V - Laser cooling and trapping of neutral atoms

* **key idea**: Light can be used to manipulate matter, due to the fact that it carries momentum. In fact, it exerts radiation pressure on material particles.

* **Early examples (radiation pressure)**:

- 17th century: Kepler proposed that the repulsion of comet tails from the sun was due to radiation pressure

- 1873: Maxwell showed that an electromagnetic field exerts a pressure $P = \frac{I}{c}$

- 1917: Einstein predicted the existence of the Recoil velocity

  If an atom of mass M absorbs a photon of momentum $\vec{p} = \hbar \vec{k}$, this photon causes the atom to recoil in the direction of the incident light and to change its velocity by $v_R = \frac{\hbar k}{M}$

  Here we assumed that the photon is resonant with the atomic transition $a \to b$.

  The atom also recoils when emitting a photon and returning from the state b to a state a (spontaneous emission of a fluorescence photon)

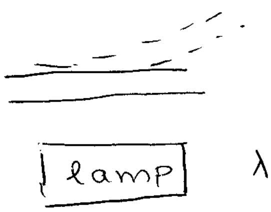
  In this case, however, the direction of the photon, and therefore of the recoil velocity, is random.

$\Rightarrow$ Absorption: V is increased in the direction of the beam

Emission: V is decreased in a random direction

$\Rightarrow$ Net result : an acceleration of the atom in the direction of the laser beam

• 1933 : R. Frisch observed a deflection in a beam of sodium atoms caused by a resonant sodium lamp. Due to the lamp's low intensity, however, only $\frac{1}{3}$ of the sodium atoms were excited



$\lambda = 5890 \mathring{A}$

• 1970s : Narrow band, tunable lasers $\Rightarrow$ higher brightness, directionality, near monochromaticity

$\Rightarrow$ Substantial increase in the radiation - pressure force

$$F = \frac{\Delta p}{\Delta t} = \frac{\hbar k}{\tau_b} = \frac{h\nu}{c\tau_b}$$

Example: Na , resonance transition $3^2S_{1/2} \rightarrow 3^2P_{3/2}$

$\tau = 16\,ns$

Radiation pressure force: $F = 2.2 \times 10^{-19}$ N

For comparison : Gravitational force

$F = mg = 9.81 \times M_{Na} = 3.5 \times 10^{-25}$ N

$$\frac{\frac{h\nu}{c\tau}}{mg} \approx 10^6$$ $\Rightarrow$ the radiation pressure force is around 10^6 times larger than the gravitational force on the surface of the earth.

- Further example of manipulation of matter with light: Optical pumping

  ⇒ useful for confining atoms and explaining certain cooling mechanisms

1950s, Alfred Kastler: One can use the resonant exchange of angular momentum between atoms and polarized photons to align / orient atomic spins, or heat/cool internal degrees of freedom.

1 - Slowing of atomic beams

Questions: how many photons does one need to **stop** a beam of sodium atoms?

Average velocity : 900 m/s
(oven at $T = 600K$)

Recoil velocity (Na resonance line at $\lambda = 5890 \mathring{A}$) ⇒ 3 cm/s

⇒ One needs ~ 30 000 counter-propagating photons

Problem : Zeeman effect

If an atom has a velocity v,

$$\vec{R} = \vec{v}\, t + \vec{R}_0$$

$$e^{i(\vec{k} \cdot \vec{R} - \omega t)} = e^{i\vec{k} \cdot \vec{R}_0}\, e^{-i(\omega - \vec{k} \cdot \vec{v})t}$$

It is as if $\omega \to \omega - \vec{k} \cdot \vec{v}$

As the atom beam is decelerated, $\vec{v}$ changes. This implies that the atom becomes non-resonant and is no longer decelerated

* Solutions:

- Chirping ⇒ the frequency of the laser field is modified to account for the Doppler shift
  (V. S. Letokhov, V. G. Minogin and D. Pavlik, Opt. Comm. 19, 72 (1976))

- Zeeman tunning technique ⇒ the Doppler shift is compensated by the Zeeman shift introduced by an inhomogeneous magnetic field
  (W. D. Philips and H. J. Metcalf, Phys. Rev. Lett. 48, 596 (1982))

$$\omega_0 \to \omega_0 + \gamma B(z)$$

* Examples

Doppler and recoil effects (Quantum mechanical discussion).

