volkov state describes a field-dressed plane wave [1]. Physically, this implies that the binding potential V has been neglected in the electron propagation, i.e., it is evolving under the sole influence of the laser field [2]. This state is associated with the Volkov Hamiltonian $H^{V}_{GA}(t) = \frac{[\vec{p} + \vec{A}(t)]^2}{2}$ [1].

Since this Hamiltonian has an analytical solution, using Volkov states reduces the numerical effort necessary for the modeling of strong-field phenomena to a great extent [2].

Explicitly, this solution is

$$\langle \vec{r} \,|\, \vec{p} + \vec{A}(t) \rangle = \frac{\exp[i\,\vec{p}\cdot\vec{r}]}{(2\pi)^{3/2}} \exp\left[-i \int_{-\infty}^{t} \frac{[\vec{p} + \vec{A}(\tau)]^2}{2} d\tau \right] \text{[1]}$$

A vanishing external field implies that $A(t) \to 0$ [1], so that the Volkov state is reduced to a field-free plane wave [1].

In the context of the strong-field approximation, the bound states are approximated by those of the field-free Hamiltonian and the continuum by Volkov states [1]. While all states (bound and continuum) of the <u>full</u> Hamiltonian are orthogonal, under these assumptions this no longer holds [1]. This leads to problems with gauge and translation invariance ("the curse of the displaced atom") [1].

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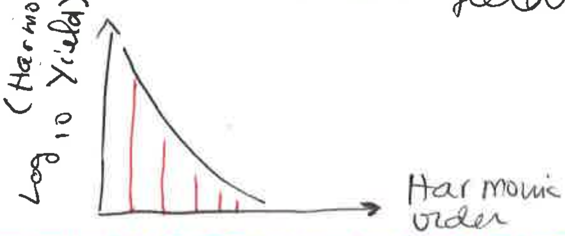
3. A typical high-harmonic spectrum exhibits a broad energy region with harmonics of comparable intensities, the "plateau" [1], followed by an abrupt decrease in the yield at the plateau's high-energy end: the "cutoff" [1]. The energy position of the cutoff is given by the law $I_p + 3.17 U_p$, for monochromatic fields or long-enough pulses [2].

Perturbation theory with regard to the laser field states that the transition probability of an n-th order process, such as the generation of a harmonic of order n, is proportional to I^N, where I = the driving-field intensity [3]. This means that, for not so strong fields, the harmonic intensities should decrease with harmonic order [1]. In other words, in the perturbative regime there will be neither a plateau nor a cutoff [1].

Schematic representations

Weak / moderate fields

1



Strong Fields

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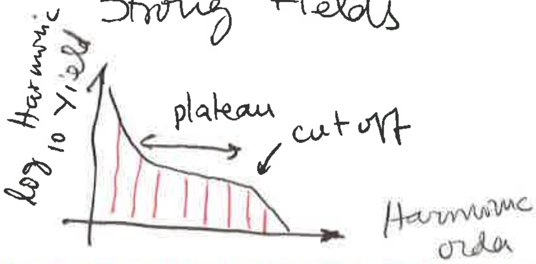

The plateau and the cutoff are explained by a three-step physical process, in which:

[1] i) an electron is freed in the continuum by tunneling OR multiphoton ionization

[1] ii) This electron propagates and is accelerated by the field

iii) It is driven back towards the core and recombines with a bound state of its parent ion [1], releasing the kinetic energy it acquired from the field in the form of high-harmonic radiation [1].

The plateau is due to the quantum interference of the possible paths along which the e^- may return and the cutoff corresponds to its maximal kinetic energy upon return [2].

17.

(b) High-order harmonic generation can be used for attosecond-pulse production for the following reasons:

(i) According to the three-step model described above, there are preferential times for which HHG takes place, i.e., the times at which the electron returns and recombines with its parent ion [1] ~~[2]~~. This implies that HHG is a time resolved process [1], and that the HHG time profile carries information about the e^- return times [1].

(ii) The time frame within which HHG occurs comprises hundreds of attoseconds [1]. This is due to the fact that the electron leaves close to a field maximum and returns close to a field crossing [1], so that the whole process takes place in a fraction of a field cycle [1]. For typical parameters employed in experiments, a field cycle is of the order of 2.7 fs [1]. Hence the above-mentioned fraction is of hundreds of attoseconds [1]

(a) and (b) imply that groups of harmonics can be combined in order to obtain pulses of attosecond duration [1].

According to perturbation theory, attosecond-pulse production ~~production~~ from groups of high-order harmonics would not be possible [1]. This is due to the fact that, if perturbation theory is applied, the transition amplitude is not time resolved [1]. Hence, in this case there are no preferential times for HHG to occur [1].

