assumption

$\boxed{1}$ We will first write $\sin^2\theta = 1 - \cos^2\theta$ in terms of spherical harmonics.
This gives

$\boxed{2}$
$$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{3}\left[1 + 4\sqrt{\frac{\pi}{5}}\, Y_2^0(\theta,\varphi)\right]$$
$$= C + D\, Y_2^0(\theta,\varphi)$$

Note that

$\boxed{1}$
$$Y_0^0 = \frac{1}{\sqrt{4\pi}} = \text{const.}$$

$\boxed{1}$ This implies that $\sin^2\theta$ can be written as a linear combination of Y_0^0, Y_2^0

$\boxed{1}$
$$\Rightarrow I_3 = \underbrace{D\int Y_m^{*\ell}(\theta,\varphi)\, Y_2^0(\theta,\varphi)\, Y_0^0(\theta,\varphi)\, d\Omega}_{(*)}$$
$$\underbrace{+ \sqrt{4\pi}\, C \int Y_m^{*\ell}(\theta,\varphi)\, Y_0^0(\theta,\varphi)\, Y_0^0(\theta,\varphi)\, d\Omega}_{(**)}$$

$$(*) = \frac{1}{\sqrt{4\pi}} \int Y_\ell^{*m}(\theta,\varphi)\, Y_2^0(\theta,\varphi)\, d\Omega = \frac{1}{\sqrt{4\pi}}\, \delta_{m,0}\, \delta_{\ell,2}$$

$\boxed{2}$
$$m = 0 \quad \Rightarrow \quad \Delta m = 0$$
$$\ell = 2 \qquad\quad \Delta \ell = +2$$

$$(**) = \frac{1}{\sqrt{4\pi}} \int Y_\ell^{*m}(\theta,\varphi)\, Y_0^0(\theta,\varphi)\, d\Omega = \frac{1}{\sqrt{4\pi}}\, \delta_{m,0}\, \delta_{\ell,0}$$

$\boxed{2}$
$$\ell = 0 \qquad \Delta m = 0$$
$$m = 0 \qquad \Delta \ell = 0 \Rightarrow \text{final state is also an s st.}$$

Total: 20