Let us consider the Hamiltonian

$$H = \underbrace{\sum_{i=1}^{n} \frac{p_i^2}{2m} + V(r_1, r_2, \ldots, r_n) + W_{rel}}_{\text{internal degrees of freedom (atom)}} + \underbrace{\frac{P^2}{2M}}_{\text{Center of mass motion}} + \underbrace{H_{int}(t, \mathbf{R})}_{\text{interaction Hamiltonian}}$$

Initial energy

$$E_i = \underbrace{\frac{\hbar^2 k_i^2}{2M}}_{\substack{\text{kinetic energy} \\ \text{(center of mass)}}} + \underbrace{E_{\alpha i}}_{\text{internal energy}}$$

Eigenstates

⇒ $|k_i, \alpha_i\rangle$ ⇒ center of mass with momentum k_i; atom in the state α_i

Final energy

$$E_f = \frac{\hbar^2 k_f^2}{2M} + E_{\alpha f}$$

$|k_f, \alpha_f\rangle \Rightarrow$ center of mass with momentum k_f, atom in the state α_f

The transition amplitude is proportional to

$$\langle \varphi_f | H_{Int} | \varphi_i \rangle = \langle k_f | e^{i\vec{k}\cdot\vec{R}} | k_i \rangle \underbrace{\langle \alpha_f | H_{int}(t) | \alpha_i \rangle}_{\text{time dependent term}}$$

(see Part I of the course for details)

$$\langle k_F | e^{i\vec{k}\cdot\vec{R}} | k_i \rangle = \int \underbrace{\langle \vec{k}_F | \vec{R} \rangle}_{\propto e^{-i\vec{k}_F\cdot\vec{R}}} e^{i\vec{k}\cdot\vec{R}} \underbrace{\langle \vec{R} | \vec{k}_i \rangle}_{\propto e^{i\vec{k}_i\cdot\vec{R}}} d^3R$$

$$\int e^{i(\vec{k}_i - \vec{k} - \vec{k}_F)\cdot\vec{R}} d^3R = \delta(\vec{k}_i + \vec{k} - \vec{k}_F) \quad \text{Conservation of momentum}$$

$$\vec{k}_i \pm \vec{k} = \vec{k}_F$$

$\oplus \Rightarrow$ absorption

$\ominus \Rightarrow$ emission

$\vec{k}_i \Rightarrow$ initial center-of-mass momentum

$\vec{k}_f \Rightarrow$ final center-of-mass momentum

i) Conservation of energy:

$$E_f - E_i = \pm \hbar\omega \qquad \begin{array}{l} + \text{ absorption} \\ - \text{ emission} \end{array}$$

$$\frac{\hbar^2}{2M}\underbrace{(k_F^2 - k_i^2)}_{(\vec{k}_f - \vec{k}_i)\cdot(\vec{k}_f + \vec{k}_i)} + E_{\alpha F} - E_{\alpha i} = \pm\hbar\omega$$

ii) Conservation of momentum:

$$\vec{k}_f - \vec{k}_i = \pm \vec{k}$$

From this expression, $\vec{k}_f + \vec{k}_i = \pm\vec{k} + 2\vec{k}_i$

Inserting in i)

$$\pm\hbar\omega = \pm\frac{\hbar^2 \vec{k}}{2M}\cdot(2\vec{k}_i \pm \vec{k}) \pm \hbar\, \omega_{Fi}$$

