Special Relativity, Lecture 2: Main consequences of Einstein's postulates from simple thought experiments

Topics: Constancy of speed of light; Clock synchronization; apparent paradoxes; relative simultaneity; time dilation; length contraction

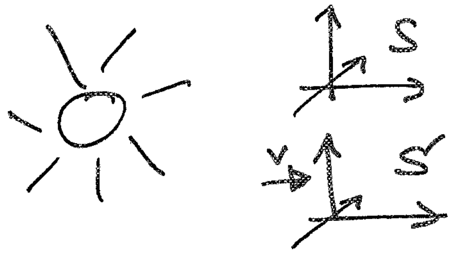
2.1. Constancy of speed of light

(E1) The laws of physics are the same in every initial frame of reference (E2) The speed of light is independent of the motion of its source

One immediate consequence of Einstein's postulates:

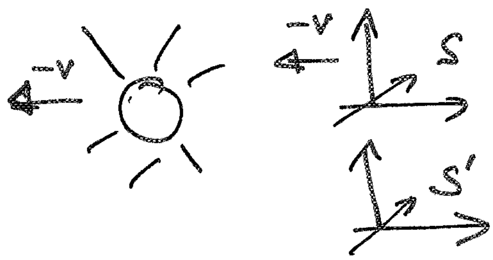
(E2)' The speed of light is the same in all frames of reference.

Proof: Consider two refrence frames, S stationary to light source, S' moving with relative velocity v.



- in S speed of light is c by postulate (E2)

What about S'? By postulate (E1) the above situation is equivalent to



- Now light source is moving in S'
- Because of (E2) we know that speed of light in S' is c.

Contrary to the intuition, an observer in S' does not measure $c-v$ as the speed of light!

As a young boy, Einstein apparently asked himself the question:

"What happens if I catch up with a light wave?"

- Using the Newtonian relativity principle, the runner would see speed of light $c-v$
- For $c-v \to 0$ the wave would appear to stand still
  $\Rightarrow$ Static fields, but static $\vec{E}$ field does not induce $\vec{B}$ field and vice versa $\Rightarrow$ no wave $\lightning$
- (E1) tells us that the physics cannot change as we move
- (E2)' tells us that the runner always sees wave with velocity c, irrespective of his own speed.

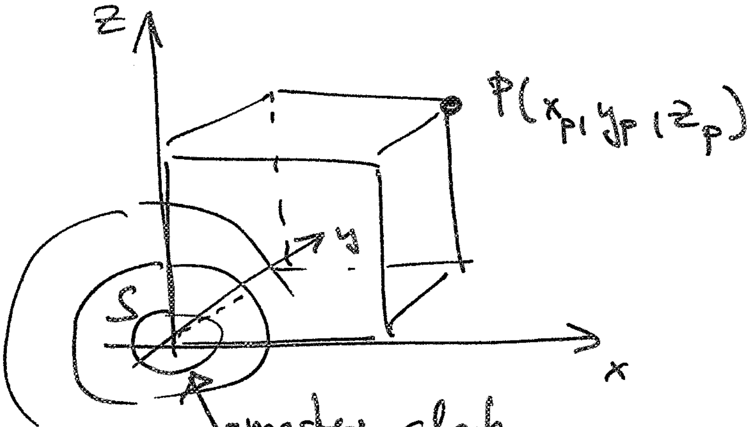
$\Rightarrow$ Can't ever catch a light wave

2.2. Clock synchronization

- To measure the time at which an event occured at a particular point in space we assumed that all of space was filled with clocks, one for each point in space
- Separate set of clocks and rulers for each reference frame (will be important in special relativity)
- We assumed that all the clocks in each reference frame were synchronized (neccessary to campare times of two events happening at two different points in space

Q: How can we synchronize clocks?

A: Use the fact that speed of light is precisely known and the same in every frame of reference



- master clock
- spherical flash of light emitted at $t_0 = 0$ indicated by this clock

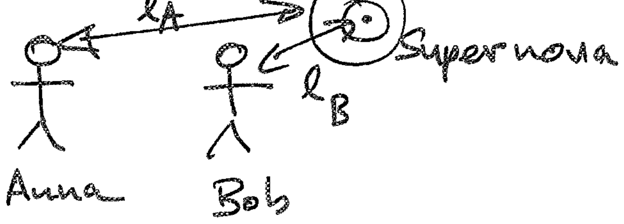
- Light wave reaches P at time $t_p = \frac{d_p}{c}$ with $d_p = \sqrt{x^2+y^2+z^2}$ the distance of P from the origin
- Precisely at this moment we set the clock in P to read t_p
- We do this ~~fall~~ for all points in space

$$t = \frac{\sqrt{x^2+y^2+z^2}}{c} \Rightarrow \boxed{x^2+y^2+z^2 = c^2t^2}$$

- connection between space an time
- we will make use of this result later when we finally derive the Lorentz transformation

