Solution - Coursework 3 - Atom-Photon Physics 2011/12

1. Address the following questions related to above-threshold ionization (ATI):

   (a) What does "above-threshold" mean? Why does this phenomenon occur?Formulate your answer in terms of the ponderomotive energy and Stark shifts. Explain why ATI does not happen for weak fields. [15]
   Above-threshold ionization means that in order to for an electron to be released in the continuum, the atom absorbs photons in excess, i.e., the total energy of the photons absorbed is higher than the atom's ionization potential [4 marks]. This happens due to the fact that the bound states are strongly distorted by the laser field, so that their Stark Shifts are of the order of the photon energies [2 marks]. Hence, it must overcome a larger binding energy than what is expected from higher-order perturbation theory with regard to to the field [2 marks]. Another way of looking at the problem is in terms of the ponderomotive energy, which is the average quiver energy or an electron in a laser field and given by the temporal average $U_p =< A^2(t) >_t /2$. [3 marks] On average, the shifts in the ionization energy are proportional to U_p. [2 mark] Above-threshold ionization does not occur for weak fields due to the fact that, in this regime, the above-mentioned shifts are negligible and the ponderomotive energy is neglected in the atom-field Hamiltonian as $A^2(t)/2 \ll \mathbf{p} \cdot \mathbf{A}(t)$[2 marks].

   (b) What are the typical features encountered in an ATI spectrum?Explain the physical mechanisms behind these features. Thereby, make a clear distinction between "direct" and "rescattered" electrons. What features can be related to the maximal photoelectron energies? [20]
   Typically, an ATI spectrum consists of a low-energy photoelectron region whose intensity drops abruptly at $2U_p$, [2 marks] followed by a plateau, i.e., a region with discrete photoelectron peaks of roughly the same strength, which may extend up to $10U_p$, where U_p is the ponderomotive energy [3 marks]. The low-energy part of the spectrum is related to the so called "direct electrons" [2 marks] in which the electrons leave the atom by tunneling or multiphoton ionization and reach the detector without further interaction with the core [2 marks]. The cutoff $2U_p$ is the maximal kinetic energy the photoelectrons may obtain in this way [2 marks]. The second part of the spectrum corresponds to the rescattered electrons [2 mark]. In this case, the electrons i) leave the atom by tunneling or multiphoton ionization, ii) propagate in the field, being accelerated by it [2 marks], and, subsequently, iii) are driven back by the field towards its parent ion, with which they rescatter elastically [3 marks]. The maximal kinetic energy they can obtain in this way is $10U_p$. [2 marks].

2. **The Volkov solution.** Consider the expression

$$\psi^{(V)}(\mathbf{r},t) = \frac{\exp[i\mathbf{p}\cdot\mathbf{r}]}{(2\pi)^{3/2}} \exp\left[-i\int_{-\infty}^{t} \frac{[\mathbf{p}+\mathbf{A}(\tau)]^2}{2} d\tau\right]. \tag{1}$$

(a) Show that it satisfies the time-dependent Schrödinger equation

$$i\frac{\partial\psi^{(V)}(\mathbf{r},t)}{\partial t} = \frac{[\mathbf{p}+\mathbf{A}(t)]^2}{2}\psi^{(V)}(\mathbf{r},t). \tag{2}$$

Differentiating $\psi^{(V)}(\mathbf{r},t)$ with respect to time, we obtain

$$\begin{aligned}\frac{\partial\psi^{(V)}(\mathbf{r},t)}{\partial t} &= -i\frac{[\mathbf{p}+\mathbf{A}(t)]^2}{2}\frac{\exp[i\mathbf{p}\cdot\mathbf{r}]}{(2\pi)^{3/2}} \exp\left[-i\int_{-\infty}^{t} \frac{[\mathbf{p}+\mathbf{A}(\tau)]^2}{2} d\tau\right] \\ &= -i\frac{[\mathbf{p}+\mathbf{A}(t)]^2}{2}\psi^{(V)}(\mathbf{r},t).\end{aligned}$$

Multiplying by i yields the time-dependent Schrödinger equation (2). [10 marks].

(b) Explain, physically, what Eq. (2) represents, together with Eq. (1). Consider now Eq. (2) in the limit of vanishing vector potential, i.e., $\mathbf{A} \to 0$. Physically, what does this mean?Provide a discussion in terms of an electron propagating in the laser field [15] Physically, Eq. (2) describes the evolution of a free particle in the presence of the external field. This reflects itself in Eq. (1), which is a field-dressed plane wave [5 marks]. In the limit $\mathbf{A} \to 0$, Eq. (2) describes the evolution of a free electron, and its solution reduces to a plane wave with energy $\mathbf{p}^2/2$, i.e.,

$$\psi^{(V)}(\mathbf{r},t) = \frac{\exp[i\mathbf{p}\cdot\mathbf{r}]}{(2\pi)^{3/2}} \exp\left[-i\frac{\mathbf{p}^2}{2}t\right].$$

[5 marks]. For that reason, within the strong-field approximation, the **field-dressed** momentum is a conserved quantity along the electron trajectory. Furthermore, the spatial extension of the binding potential and effects such as the distortion it causes upon the recolliding wavepacket and Coulomb focusing are not taken into account [5 marks].

(c) Why are Volkov waves useful in the theoretical modeling of strong-field phenomena? [10]
The Volkov solution has two advantages: i) it treats the laser-field non-pertubatively, and it is thus applicable to the strong-field regime; ii) It is obtained analytically from the Schrödinger equation (2) [5 marks].

Since the full Schrödinger equation of an electron under the influence of a binding potential and a strong laser field is not analytically solvable, approximating field-dressed continuum waves by Volkov states considerably simplifies the modeling of strong-field phenomena [5 marks].

3. Answer the following questions related to laser cooling:

   (a) Why can one employ a counter-propagating laser beam to cool atoms? Explain this in terms of momentum transfer from the field to an atom and vice-versa. [15]

   The photons from the counter-propagating laser field carry momentum. This means that if an atom of mass m absorbs a photon of momentum $\mathbf{p} = \hbar\mathbf{k}$, this will cause the atom to recoil in the direction of the incident light and to change its velocity by $v_R = \hbar k/m$ [6 marks]. The atom also recoils when re-emitting the absorbed photon. In this case, however, the direction of the emitted photon, and therefore of the recoil velocity, will be random [6 marks]. Hence, over many absorption-re-emission cycles the latter contributions will average out and there will be a net momentum transfer from the field to the atom in the direction of the laser beam [3 marks].

   (b) What is the Doppler cooling limit and why does it occur?Base your explanation on the momentum and energy transfers over one absorption-spontaneous emission cycle of a photon of momentum $\mathbf{k}$ and an atom of velocity $\mathbf{v}$. [15]

   Ideally, all atoms would cool to vanishing velocity. In real life, however, the light beam employed to cool atoms also causes heating. This occurs due to the fact that each absorption/re-emission cycle follows discrete steps and can be understood as a random walk process [5 marks]. At each cycle, the light field loses $2\hbar\omega_R$ of energy, where ω_R is the frequency associated with the recoil process. This implies that the atoms heat up by $2\hbar\omega_R$ at each cycle [5 marks]. In the steady state situation, the cooling rate is equal to the heating rate. The temperature obtained under this condition is known as the Doppler cooling limit [5 marks].