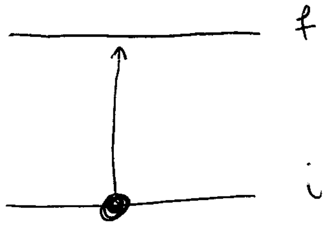
$\frac{E_{\alpha f} - E_{\alpha i}}{\hbar}$

since $\frac{\hbar \vec{k}_i}{M} = \vec{v}_i$

$$\boxed{\omega = \vec{k}\cdot\vec{v}_i \pm \frac{\hbar k^2}{2m} \pm \omega_{FI}}$$

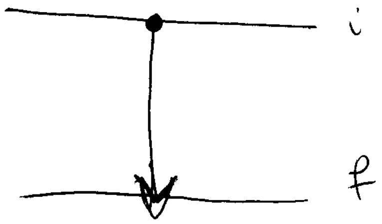
\+ absorption
\- emission

Absorption



$$\omega = \underbrace{\omega_{FI}}_{\text{transition freq.}} + \overbrace{\vec{k}\cdot\vec{v}_i}^{\text{Doppler term}} + \overbrace{\frac{\hbar k^2}{2M}}^{\text{Recoil effect}}$$

Emission



$$\omega = \omega_{iF} + \overbrace{\vec{k}\cdot\vec{v}_i}^{\text{Doppler term}} \overbrace{- \frac{\hbar k^2}{2M}}^{\text{recoil effect}}$$

Orders of magnitude

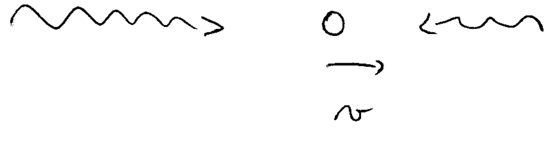
Doppler term: $\frac{\omega v}{c}$ $\quad \frac{10^3}{3\times 10^8} \sim 10^{-6}$

Recoil effect: $\frac{\hbar^2\omega^2}{2Mc^2} = \omega\frac{\hbar\omega}{2Mc^2} \sim 10^{-10}$ (very small)

2.- Doppler Cooling

- T. W. Hänsch and A. L. Schawlow, Opt. Comm. 13, 68 (1975)
- D. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. 20, 637 (1975)
(ions)

Key idea: If an atom is between two laser beams with frequency $\omega < \omega_0$, then it will absorb more photons from the counter-propagating beam

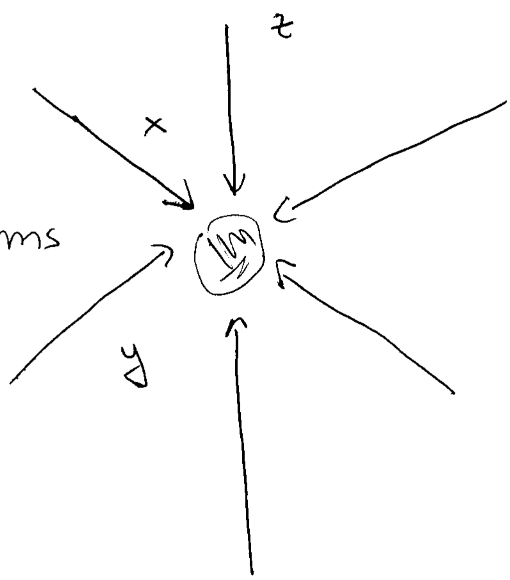


$$\omega = \omega_0 + \vec{k}\cdot\vec{v} \begin{cases} kv \text{ (co-propagating beam)} \\ -kv \text{ (counter-propagating beam)} \end{cases}$$

(*) Optical molasses

(S. Chu, J. E. Bjorkholm, A. Cable and A. Ashkin, Phys. Rev. Lett. 55, 48 (1985)) $\rightarrow 10^5$ Na atoms were cooled to $T \sim 240\,\mu K$

3 sets of counter-propagating laser beams with ~~red~~ detuned frequency $\omega_L < \omega_0$



$\sim 10^{10}$ atoms

- The atoms will absorb more photons from the counter-propagating beam
- For small velocities

$$F = -\alpha v$$

(A damping force)