2.3. Seeing and Measuring

Everyone in the same inertial frame of reference will agree on results. They may <u>see</u> things differently, but a little calculation at the end will reduce to the same



Bob <u>sees</u> supernova first since distance is shorter

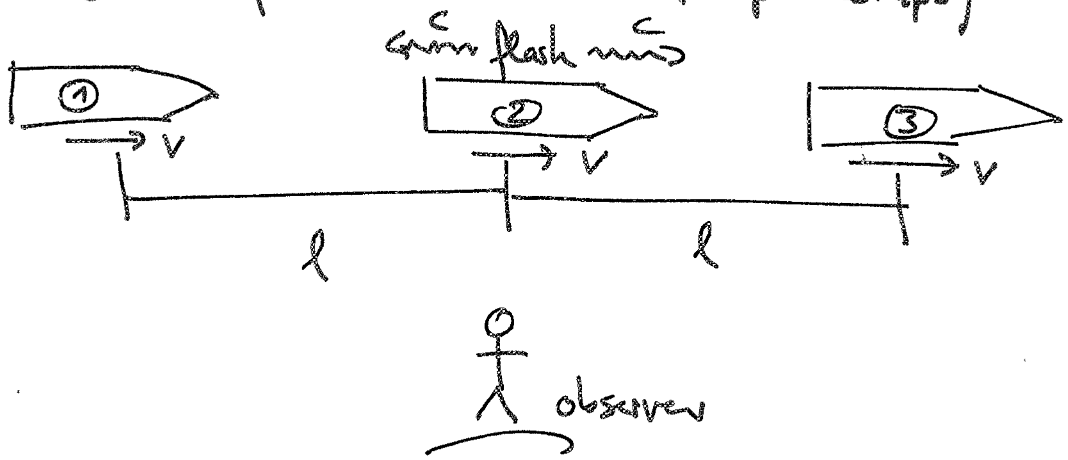
<u>But</u>:
- Synchronize watches, record times t_A and t_B when they see the supernova, and know distances l_A and l_B

- using $t_A - t_B = \frac{l_A - l_B}{c}$, Anna and Bob conclude that flash happened at the same time

They <u>measure</u> the supernova to happen at the same time

2.4. Apparent paradoxes

Repeat the sound experiment (whistle on a train) with light (flash on a train of spaceships)



<u>In the spaceship frame</u>:

light from 2 reaches 1 and 3 at the <u>same</u> time

$$t_1 = t_3 = \frac{l}{c}$$

<u>in the observer frame</u>:

- 1 and light front approach at $v+c$ (relative speeds can be observed to be $> c$)
- wavefront catches up 3 at ~~$v-c$~~ $c-v$

$\Rightarrow$ light reaches 1 before 3

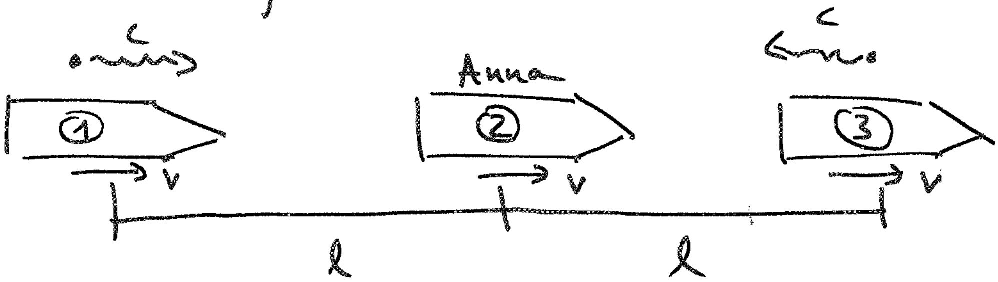
- How can 1 and 3 think it takes the same time and observer ~~thi~~ thinks it takes a different time?
- Most of Special Relativity is about learning to resolve such apparent paradoxes

2.5. Relative Simultaneity

> Two events at different locations that happen at the same time in one frame of reference will not be simultaneous in a frame of reference moving relative to the first.

