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(a)

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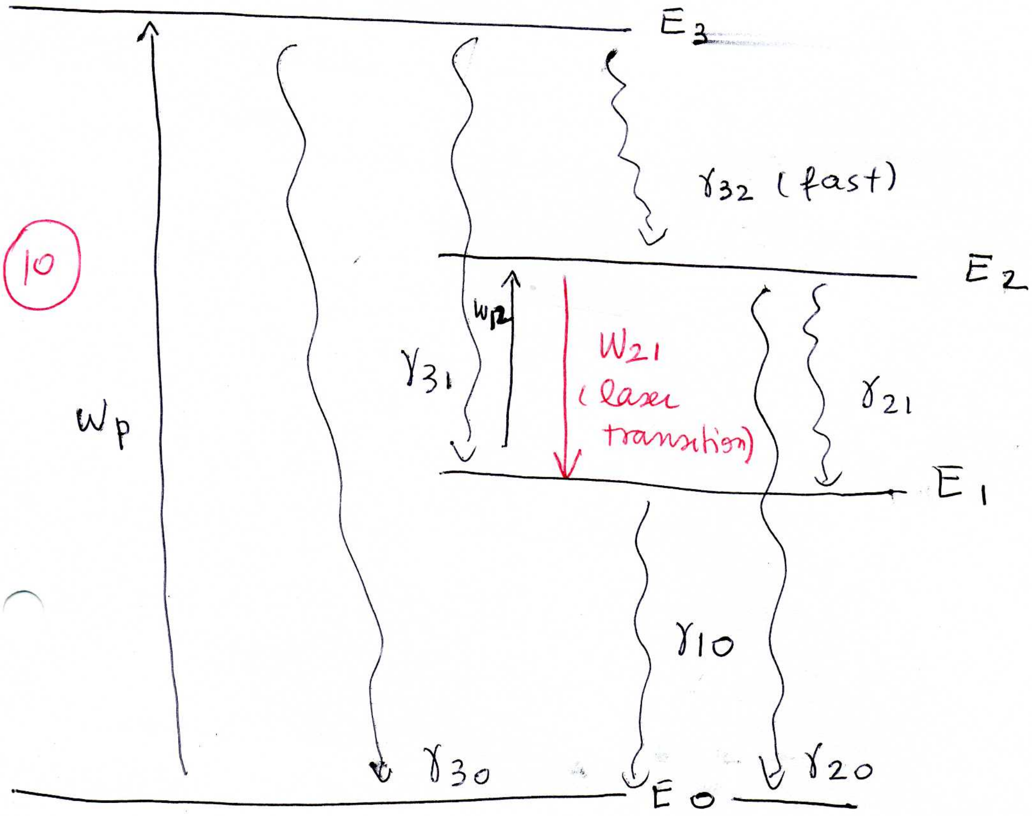

(b) $N_3 << N_0 \Rightarrow$ γ_{30} can be neglected

Stimulated emission from 3 to 1 and 3 to 0 can be neglected.

Number of atoms/time pumped from 0 to 3 :

$$\left.\frac{dN_3}{dt}\right|_{pump} = W_p (N_0 - N_3) \approx W_p N_0$$

A fraction of these atoms populate the level 2. We assume that this level will populated by

$$\underbrace{R_p}_{\substack{\text{effective}\\ \text{pumping}\\ \text{rate @ level 2}}} = \underbrace{\eta_p}_{\substack{\text{quantum}\\ \text{efficiency}}} W_p N_0$$

Rate equations for the excited levels E_2 and E_1

$$\frac{dN_2}{dt} = R_p - W_{21}(N_2 - N_1) - \gamma_2 N_2 \qquad \gamma_2 = \gamma_{21} + \gamma_{20}$$

$$\frac{dN_1}{dt} = W_{21}(N_2 - N_1) + \gamma_{21} N_2 - \gamma_{10} N_1$$

Steady-state solutions as functions of W_{21}, R_p

$$0 = R_p - W_{21}(N_2 - N_1) - \gamma_2 N_2 \qquad (*)$$

$$0 = W_{21}(N_2 - N_1) + \gamma_{21} N_2 - \gamma_{10} N_1 \qquad (**)$$

Using $(**)$: $W_{21} N_2 + \gamma_{21} N_2 = W_{21} N_1 + \gamma_{10} N_1$

$$(W_{21} + \gamma_{21}) N_2 = (W_{21} + \gamma_{10}) N_1$$

$$\Rightarrow \quad N_1 = \frac{W_{21} + \gamma_{21}}{W_{21} + \gamma_{10}} N_2$$

Inserting into $(*)$:

$$R_p - W_{21}\left(\frac{\cancel{W_{21}} + \gamma_{10} - (\cancel{W_{21}} + \gamma_{21})}{W_{21} + \gamma_{10}}\right) N_2 - \gamma_2 N_2 = 0$$

$$R_p (W_{21} + \gamma_{10}) = [\,W_{21}\gamma_{10} \underbrace{- W_{21}\gamma_{21} + \gamma_2 W_{21}}_{= \gamma_{20} W_{21}} + \gamma_{10}\gamma_2\,] N_2$$

$$\Rightarrow \quad N_2 = \frac{W_{21} + \gamma_{10}}{W_{21}(\gamma_{10} + \gamma_{20}) + \gamma_{10}\gamma_2} R_p$$

$$N_1 = \frac{W_{21} + \gamma_{21}}{W_{21}(\gamma_{10} + \gamma_{20}) + \gamma_{10}\gamma_2} R_p$$

$$N_2 - N_1 = \frac{\gamma_{10} - \gamma_{21}}{\gamma_{10}\gamma_2} \frac{R_P}{1 + [(\gamma_{10}+\gamma_{20})/\gamma_{10}\gamma_2] W_{21}}$$

If we call

$$\tau_{eff} = \frac{(\gamma_{10} + \gamma_{20})}{\gamma_{10}\gamma_2} \quad \text{and} \quad \Delta N_0 = \frac{(\gamma_{10} - \gamma_{21})}{\gamma_{10}\gamma_2} R_P$$

$$\boxed{\Delta N_{21} = \Delta N_0 \frac{1}{1 + W_{21}\tau_{eff}}}$$

(c) Saturation intensity $\Delta N_{21} = \frac{1}{2}\Delta N_0$

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$$\frac{(\gamma_{10} - \gamma_{21}) \cancel{R_P}}{2\,\cancel{\gamma_{10}\gamma_2}} = \cancel{R_P} \frac{(\cancel{\gamma_{10} - \gamma_{21}})}{\cancel{\gamma_{10}\gamma_2}} \cdot \frac{1}{1 + \tau_{eff} W_{21}}$$

$$\Rightarrow 1 + \tau_{eff} W_{21} = 2 \qquad \boxed{W_{21} = \frac{1}{\tau_{eff}} = \frac{\gamma_{10}\gamma_2}{\gamma_{10} + \gamma_{20}}}$$

(d) The saturation intensity does not depend directly on the pumping rate, as seen above. It depends only on the relaxation lifetimes of the atomic levels. Physically, this means that increasing the pump intensity will not influence the saturation intensity of the material.

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