$\Rightarrow$ The atoms move in a "viscous" medium created by the laser fields (optical molasses)

This can be seen if we consider the average radiation pressure force in a two-level atom, given by

$$\vec{F} = \frac{\hbar \Gamma \Omega^2 \vec{k}}{\Gamma^2 + 2\Omega^2 + 4[\Delta - \vec{k}\cdot\vec{v}]^2}$$

(R. J. Cook, Phys Rev A 20, 224 (1979))

$\Delta \equiv \omega_L - \omega_0$ (detuning)

$\Omega \equiv$ Rabi frequency $= \frac{-eE_0\langle e|r|g\rangle}{\hbar}$ (excited state, gd state)

$\Gamma \equiv$ linewidth of the transition in question

$\vec{v} \equiv$ atom velocity

$\vec{k} \equiv$ wave vector of the laser beam

$$F_x = F_{1x} + F_{2x} = \hbar \Gamma \Omega^2 k \left[\frac{1}{\Gamma^2 + 2\Omega^2 + 4(\Delta - k v_x)^2} - \frac{1}{\Gamma^2 + 2\Omega^2 + 4(\Delta + k v_x)^2} \right]$$

Similar forces along the y and z axis

Expanding F_x in series,

$$F_x = \frac{\hbar \Gamma \Omega^2 k^2 \, 16 \Delta v_x}{\left[\Gamma^2 + 4\Delta^2 + 2\Omega^2\right]^2} + O(v^3) = -\beta v_x$$

negative as Δ is negative

$S_0 = \dfrac{2\Omega^2}{\Gamma^2}$ is known as the saturation term.

If the driving-field intensity is low, $S_0 \ll 1$
For high intensities, it causes F to saturate.

⊛ The Doppler cooling limit

Ideally, all atoms would cool to $v = 0 \rightarrow T = 0\,K$. However, the light beam also causes heating. This occurs due to the discrete steps in momentum experienced by the atom in absorption/re-emission ⇒ this is a random walk process.

The kinetic energy of the atom will change by

$$E_r = \frac{\hbar^2 k^2}{2m} = \hbar \omega_R \quad \text{at each absorption/emission}$$

Freq. associated with recoil

The average frequency:

i) at each absorption: $\omega_{ab} = \omega_0 + \omega_R$
ii) at each emission: $\omega_{ab} = \omega_0 - \omega_R$

⇒ the light field loses $2\hbar\omega_R$ of energy in each absorption/emission cycle.

$\Rightarrow$ The atoms heat up by $2\hbar\omega_r$ at each cycle.

- Steady-state situation : cooling rate = heating rate

cooling rate : $F v = \beta v^2$

Heating rate: $2\hbar\omega_R\Gamma \cdot 2$

to account for absorption and emission

$$\boxed{\beta v^2 = 4\hbar\omega_R\Gamma}$$

It is possible to show that the kinetic energy of the atom depends on the detuning and that it will have a minimum for $\Delta = -\frac{1}{2}\gamma$

$\Rightarrow T_0 = \dfrac{\hbar\Gamma}{2k_b}$ ($k_b \equiv$ Boltzmann const.) Na: $T_0 \approx 240\mu K$

T_0 is known as the Doppler cooling limit

Details: D. Wineland and W. Itano, Phys Rev A 20, 1521(1979); V. S. Letokhov and V. G. Minogin, Phys Rev A 73, 1(1981)

3 - Trapping of Atoms - the Magneto-Optical Trap

- In the optical molasses scheme, one cannot say that the atoms are trapped, as there is no potential minimum confining the atoms.
- One can build a Magneto-Optical trap by inserting an inhomogeneous magnetic field in the previous scheme

Proposed: D. E. Pritchard, E. L. Raab, V. Bagnato, C. E. Wieman, R. N. Watts, PRL 57, 310 (1986)

Built: E. L. Raab, M. Prentiss, A. Cable, S. Chu and D. E. Pritchard, Phys Rev Lett. 59, 2631 (1987)

Principle

Two-level atom:

Ground state: $J = 0$ ($m_J = 0$)

Excited state: $J = 1$ ($m_J = -1, 0, 1$)

\+ Inhomogeneous magnetic field $B(z) = b\,z$

$\Rightarrow$ $m_J = 1$ and $m_J = -1$ sublevels experience zeeman shifts which are linear in z.