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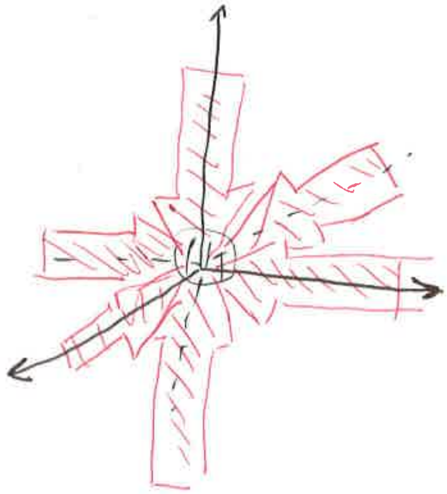
(3) Above-threshold ionization means that an atom absorbs more photons than the necessary amount for it to ionize [1]. According to the predictions of perturbation theory, the photoelectron spectra will predominantly exhibit a peak around vanishing electron momentum, and smaller peaks for higher momenta [1]. For intensities around 10^{13} W/cm^2, however, the peaks at higher photoelectron energy will be more prominent [1]. This occurs because, in this intensity range, the Stark shifts are of the order of the photon energy and effectively shift the ionization threshold. [1] 4

(4) (6)

An optical molasses is obtained by applying three orthogonal pairs of red-detuned laser beams to a medium (e.g. a gaseous sample) [2] (see schematic Representation)

[1]



The red detuning will ensure that an atom of velocity $\vec{v}$ in this medium will always be resonant with the counter-propagating beam [1]. This happens due to the fact that a moving atom will be resonant with such a beam if $\underbrace{\omega_L}_{\text{laser freq.}} = \underbrace{\omega_0}_{\text{transition freq.}} - kv$, [1] as a direct consequence of the Doppler effect [1]. Hence, by absorbing more photons from the counterpropagating beam the atoms will be cooled [1].

The three orthogonal pairs will ensure that the atoms are cooled in all directions [1]. For small velocities [1], the atoms will be subjected to a damping force $\vec{F} = -\alpha \vec{v}$ [1] where α is proportional to the detuning [1]. Consequently, one may say that the atoms move in a "viscous" medium created by the laser field. Hence the name "Optical molasses" [2].

This set up cools, but does not confine atoms [1], as there is no potential-energy minimum in it [1]. 15 7

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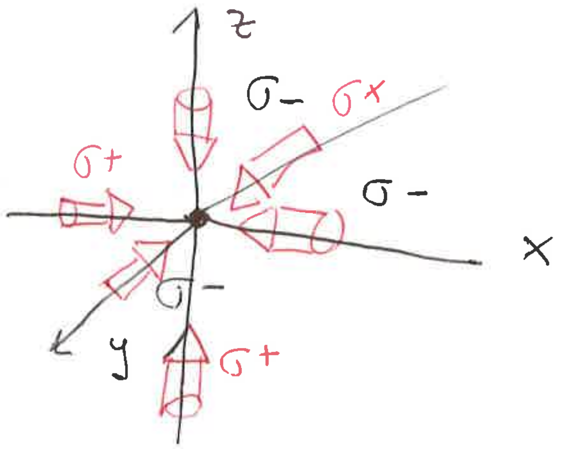
5 The key idea behind a magneto-optical trap is to confine atoms in a potential minimum[1] using

[1] i) An inhomogeneous magnetic field

ii) Three orthogonal pairs of red-detuned laser beams propagating towards the center of the trap and with appropriately chosen polarizations [1].

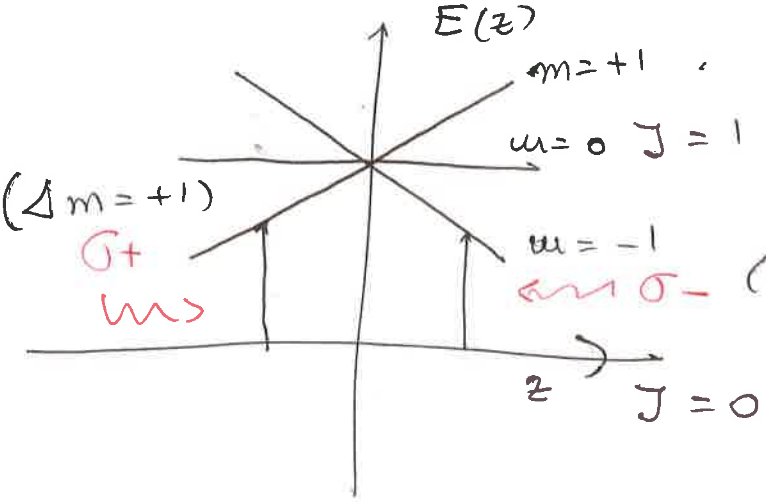
[1]



We will explain the trapping mechanism considering a two-level atom, whose ground and excited states exhibit angular momenta $J=0$ ($m_J=0$) and [1] $J=1$ ($m_J=-1,0,+1$) [1]. The $m=\pm 1$ sublevels will suffer Zeeman shifts [1]. In the z, y and x directions, these shifts will be $\Delta E = \mu m_J b z$, $\mu m_J b y$ and $\mu m_J b x$, respectively [2]. We will focus on what happens along z.

