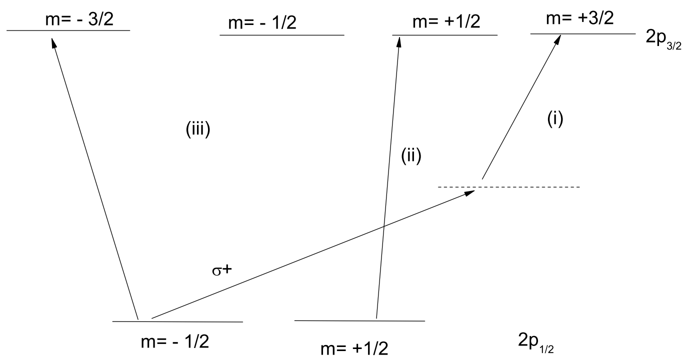
# Coursework 1 - Atom Photon Physics

Deadline: 18th of November 2011

1. (25/100) Consider the electric quadrupole operator $xz$, coupling an arbitrary initial state of principal quantum number $|n_1, l_1, m_1 >$ and a final state $|n_2, l_2, m_2 >$, where $n_i$, $l_i$ and $m_i$ $(i = 1, 2)$ denote the principal, with angular momentum and magnetic quantum numbers, respectively. Derive the electric quadrupole selection rules and find for which values of $\Delta m = m_2 - m_1$ and $\Delta l = l_2 - l_1$ the transition matrix element is non-vanishing.

2. (a) (20/100)Consider the transition $3p_{1/2} -> 4f_{5/2}$. Determine the m sublevels coupled by the components $x^2$, $xz$ and $z^2$ of the electric quadrupole operator. Justify your answer in terms of selection rules.

   (b) (15/100) Consider the three electric dipole transitions below. Are all transitions allowed? Why or why not? What are the missing polarizations in the photons so that all transitions are allowed?Justify your answer based on selection rules.




   (c) (5/100) What are the values for m so that the magnetic dipole transition $2p -> 3p$ in Hydrogen may occur?Neglect spin-orbit coupling. Justify your answer in terms of selection rules.

3. The "oscillator strength" $f_{ao}$ of an atomic transition from a state $|\psi_a\rangle$ to a state $|\psi_0\rangle$ is given by

$$f_{ao} = \frac{2m\omega_{ao}}{\hbar} \left|\langle\psi_0|\, \hat{z}\, |\psi_a\rangle\right|^2 ,$$

where $\omega_{ao}$ is the transition frequency between both levels. Address the following issues related to this concept.

(a) (20/100) Show that the expectation value expectation value of the dipole moment operator $e\hat{z}$ for a linearly polarized field $\mathbf{E}(t) = E_0 \cos(\omega t)\hat{e}_z$ computed using the time-dependent state

$$|\psi(t)\rangle = |\psi_0\rangle + \sum_{n \neq 0} \frac{eE_0\omega_{n0}}{2\hbar\omega} \langle\psi_n| \hat{z} |\psi_0\rangle F(\omega, \omega_{n0}) |\psi_n\rangle ,$$

with

$$F(\omega, \omega_{n0}) = \left\{ \frac{e^{-i\omega_{n0}t} - e^{i\omega t}}{\omega_{n0} + \omega} - \frac{e^{-i\omega_{n0}t} - e^{-i\omega t}}{\omega_{n0} - \omega} \right\},$$

obtained up to first-order perturbation theory in the field and retaining only the terms linear in $E_0$ gives

$$\langle D(z)\rangle (t) = \frac{2e^2}{\hbar} E_0 \cos(\omega t) \sum_n \frac{\omega_{n0} \left|\langle\psi_0| \hat{z} |\psi_n\rangle\right|^2}{\omega_{n0}^2 - \omega^2}.$$

(b) (10/100) Consider now the classical forced harmonic oscillator whose equation of motion is given by

$$m\ddot{z} + m\omega_0^2 z = -eE_0 \cos(\omega t).$$

Compute the explicit expression for the induced dipole $ez$.

(c) (5/100) How do you relate the results obtained in (a) and (b) to the concept of oscillator strength?What do they mean, physically?