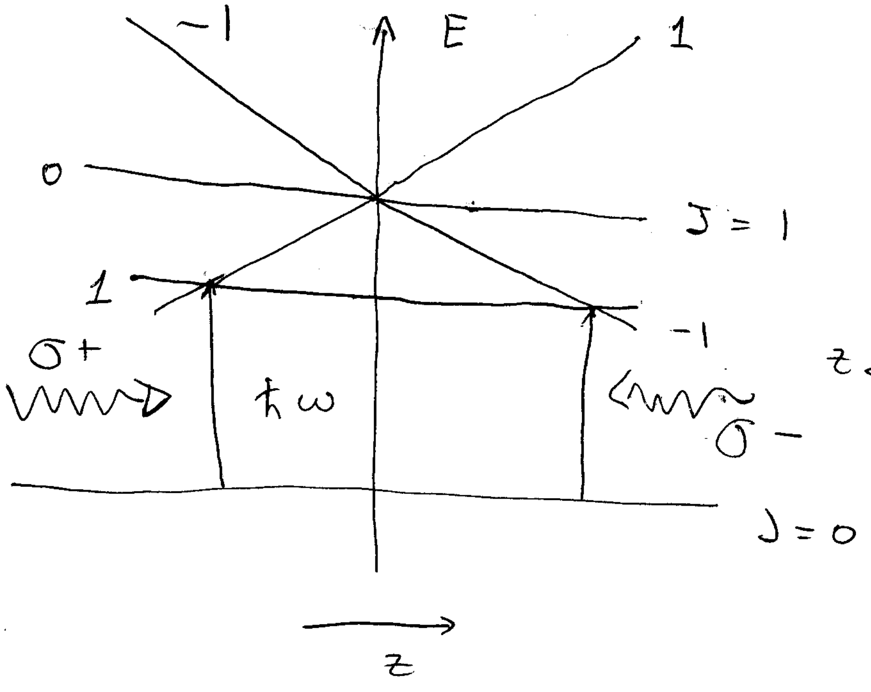
$$\Delta E = \mu\, m_J\, B(z)$$

Let us now consider two red detuned circularly polarized fields:

σ^- propagating along $-\hat{z}$

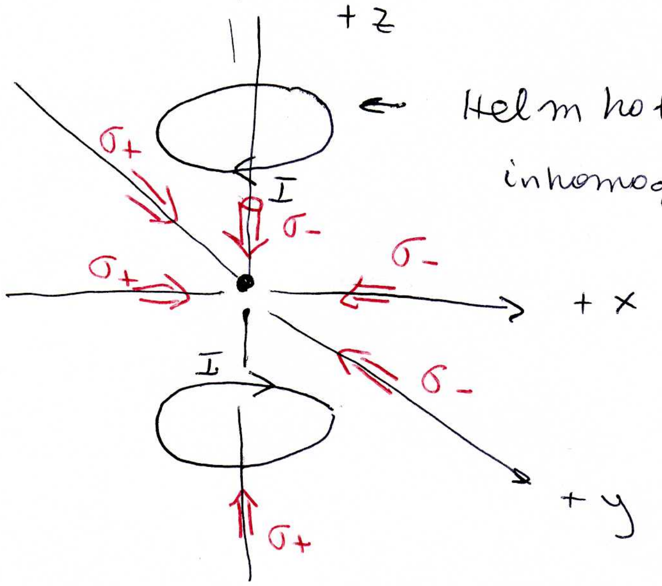
σ^+ propagating along $+\hat{z}$

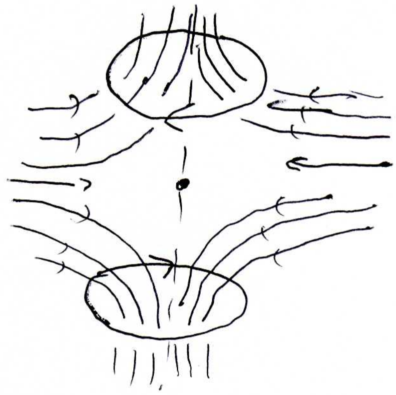
The atom will absorb more photons from the counter-propagating beam