If we consider a σ- field propagating along $-\hat{z}$ and a σ+ field propagating along $+\hat{z}$ such that $\omega_{laser} < \omega_0$) [2 marks for getting the polarizations right], the atom will be in resonance with the former for $z>0$ and with the latter for $z<0$ [1]. Hence, the atoms will absorb photons from the ~~cont~~ counterpropagating beam and be trapped in a potential minimum [1].

(8)



[2]

← Important to indicate the selection rules; if not here, in the text.

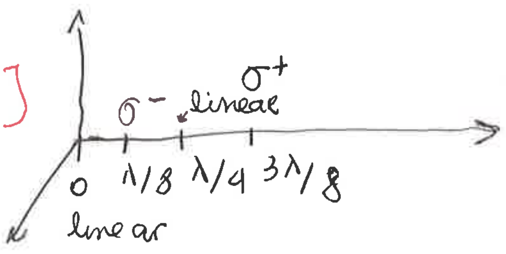
(15)

(6) The key idea behind the sisyphus cooling mechanism is that a moving atom within an optical molasses "climbs" potential-energy hills following a light-shifted Zeeman sublevel adiabatically, before being optically pumped in another sublevel [2]. During this process, the moving atom loses kinetic energy [1]. This pumping occurs due to an existing polarization gradient [1], which causes the Zeeman sublevels and their light shifts to ~~to~~ depend on the position of the atoms within the molasses [1].

In the example requested, we have (a) two counter-propagating orthogonally polarized fields of linear polarization; (b) an atom whose ground state has $J = 1/2$ ($m_J = \pm 1/2$) and whose excited state has $J = 3/2$ ($m_J = \pm 3/2, \pm 1/2$).

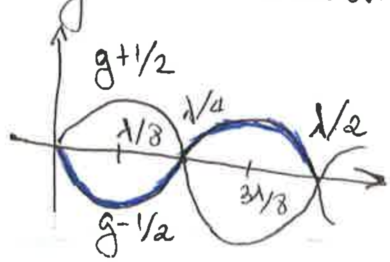
In this case, the polarization of the resulting field will change as shown below

[1]



It is not necessary to prove it, but the student should at least state it.

For simplicity, we will neglect the light shifts of the excited state and consider those of the ground state. They are shown below for each Zeeman sublevel



We will now discuss how the ~~are~~ atoms at $z = \frac{\lambda}{8}$ will be optically pumped. Excitation will depend on the selection rules determined by the local polarization σ^- (i.e., $\Delta m = -1$), and the probability that it will decay to a specific sublevel by spontaneous emission will be the square of the given Clebsch Gordan coefficients [3].

If the atom is initially in $g_{-1/2}$, $\Delta m = -1$ means that it will be excited to $e_{-3/2}$. From there, according to the figure provided it may only decay to $g_{-1/2}$ [2].

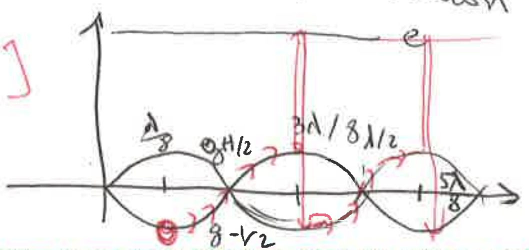
In contrast, if it is initially in $g_{+1/2}$, it will be excited to $e_{-1/2}$ and may decay both to $g_{+1/2}$ and $g_{-1/2}$. It is however 2x more probable that it will decay to the latter sublevel [3]. Hence, population will mainly accumulate in $g_{-1/2}$. [1]

The same argument may be applied to atoms at $z = \frac{3\lambda}{8}$, with the difference that the local polarization will be σ^+ and population will mainly accumulate in $g_{+1/2}$. [1]

On average, optical pumping will occur within a finite time τ_p [1]. If one considers atoms initially at $z = \frac{\lambda}{8}$ and with an average velocity v such that $v\tau_p \sim \lambda$, this atom will follow $g_{-1/2}$ ~~adiabatical~~ adiabatically until the top of the hill (at $z = \frac{3\lambda}{8}$). [2] At this point, the driving-field polarization is σ^+, so that the population will be mostly pumped into $g_{+1/2}$ [1].

A schematic representation is provided below

[1]



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