<u>Proof</u>: (by contradiction)

setup:




Bob the observer

- 1, 2, and 3 synchronize watches in advance
- 1 and 3 <u>simultaneously</u> flash torches towards Anna (at $t=0$)

(E2) $\Rightarrow$ Anna sees both flashes arrive at $t = \frac{l}{c}$

Note: the coincidental arrival of the two beams is a <u>single</u> event

<u>Assumption</u>: Bob measures both beams to leave simultaneously

Bob sees the beam from 1 approach Anna with relative velocity $c-v$, the beam from 3 with $c+v$

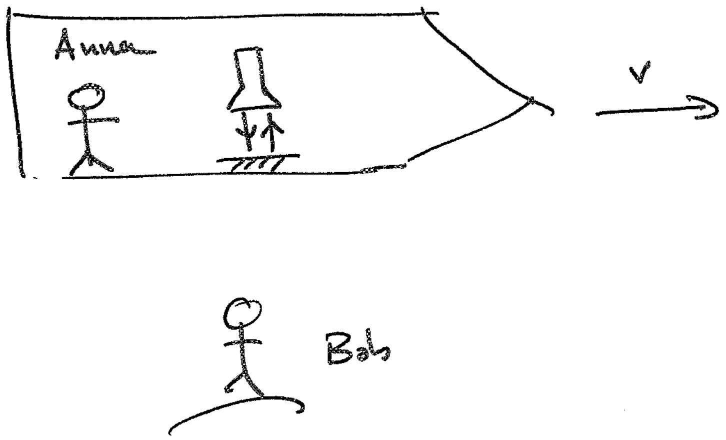
$\Rightarrow$ Bob measures the beam from 3 to arrive <u>before</u> the beam from 1

↯ contradiction!   A <u>unique event</u> is <u>unique in all frames</u>!

$\Rightarrow$ the assumption is wrong

$\Rightarrow$ B sees the beam from 1 leave before the beam from 3

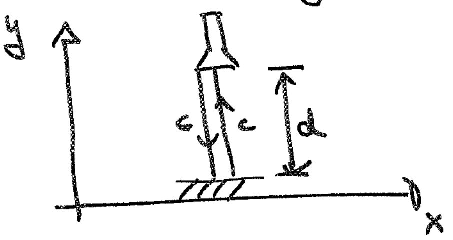
2.6. Time Dilation



Anna flashes a torch at a mirror on the floor and measures the time for the reflection to return

Anna (frame S)

(because of (E1) Anna's experiment is the same as if the spaceship was not moving)

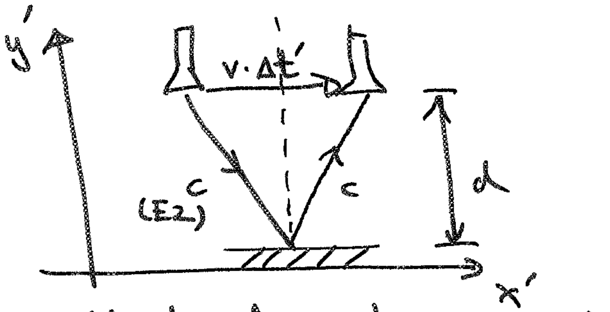


light travels distance $2d$

$$\Delta t = \frac{2d}{c}$$

Bob (frame S')

Say Bob measures the reflection to take $\Delta t'$



light travels distance $2d'$

$$\Delta t' = \frac{2d'}{c} = \frac{2}{c}\sqrt{d^2 + \left(\frac{v\Delta t'}{2}\right)^2}$$

$$\Rightarrow (\Delta t')^2 = \frac{4}{c^2}\left(d^2 + \frac{v^2(\Delta t')^2}{4}\right)$$

$$= (\Delta t)^2 + \frac{v^2}{c^2}(\Delta t')^2$$

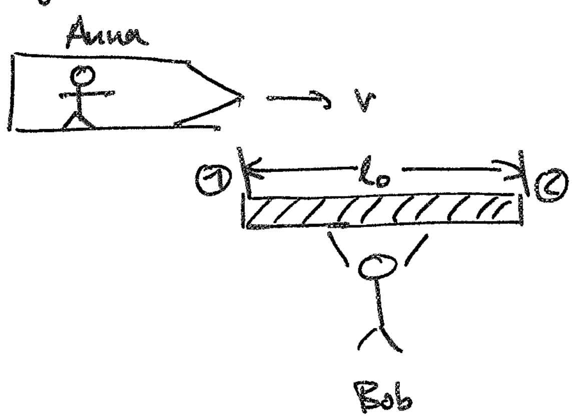
$$\Leftrightarrow \left(1 - \frac{v^2}{c^2}\right)(\Delta t')^2 = (\Delta t)^2 \Rightarrow \boxed{\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} < 1 \Rightarrow \Delta t' > \Delta t \quad \text{(time dilation)}$$