$z > 0$: The atom will absorb more photons from the σ^- beam

$z < 0$: the atom will absorb more photons from the σ^+ beam





4 - Sisyphus cooling

(J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B 6, 2023 (1989); see C. Cohen-Tannoudji and W. D. Phillips, Physics Today, Oct. 1990 for a more accessible discussion).

(*) Problems with Doppler cooling / two level picture:

- The confinement times in an optical molasses were optimized for much larger detunings than those predicted by the theory
- Sodium atoms could be cooled far below the Doppler limit ($T \sim 40\,\mu K$ instead of $T \sim 240\,\mu K$)

(*) Key issues:

- The two-level picture breaks down: Alkali atoms have several Zeeman sublevels in the gd state
  ⇒ Optical pumping can transfer atoms from g_m to $g_{m'}$
- The optical interaction induces energy shifts in g (light shifts).

$\Rightarrow$ The light shifts depend on the laser polarization, strength and vary for each Zeeman sublevel

- Within the molasses, there exist polarization gradients: The population of the sublevels and the light shifts depend on the position of the atom in the laser wave

(*) Examples - polarization gradients

Let us consider two plane waves propagating along the Oz axis.

$\mathcal{E}_0, \mathcal{E}_0' \equiv$ amplitudes (real)

$\hat{\epsilon}, \hat{\epsilon}' \equiv$ polarizations

$\omega_L \equiv$ frequency (same for both)

- Total electric field : $\vec{E}(z,t) = \mathcal{E}^+(z)\exp(-i\omega_L t) + c.c.$

$$\mathcal{E}^+(z) = \mathcal{E}_0\hat{\epsilon}\, e^{ikz} + \mathcal{E}_0'\hat{\epsilon}'\, e^{-ikz}$$

(a) - The $\sigma^+ - \sigma^-$ configuration

$$\hat{\epsilon} = \hat{\epsilon}_+ = \frac{-1}{\sqrt{2}}(\hat{\epsilon}_x + i\hat{\epsilon}_y)$$

$$\hat{\epsilon}' = \hat{\epsilon}_- = \frac{1}{\sqrt{2}}(\hat{\epsilon}_x - i\hat{\epsilon}_y)$$

$$\mathcal{E}^+(z) = \mathcal{E}_0\left(\frac{-1}{\sqrt{2}}(\hat{\epsilon}_x + i\hat{\epsilon}_y)\right)e^{ikz} + \mathcal{E}_0'\left(\frac{1}{\sqrt{2}}(\hat{\epsilon}_x - i\hat{\epsilon}_y)\right)e^{-ikz}$$

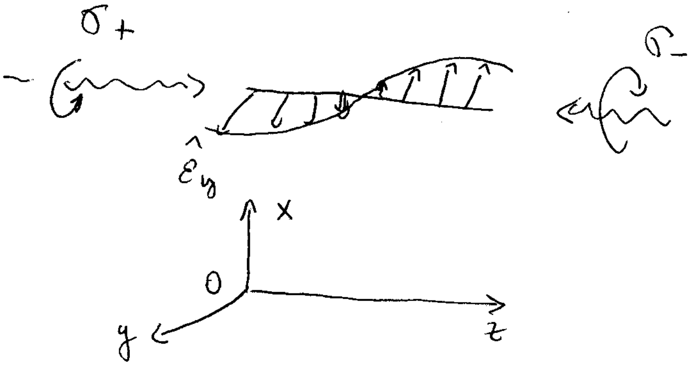
$$\Rightarrow \mathcal{E}^+(z) = \frac{1}{\sqrt{2}}(\mathcal{E}_0' - \mathcal{E}_0)\hat{\epsilon}_X - \frac{i}{\sqrt{2}}(\mathcal{E}_0' + \mathcal{E}_0)\hat{\epsilon}_Y$$

