Solution - Coursework 2 - Atom-Photon Physics 2012/13

1. Address the following issues regarding lasers:

   (a) (10/100) What are the pre-requisites for a laser and why? Provide examples.
   The pre-requisites for a laser are (i) Stimulated emission; (ii) Population inversion; (iii) a cavity [1 mark].
   Justification:

   In stimulated emission, the system decays from an upper to a lower energy level emitting a photon in the presence of a radiation field [1 mark]. Because a photon is being added to a pre-existing radiation mode, the photon in question exhibits the same properties (frequency, phase, polarization, etc) as that of the incident light [2 marks]. This is essential in order to guarantee the properties of the light generated by a laser (coherence, directionality, etc) [1 mark].

   Population inversion is important because, if the system is in thermal equilibrium, it acts as an absorber, and it must act as an amplifier [1 mark]. This is easy to see if one considers a two level system whose energies are E_1, E_2 and whose populations are N_1, N_2 (the indexes 1 and 2 relate to the lower and upper level, respectively). Due to the principle of the detailed balancing, the stimulated emission rate/atom (here called W) is equal to the absorbtion rate/atom [1 mark]. In total, the absorbed and emitted energy densities will be $\rho_{abs} = WN_1$ and $\rho_{em} = WN_2$, so we need $N_2 > N_1$ [1 mark]. However, in thermal equilibrium $N_2/N_1 = \exp[-(E_2 - E_1)/(k_bT)]$, where k_b =Boltzmann const. and T = absolute temperature, so that we will need a pumping mechanism [1 mark].

   Finally, we need to place the medium in a cavity to change the amplifier into an oscillator. A cavity will allow positive feedback and lead to a stationary wave as the e.m. waves bounce back and forth.[1 mark]

   (b) (10/100) What kind of modes may a laser cavity have? Why? Discuss and explain these modes

   A laser cavity may have transverse modes and longitudinal modes [1 mark]. These modes are stationary patterns caused by the boundary conditions imposed by the cavity [1 mark]. Transverse modes occur perpendicular to the propagation direction of the wave associated with the laser field [1 mark], and are due to the fact that the laser beam has a transverse intensity variation [1 mark].

   Longitudinal modes occur in the propagation direction of the beam [1 mark]. They may also occur for plane waves, and are easily determined by the condition that the pattern obtained for the electromagnetic wave after a round trip in a cavity is the same [2 marks]. This means that, for a wave propagating in the z direction, $\exp[ik(z+2L)] = \exp[i2n\pi]\exp[ikz]$, where L is the length of the cavity and k the radiation wavevector, i.e., $L = n\pi/k = \lambda/2$, where λ is the wavelength of the radiation [3 marks].

(The marking of this question is flexible depending on how much info was given on the transverse modes; note that, in comparison to the 2010 coursework no info about spherical mirrors was asked; hence only 10 marks).

2. (15/100) Explain how the Doppler effect is removed in two-photon absorption spectroscopy.

In two-photon absorption spectroscopy, a two photon transition is used to eliminate Doppler broadening [1 mark]. An atom with velocity v interacts with two light beams of the same frequency ω_L [1 mark]such that $2\omega_L$ is resonant with the atomic transition frequency [1 mark], and opposite propagation directions [1 mark]. The atom in question undergoes a two-photon transition by absorbing one photon from each beam [1 mark].

Due to the Doppler effect, the atom "sees" the following frequencies: (i) photon 1: $\omega_{seen}^{(1)} = \omega_L(1 + v/c)$ and (ii) photon 2: $\omega_{seen}^{(2)} = \omega_L(1 - v/c)$ [3 marks]. At resonance, the energy difference between the excited and bound state of the atom is $\hbar\omega = \hbar\omega_L(1 + v/c) + \omega_L(1 - v/c)$ [1 mark], so that the Doppler shift cancels out and the Doppler broadening is eliminated [1 mark].

This can also be seen from the line profile for two-photon absorption, which, for an atom moving with velocity v reads [5 marks for the explanation below]

$$g(\omega, \omega') = \frac{\Gamma_b/(2\hbar)}{[\omega_0 - \omega(1 + \mathbf{v}\cdot\hat{k}/c) - \omega'(1 + \mathbf{v}\cdot\hat{k}'/c)]^2 + [\Gamma_b/(2\hbar)]^2}, \tag{1}$$

where ω_0 is the transition frequency, Γ_b the linewidth of the excited level and $\hat{k}, \hat{k}'$ the unitary propagation vectors of the laser beams and $\mathbf{v}$ the atom velocity. The terms in v are related to the Doppler effect. These terms will cancel out if $\omega = \omega'$ (beams of the same frequency) and $\hat{k} = -\hat{k}'$ (counter-propagating beams). In this case, the line profile becomes

$$g(\omega, \omega) = \frac{\Gamma_b/(2\hbar)}{[\omega_0 - 2\omega]^2 + [\Gamma_b/(2\hbar)]^2}. \tag{2}$$

(there is some flexibility in the marking depending on whether more emphasis was placed in the quantitative explanation or on the qualitative one)

3. (15/100) Explain hole burning and saturation-absorption spectroscopy. Why is it useful to study transitions masked by the Doppler broadening?

The key idea behind hole burning is that a strong laser can be used to burn a "hole" in the Doppler-broadened profile of a spectral line [1 mark]. This occurs due to the fact that, for strong enough fields in a cavity, stimulated transitions will reduce the population inversion density [1 mark]. Hence, the light intensity cannot increase indefinitely [1 mark]. This hole will be centered at the frequency of the applied signal and exhibit a Lorentzian shape much narrower than that of the broadened line [2 marks]. This happens because the strong signal will only saturate the population difference of a group of atoms whose Doppler-shifted frequencies $\omega_1 = \omega_0(1 \mp v/c)$ are resonant with it.[2 marks]

This will be a much smaller group of atoms than those present in the gas inside the cavity [1 mark]. The atoms in this gas will obey a Maxwellian velocity distribution. According to this distribution, the number of atoms dN whose velocity is between v and $v + dv$ reads

$$dN = N_0 \exp\left[-Mv^2/(2k_bT)\right]dv, \tag{3}$$

where M is the atomic mass, k_b the Boltzmann const. and T the absolute temperature [1 mark]. This will lead to a Gaussian intensity profile for the light emitted/absorbed by the gas, centered at $\omega = \omega_0$ [1 mark].

In saturation absorption spectroscopy, one employs a strong saturating beam at the rest transition frequency $\omega = \omega_0$ to burn a hole in the absorption profile [1 mark]. Subsequently, a weak probe beam resonant with of the same frequency is applied [1 mark]. Because absorption around ω_0 has decreased [1 mark] and both beams are interacting with the same group of atoms [1 mark], there will be an increase in the intensity of the probe beam around this frequency.[1 mark]

(there is some flexibility in the marking depending on the quality of the explanation, if diagrams have been used, etc).