Time dilation: The smallest time Δt between two events is measured in the inertial frame in which they are at the same location. All other observers moving with relative velocity v measure a longer time interval

$$\Delta t' = \Delta t \Big/ \sqrt{1 - \frac{v^2}{c^2}} > \Delta t .$$

2.7 Length contraction

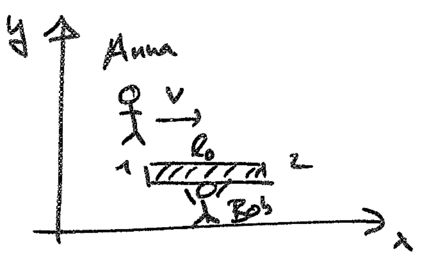


Bob knows velocity v (don't ask how)

They will use the following events to determine the length of the rod

① Anna passes end 1
② Anna passes end 2

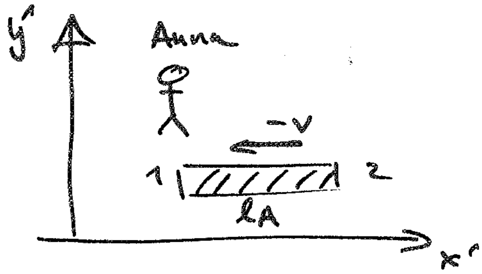
Bob (frame S):



time for Anna to travel length l_0:

$$t_{12} = t_2 - t_1 = \frac{l_0}{v}$$

Anna (frame S'):



time between 1 and end 2 passing

$$t'_{12} = \frac{l_A}{v}$$

From section 2.6. we know that

$$t_{12} = t'_{12} / \sqrt{1 - \frac{v^2}{c^2}}$$

Q: Why this way?
A: in frame S' (Anna's rest frame) the two events happen at the same location

$$\Rightarrow \quad \frac{l_0}{v} = \frac{l_A}{v} \Big/ \sqrt{1 - \frac{v^2}{c^2}} \quad \Rightarrow \quad \boxed{l_A = l_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

$l_A < l_0$ (length contraction)

Length Contraction: An object will be measured to have its greatest length l_0 (or two events will have their greatest separation l_0) when it is observed in its rest frame. All other observers moving with relative velocity v measure that the object is contracted to the length

$$l_v = l_0 \sqrt{1 - \frac{v^2}{c^2}} < l_0$$

Proper length l_0: The length of an object at rest is defined as its rest length or proper length.

2.8. Crucial differences with the Galilean transformation

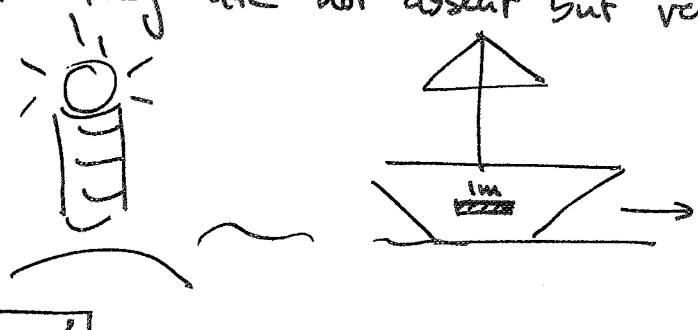
- SR is about transforming coordinates between inertial frames of reference
  (Note: In one reference frame, two observers always measure the same thing even though they might see things differently)

Einsteins postulates:		Main Consequences:
1) Same as Galilei's relativity principle 2) Speed of light is the same in all inertial frames of reference	$\Rightarrow$	1) Relative Simultaneity 2) Time dilation 3) Length contraction

- The consequences are in contradiction with the Galilean transformation under which time and spatial differences remain the same

- New transformation: Lorentz transformation (next lecture)

- Q: In the situation of Galilei's ship, why can we use the Galilean transformation and forget about relativistic effects like length contraction?
  A: They are not absent but very tiny!



$$\left.\begin{aligned} v &= 50\,\frac{km}{h} \simeq 0{,}014\,\frac{km}{s} \\ c &\simeq 300.000\,\frac{km}{s} \end{aligned}\right\} v \ll c$$

$\sqrt{1-\frac{v^2}{c^2}} \simeq 1-10^{-15}$ $\Rightarrow$ For $l_0 = 1m$ the lengt contraction is

$\Delta l = l_0 - l_v = 10^{-15} m$ ($\simeq$ size of atomic nucleus)