$$\hat{\epsilon}_X = \hat{\epsilon}_x\cos(kz) - \hat{\epsilon}_y\sin(kz)$$

$$\hat{\epsilon}_Y = \hat{\epsilon}_x\sin(kz) + \hat{\epsilon}_y\cos(kz)$$

X, Y are orthogonal and are deduced from x, y by a rotation around Oz ($\varphi = -kz$)

$\mathcal{E}^+(z)$ is elliptically polarized and rotates around Oz



$\mathcal{E}_0 = \mathcal{E}_0'$

$\rightarrow$ $\mathcal{E}^+(z)$ is linearly polarized along ϵ_y

(Helix with a pitch λ)

(b) The lin $\perp$ lin configuration

$\hat{\epsilon} = \hat{\epsilon}_x$; $\hat{\epsilon}' = \hat{\epsilon}_y$ (two counter-rotating waves of orthogonal linear polarization)

$$\mathcal{E}^+(z) = \mathcal{E}_0\sqrt{2}\left(\cos kz\,\frac{(\hat{\epsilon}_x+\hat{\epsilon}_y)}{\sqrt{2}} - i\sin kz\,\frac{(\hat{\epsilon}_y-\hat{\epsilon}_x)}{2}\right)$$

The ellipticity changes when one moves along Oz

$z = 0$: linear polariz. along $\frac{\hat{\epsilon}_x+\hat{\epsilon}_y}{\sqrt{2}}$ ($\cos kz = 1$; $\sin kz = 0$)

$z = \lambda/8$: $$\mathcal{E}^+(z) = \mathcal{E}_0\sqrt{2}\left(\underbrace{\cos\left(\frac{\pi}{4}\right)}_{\frac{1}{\sqrt{2}}}\frac{(\hat{\epsilon}_x+\hat{\epsilon}_y)}{\sqrt{2}} - i\overbrace{\sin\left(\frac{\pi}{4}\right)}^{1/\sqrt{2}}\frac{(\hat{\epsilon}_y-\hat{\epsilon}_x)}{\sqrt{2}}\right)$$

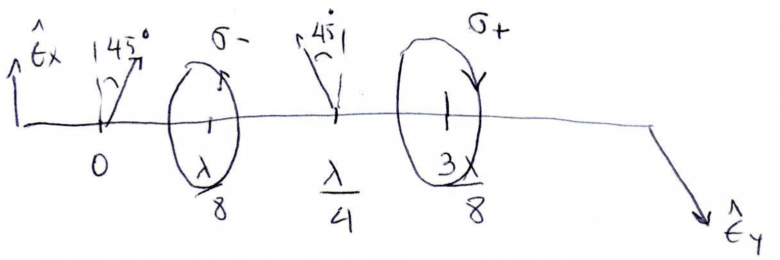
$$\mathcal{E}^+(z) = \mathcal{E}_0\left(\frac{(\hat{\epsilon}_x+\hat{\epsilon}_y)}{\sqrt{2}} - i\frac{(\hat{\epsilon}_y-\hat{\epsilon}_x)}{\sqrt{2}}\right) \Rightarrow \sigma^- \text{ polarization}$$

$z = \frac{\lambda}{4}$: $\mathcal{E}^+(z) = -i\mathcal{E}_0\cancel{\sqrt{2}}\frac{(\hat{\epsilon}_y-\hat{\epsilon}_x)}{\cancel{\sqrt{2}}}$ linearly polarized along $\hat{\epsilon}_x - \hat{\epsilon}_y$

$$z = \frac{3\lambda}{8} \Rightarrow \mathcal{E}^+(z) = \mathcal{E}_0\sqrt{2}\left[\overbrace{\cos\left(\frac{3\pi}{4}\right)}^{-1/\sqrt{2}}\frac{(\hat{\epsilon}_x+\hat{\epsilon}_y)}{\sqrt{2}} - i\overbrace{\sin\left(\frac{3\pi}{4}\right)}^{1/\sqrt{2}}\frac{(\hat{\epsilon}_y-\hat{\epsilon}_x)}{\sqrt{2}}\right]$$

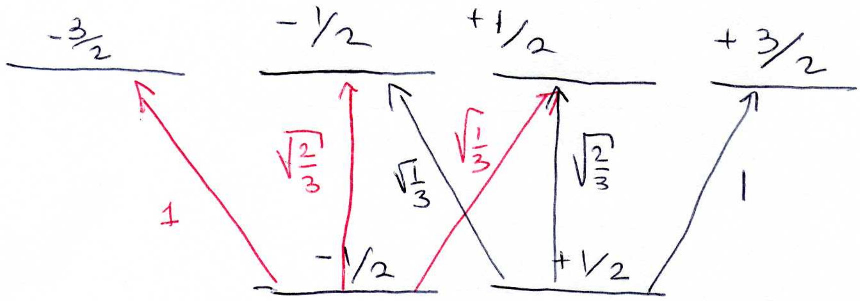
$$= -\mathcal{E}_0\left(\frac{\hat{\epsilon}_x+\hat{\epsilon}_y}{\sqrt{2}} + i\frac{(\hat{\epsilon}_y-\hat{\epsilon}_x)}{\sqrt{2}}\right) \Rightarrow \sigma^+ \text{ polarization}$$

$z = \lambda/2 \Rightarrow$ linear along $-\hat{\epsilon}_1 = -\frac{(\hat{\epsilon}_x+\hat{\epsilon}_y)}{\sqrt{2}}$



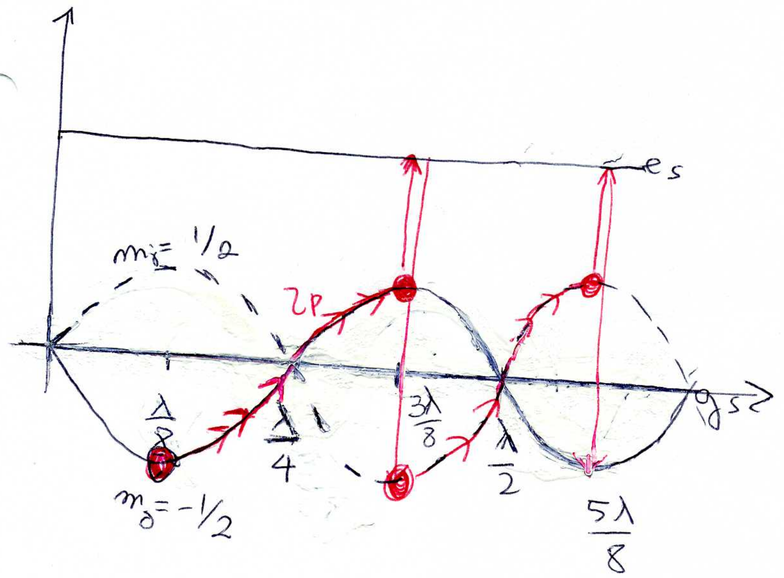
Let us now consider an atom whose gd state has $J = 1/2$ ($m_J = \pm 1/2$) and an excited state with $J = \frac{3}{2}$ ($m_J = \pm 3/2, \pm 1/2$)

- Further assumptions:
  - light shifts occur for the gd state
  - light shifts can be neglected for the excited state



Numbers = Clebsch Gordan coefficients

light shifts: depend on the sublevel and on the field polarization



- A moving atom "climbs a hill" in a time τ_p. We assume $v \tau_p \sim \lambda$